

# **Authentication Tests and the Structure of Bundles**

**Joshua D. Guttman  
F. Javier Thayer**

**September 2000**

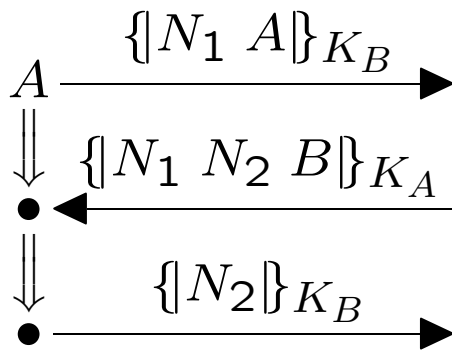
**MITRE**

# Today's Lecture

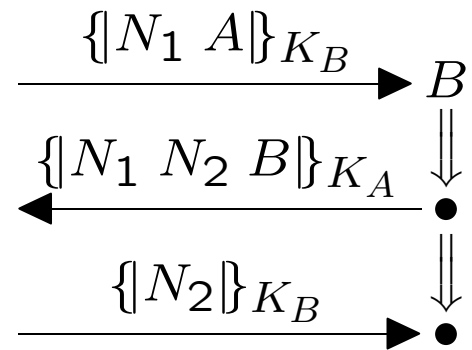
- Authentication Tests:
  - How to find out what a protocol achieves
  - How to prove it achieves that
  - Methods to establish
    - Secrecy (especially of keys)
    - Authentication
- Justifying authentication tests
  - Equivalence of bundles
  - Graph operations to simplify bundles
    - Well-behaved bundles
  - Paths through bundles
  - Transforming edges and pedigrees
  - The secrecy theorem
  - Authentication test theorems

**MITRE**

# Needham-Schroeder-Lowe Protocol



NSLInit[ $A, B, N_1, N_2$ ]



NSLResp[ $A, B, N_1, N_2$ ]

**MITRE**

# Secrecy of Keys in Needham-Schroeder-Lowe

- Some keys  $K_{\mathcal{P}}$  are known initially to penetrator
  - Any public key
  - Private keys of malicious principals
  - Private keys carelessly disclosed or stored in compromised devices
- No keys are disclosed in protocol

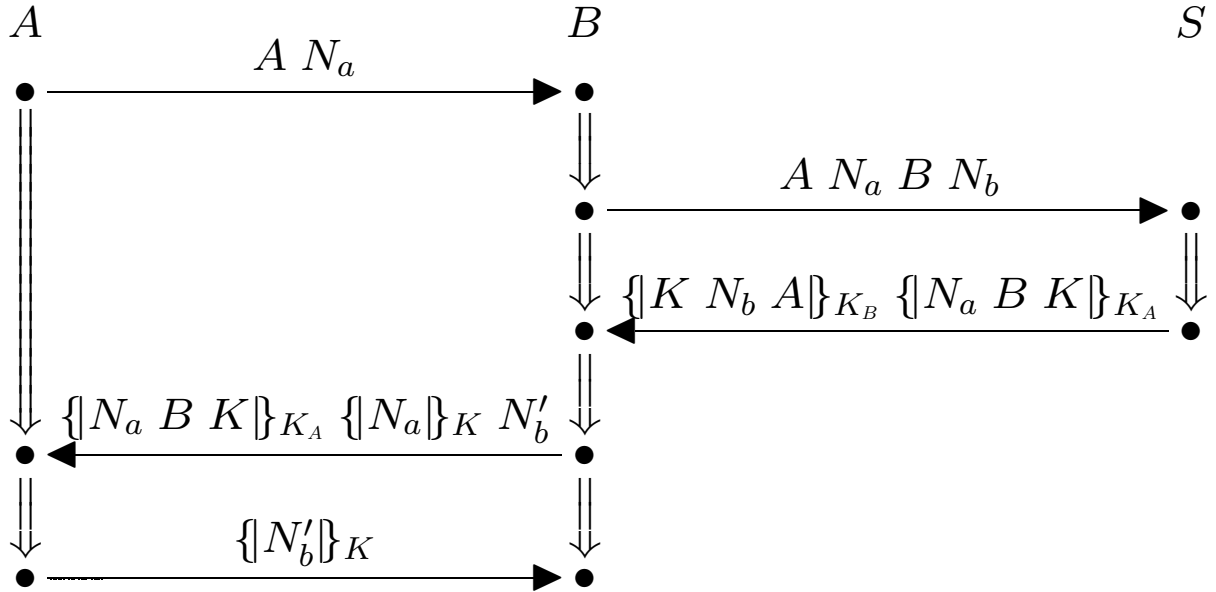
$$K \sqsubset t$$

implies  $t \neq \text{term}(n)$   
for any regular node  $n$

- What the penetrator does not know, he cannot learn
  - Any key not in  $K_{\mathcal{P}}$
  - All other keys immediately safe, called  $S_0$

**MITRE**

# Carlsen Protocol



$S$ 's behavior:

$\text{CServ}[A, B, N_a, N_b, K]$

$A$ 's behavior:

$\text{CInit}[A, B, N_a, K, N'_b]$

$B$ 's behavior:

$\text{CResp}[A, B, N_a, N_b, K, N'_b, H]$

with trace

$$\begin{aligned}
 & -A \ N_a \quad + \ A \ N_a \ B \ N_b \quad - \ \{K \ N_b \ A\}_{K_B} \ H \\
 & \quad + \ H \ \{N_a\}_K \ N'_b \quad - \ \{N'_b\}_K
 \end{aligned}$$

**MITRE**

# Secrecy in Carlsen, I

## Safety

- Some keys  $K_{\mathcal{P}}$  known initially to penetrator
  - Long-term keys: malice or compromise
  - Old compromised session keys
- Protocol disclosures on regular nodes
  - Long-term keys never disclosed in protocol  
 $K_A \in S_0$  unless  $K_A \in K_{\mathcal{P}}$
  - $H$  terms: No new disclosure
    - Retransmission preserves safety
  - Session keys packed via long-term keys
- Keys in new context: protection by safe key gives derivative safety
  - If  $K$  occurs new only in  $\{\dots K \dots\}_{K_A}$
  - and  $K_A \in S_i$
  - then  $K \in S_{i+1}$

**MITRE**

# Components, “New”

- Term  $t_0$  is a component of  $t$ , written  $\boxed{t_0} \sqsubset t$

If  $t \in T \cup K$ , then  $\boxed{t} \sqsubset t$

If  $t = \{h\}_K$ , then  $\boxed{t} \sqsubset t$

$\boxed{t_0} \sqsubset g$  implies  $\boxed{t_0} \sqsubset g h$

$\boxed{t_0} \sqsubset h$  implies  $\boxed{t_0} \sqsubset g h$

Largest non-concatenated part

- E.g.:  $N_b \{K N_b A\}_{K_B} \{N_a B K\}_{K_A}$

$\boxed{N_b}$

$\boxed{\{K N_b A\}_{K_B}}$

$\boxed{\{N_a B K\}_{K_A}}$

- Penetrator controls concatenation fully

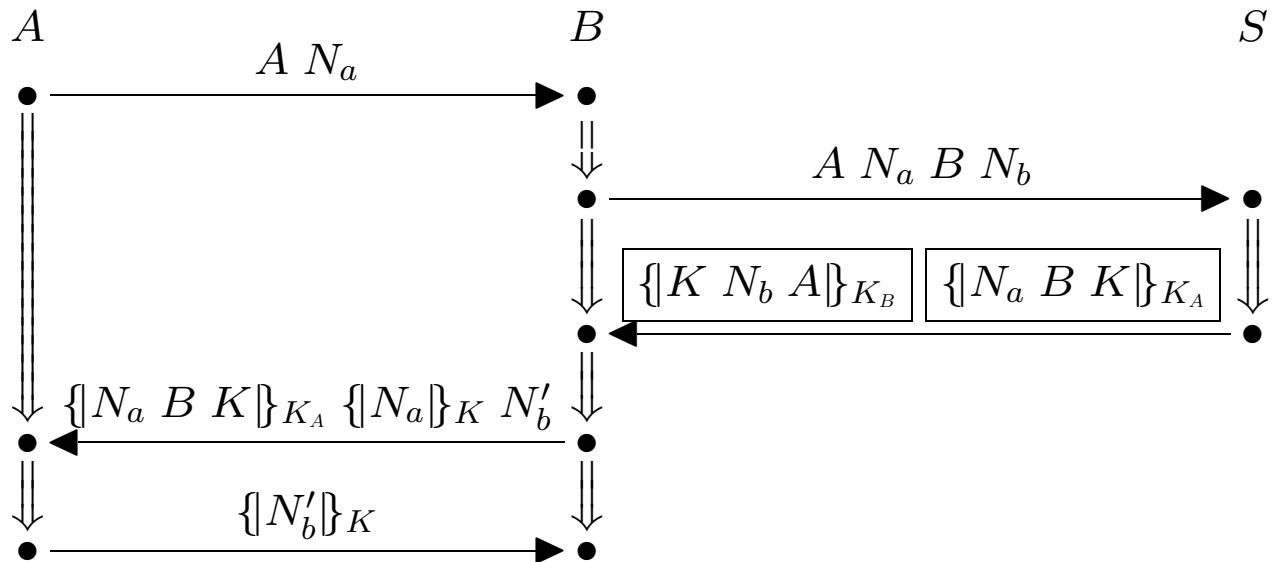
- $\boxed{t_0}^{\text{new}} \sqsubset \text{term}(n)$  means

–  $\boxed{t_0} \sqsubset \text{term}(n)$

– If  $n' \Rightarrow^+ n$  then  $\boxed{t_0} \not\sqsubset \text{term}(n')$

**MITRE**

# Carlsen: New Components with Keys as Subterms



- Key server (probabilistically) chooses session keys
  - Never used previously
  - Disjoint from long-term keys  $K_A$
  - Not in  $K_{\mathcal{P}}$
- $\text{CServ}[\ast\ast, K]$  has at most one strand
- $K_A, K_B \in S_0$  implies  $K \in S_1$

MITRE



# Key Safety

- $K$  unsafe if  $K \notin S_i$  for all  $i$ 
  - Define  $S = \bigcup_i S_i$
- For almost all protocols, either
  - $K \in S_0$ , or
  - $K \in S_1$ , or
  - $K \notin S$   $K$  unsafe
- Theorem (proof later):
  - If  $n$  a node in bundle  $\mathcal{C}$
  - and  $\text{term}(n) = K$
  - then  $K \notin S$
- “Syntactic” property of protocol entails dynamic property of executions (bundles)
  - $S$  depends on individual regular strands

**MITRE**

# Authentication Tests

**MITRE**

# NSL: Responder's Guarantee

- Suppose:

$K_A^{-1}$  safe

$N_2$  uniquely originating

- Responder's edge  $\{N_1 \ N_2 \ B\}_{K_A} \Rightarrow \{N_2\}_{K_B}$  is a test

- Penetrator can't decrypt  $\{N_1 \ N_2 \ B\}_{K_A}$
- Super-encrypting does no good
- Penetrator's only choice:  
discard it or deliver it?

- If responder receives  $\{N_2\}_{K_B}$  then test value was delivered

- But to whom? Which regular strands will receive, change  $\{N_1 \ N_2 \ B\}_{K_A}$ ?
- Only regular strand  
NSLInit[A, B, N<sub>1</sub>, N<sub>2</sub>], at node 2

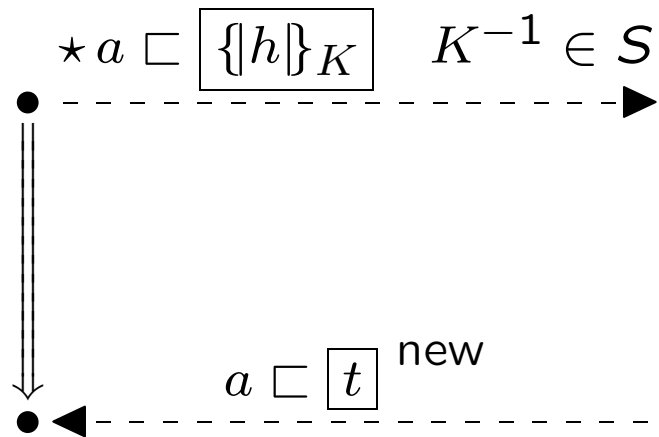
**MITRE**

# The Anatomy of the Case, I

- Find values originating uniquely on  $s_r \in \text{NSLResp}[A, B, N_a, N_b]$ 
  - $N_b$  only, on node  $n_0 = s_r \downarrow 2$   
in component  $\{N_a \ N_b \ B\}_{K_A}$
- Find negative (receiving) nodes containing  $N_b$   
 $n_1 = s_r \downarrow 3$  with term  $\{N_b\}_{K_B}$
- Check:
  - $K_A^{-1}$  is safe
  - $\{N_a \ N_b \ B\}_{K_A}$  not a subterm of a regular node
  - $N_b$  occurs in only one component of  $n_0$
- Therefore,  $n_0 \Rightarrow n_1$  is an “outgoing test”

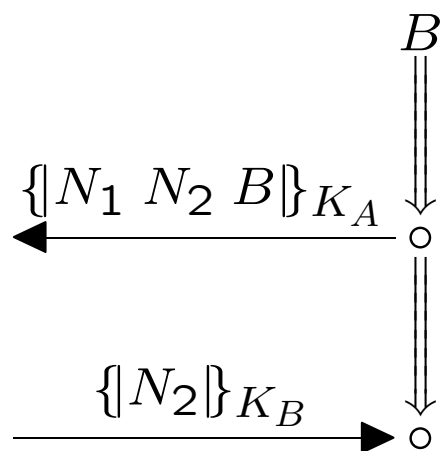
MITRE

# Outgoing Test



$a$  uniquely originates at  $\star$   
 $\boxed{t}$  means a component

# NSL Responder Test



**MITRE**

# Transforming Edge

- $n \Rightarrow^+ n'$  is a transforming edge for  $a$  if:
  - $n$  negative
  - $n'$  positive
  - $a \sqsubset \boxed{t} \sqsubset \text{term}(n')$
  - $t$  new at  $n'$
- A transforming edge does cryptographic work
  - Creates, transmits new component
- Outgoing test entails *regular* transforming edge

**MITRE**

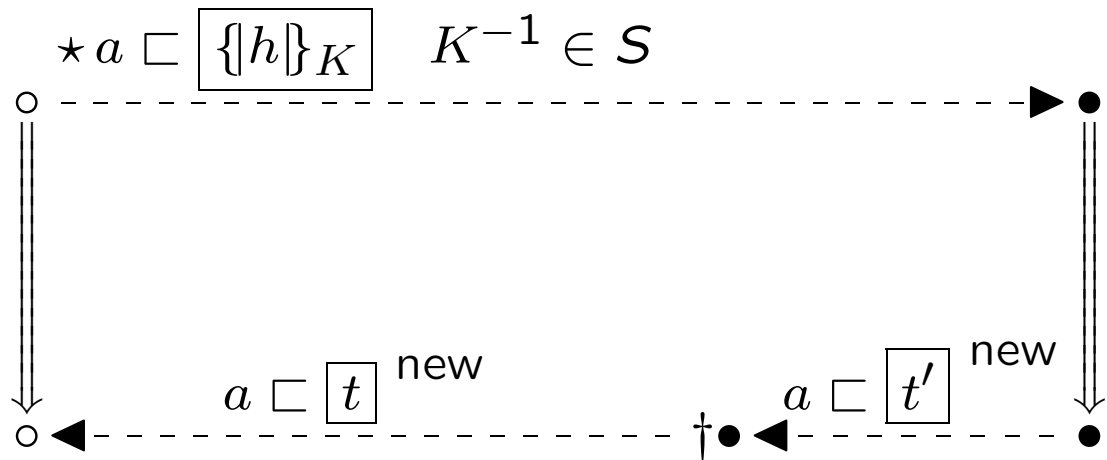
# The Anatomy of the Case, II

- Since  $n_0 \Rightarrow n_1$  is an outgoing test, there's a regular transforming edge  $m_0 \Rightarrow m_1$  such that
  - $\boxed{\{N_a \ N_b \ B\}_{K_A}} \sqsubset \text{term}(m_0)$
  - $m_0$  negative (receiving)
  - $m_1$  contains  $N_b$  in a new component
- Inspecting protocol,  $m_0 = s_i \downarrow 2$ , where  $s_i \in \text{NSLInit}[A, B, N_a, N_b]$ , so
  - $m_1 = s_i \downarrow 3$
  - $s_i$  has  $\mathcal{C}$ -height 3
- This is the NSL responder's guarantee

MITRE



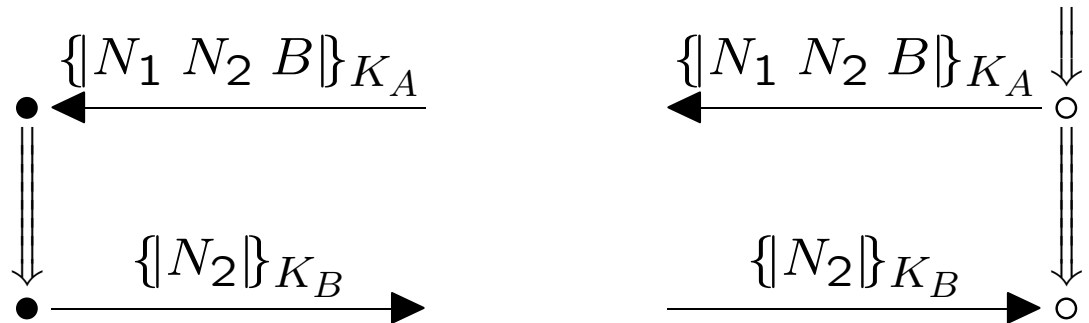
# Outgoing test Authentication



“●” means the test shows this regular node exists  
 † this node depends on extra conditions

**MITRE**

# NSL Responder Authentication

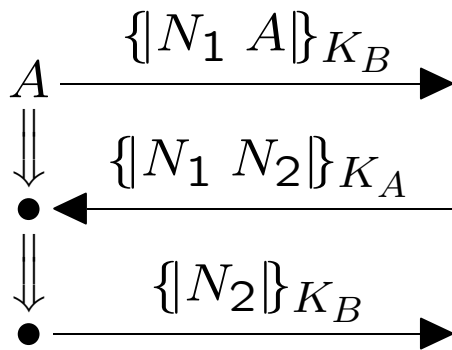


Outgoing test establishes

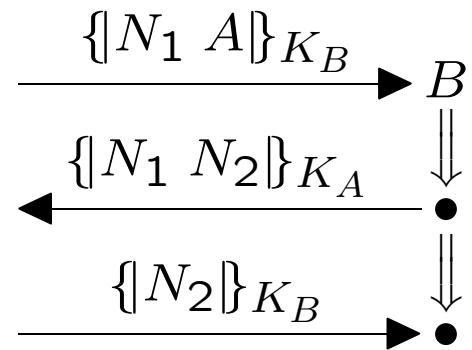
- nodes present and non-penetrator

**MITRE**

# Original Needham-Schroeder



NSInit[ $A, B, N_1, N_2$ ]



NSResp[ $A, B, N_1, N_2$ ]

**MITRE**

# Original NS Responder's Guarantee

- Suppose again:
  - $K_A^{-1}$  safe
  - $N_2$  uniquely originating
- Responder's edge  $\{N_1 \ N_2\}_{K_A} \Rightarrow \{N_2\}_{K_B}$  is a test
  - Penetrator can't decrypt  $\{N_1 \ N_2\}_{K_A}$
  - Super-encrypting does no good
  - Penetrator's only choice:  
discard it or deliver it?
- If responder receives  $\{N_2\}_{K_B}$ , then test value was delivered
  - But to whom?
  - Only regular strand  $\text{NSInit}[A, *, N_1, N_2]$  can receive  $\{N_1 \ N_2\}_{K_A}$  and change it
- Whoops: What if  $* \neq B$ ?
  - Unintended service!

**MITRE**

# Anatomy of Original NS

- Part I identifies outgoing test, as in NSL
- Since  $n_0 \Rightarrow n_1$  is an outgoing test, there's a regular  $m_0 \Rightarrow m_1$  such that
  - $\boxed{\{N_a \ N_b\}_{K_A}} \sqsubseteq \text{term}(m_0)$
  - $m_0$  negative (receiving)
- Inspecting protocol,  $m_0 = s_i \downarrow 2$ , where  $s_i \in \text{NSInit}[A, *, N_a, N_b]$ , so
  - $m_1 = s_i \downarrow 3$
  - $s_i$  has  $\mathcal{C}$ -height 3
- This is the NS responder's guarantee;  $B$  unconstrained

**MITRE**

# NSL Initiator's Guarantee, I

- Suppose:

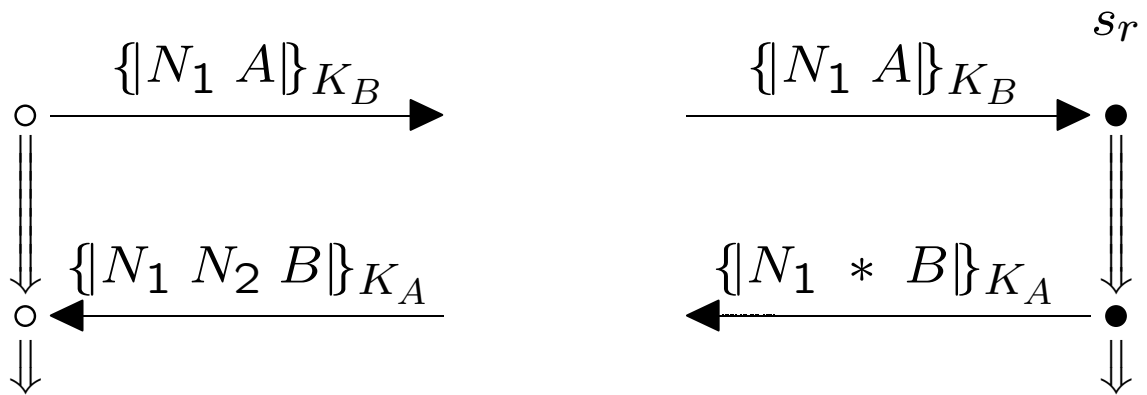
$K_B^{-1}$  safe

$N_1$  uniquely originating

- Initiator's edge  $\{N_1 \ A\}_{K_B} \Rightarrow \{N_1 \ N_2 \ B\}_{K_A}$  is a test
  - Penetrator can't decrypt  $\{N_1 \ A\}_{K_B}$
  - Super-encrypting does no good
  - Penetrator's only choice:  
discard it or deliver it?
- If initiator receives  $\{N_1 \ N_2 \ B\}_{K_A}$  then it was delivered
  - But to whom? Which regular strands will receive, change  $\{N_1 \ A\}_{K_B}$ ?
  - Only regular strand  
 $s_r \in \text{NSResp}[A, B, N_1, *]$ ,  
at node 1

**MITRE**

# NSL Initiator Authentication, I



**MITRE**

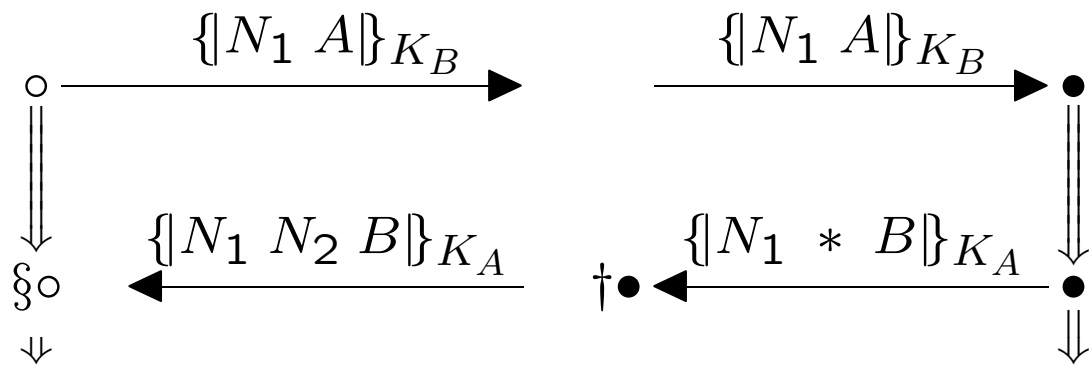
# NSL Initiator's Guarantee, II

- Suppose  $K_A^{-1}$  also safe
- Penetrator choice:  
discard or deliver  $\{N_1 * B\}_{K_A}$ 
  - Must have delivered it to a regular strand,  
an initiator strand  $\text{NSInit}[A, B, N_1, *]$
  - But  $N_1$  originates uniquely  
on a strand in  $\text{NSInit}[A, B, N_1, N_2]$
- So  $* = N_2$  and  
 $s_r \in \text{NSResp}[A, B, N_1, N_2]$
- Uses additional node in  
Outgoing Test Authentication

**MITRE**



# NSL Initiator Authentication, II

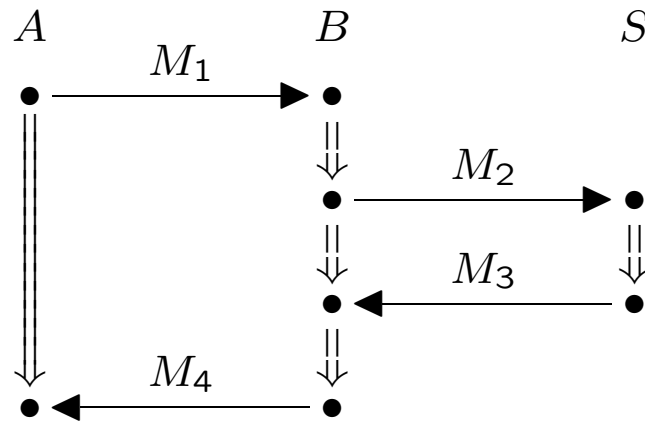


$$\xi \circ = \dagger \bullet$$

because of unique origination of  $N_a$

**MITRE**

# Exercise: Otway-Rees



$$M_1 = M \ A \ B \ \{N_a \ M \ A \ B\}_{K_{AS}}$$

$$M_2 = M \ A \ B \ \{N_a \ M \ A \ B\}_{K_{AS}} \ \{N_b \ M \ A \ B\}_{K_{BS}}$$

$$M_3 = M \ \{N_a \ K_{AB}\}_{K_{AS}} \ \{N_b \ K_{AB}\}_{K_{BS}}$$

$$M_4 = M \ \{N_a \ K_{AB}\}_{K_{AS}}$$

What authentication properties does this protocol achieve?

**MITRE**

# The Server's Guarantee

Unsolicited Test:



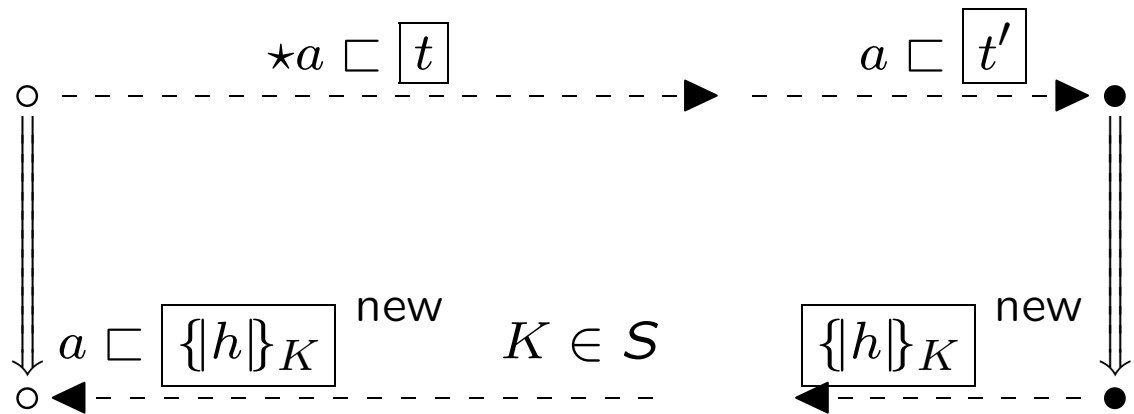
## In Otway-Rees

Suppose  $s_s \in \text{Serv}[A, B, M, N_a, N_b, *]$   
has  $\mathcal{C}$ -height 1:

- $K_A \notin K_{\mathcal{P}}$  implies  
some  $s_i \in \text{Init}[A, B, M, N_a, *]$   
has  $\mathcal{C}$ -height 1
- $K_B \notin K_{\mathcal{P}}$  implies  
some  $s_r \in \text{Resp}[A, B, M, N_b, *]$   
has  $\mathcal{C}$ -height 1

**MITRE**

# Incoming Test Authentication



**MITRE**

# Justifying Authentication Tests

**MITRE**

# Goals for this Hour

- Justify authentication test method
  - Use three ideas
    - Use equivalence relation on bundles  
Security goals invariant under equivalence
    - Focus on “well-behaved” bundles  
For every bundle, an equivalent well-behaved bundle exists
    - Consider paths through bundles
- Tomorrow: Apply same proof methods to protocol mixing

**MITRE**

## Definition: Bundles

A subgraph  $\mathcal{C}$  of  $G_\Sigma$  is a *bundle* if  $\mathcal{C}$  is finite and causally well-grounded, which means:

1. If  $n_2 \in \mathcal{C}$  negative,  
there is a unique  $n_1 \rightarrow n_2$  in  $\mathcal{C}$   
(everything heard was said)
2. If  $s \downarrow i + 1 \in \mathcal{C}$ , then  
 $s \downarrow i \Rightarrow s \downarrow i + 1$  in  $\mathcal{C}$   
(everyone starts at the beginning)
3.  $\mathcal{C}$  is acyclic  
(time never flows backward)

Causal partial ordering  $n_1 \preceq_{\mathcal{C}} n_2$  means  
 $n_2$  reachable from  $n_1$  via arrows in  $\mathcal{C}$

Induction: If  $S \subset \mathcal{C}$  is a non-empty set  
of nodes, it contains  $\preceq_{\mathcal{C}}$ -minimal members

**MITRE**

# Equivalent Bundles

- Bundles  $\mathcal{C}, \mathcal{C}'$  are equivalent iff they have the same regular nodes
  - Written  $\mathcal{C} \equiv \mathcal{C}'$
  - Penetrator nodes may differ arbitrarily
  - Ordering  $\preceq$  may differ arbitrarily
- Authentication goals invariant under equivalence
- Secrecy goals may be expressed in invariant form

Define  $v$  “uncompromised” in  $\mathcal{C}$   
to mean:

if for all  $\mathcal{C}' \equiv \mathcal{C}$  and  $n \in \mathcal{C}'$ ,

then  $v \not\sqsubseteq_{\emptyset} \text{term}(n)$

- “Regular nodes” means non-penetrator nodes  
 $v \sqsubseteq_{\emptyset} t$  concatenating  $v$  to other terms yields  $t$   
( $v$  is visible in  $t$ ,  
not protected by encryption)

**MITRE**



# Paths and Normality

**MITRE**

# Graph Operations

- A graph operation may:
  - Delete penetrator strands
  - Add edges  $n \rightarrow n'$   
with  $\text{term}(n) = +a$ ,  $\text{term}(n') = -a$
  - Delete edges  $n \rightarrow n'$
- A graph operation yields graph  $\mathcal{C}'$ 
  - $\mathcal{C}'$  not necessarily a bundle
  - But if it is a bundle, then  
 $\mathcal{C}' \equiv \mathcal{C}$

**MITRE**

# Loneliness

- A lonely node in a graph has no edge
  - No incoming edge if negative
  - No outgoing edge if positive
- In definition of bundle:
  - Lonely negative nodes are ruled out:  
You can't hear something if nobody says it
  - Lonely positive nodes are allowed:  
Nobody hears what you say

# Gregariousness

- A gregarious node in a graph has
  - Several incoming edges if negative
  - Several outgoing edges if positive
- In definition of bundle:
  - Gregarious negative nodes are ruled out:  
Hear the soloists, not the choir
  - Gregarious positive nodes are allowed:  
Many people hear your words

**MITRE**

# When are Graph Operations OK?

Suppose  $\mathcal{C}'$  is obtained from bundle  $\mathcal{C}$  by a graph operation such that

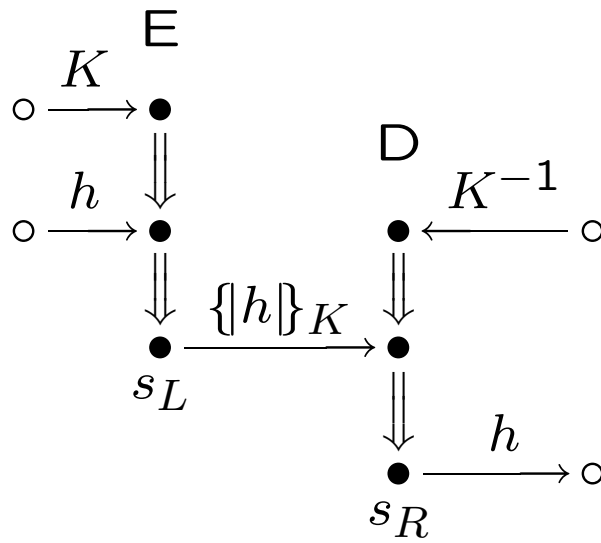
- For any edge new  $n \mapsto n'$  of  $\mathcal{C}'$ ,  $n \preceq_{\mathcal{C}} n'$
- $\mathcal{C}'$  has no lonely or gregarious negative nodes

Then

- $\mathcal{C}'$  is a bundle
- $\mathcal{C}' \equiv \mathcal{C}$
- The ordering  $\preceq_{\mathcal{C}'}$  on  $\mathcal{C}'$  weakens the ordering  $\preceq_{\mathcal{C}}$  on  $\mathcal{C}$

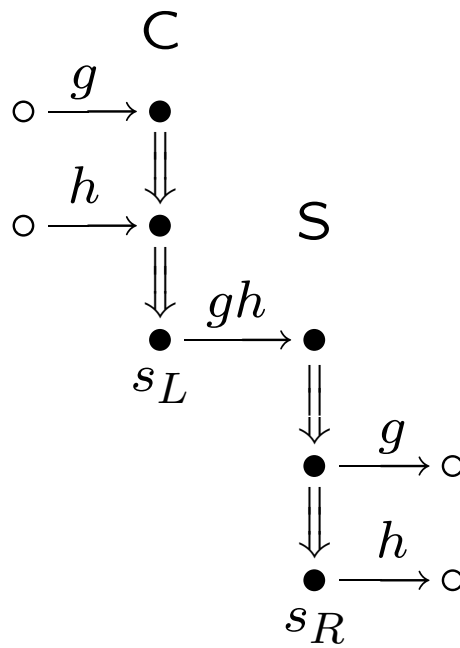
**MITRE**

# E-D Redundancies



MITRE

# C-S Redundancies



**MITRE**

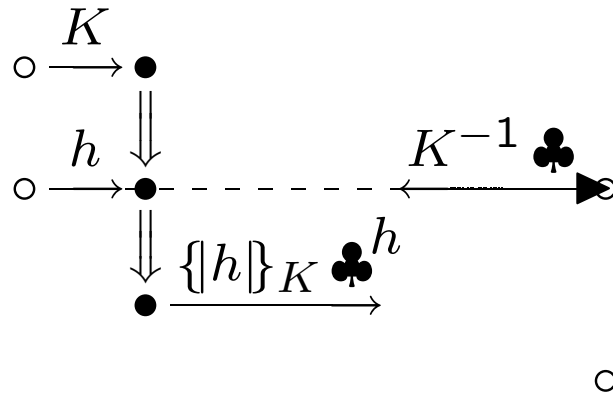
# Redundancy Elimination

- Any bundle  $\mathcal{C}$  is equivalent to a bundle  $\mathcal{C}'$  with no redundancies. Moreover,
  - Penetrator nodes of  $\mathcal{C}'$  is a subset of penetrator nodes of  $\mathcal{C}$
  - The ordering  $\prec_{\mathcal{C}'}$  weakens the ordering  $\prec_{\mathcal{C}}$
- Proof: Next two slides
- Consequence: Can assume attacker always
  - First Takes things apart
  - Next Puts things together
  - Then Delivers results

**MITRE**



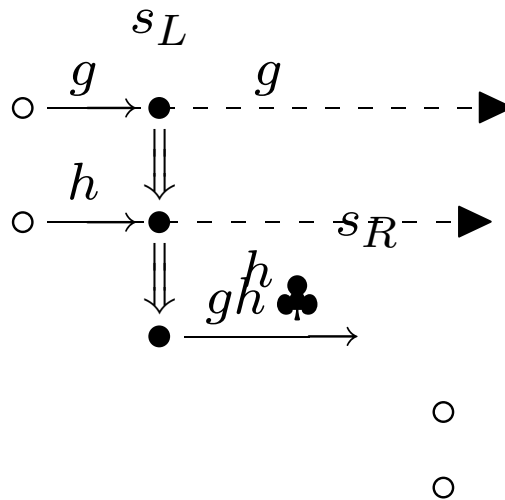
# E-D Redundancy Elimination



♣ Discarded message

MITRE

# C-S Redundancy Elimination



♣ Discarded message

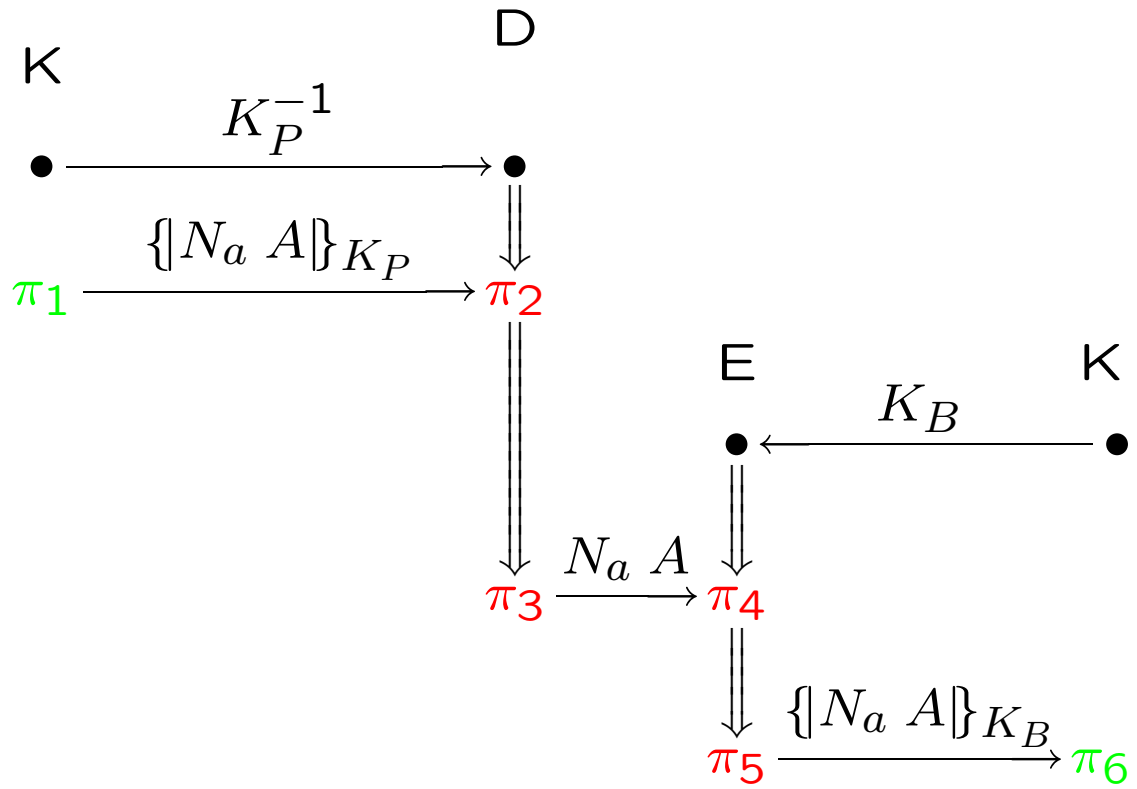
**MITRE**

# Paths

- $m \Rightarrow^+ n$  means  
 $n$  occurs after  $m$  on the same strand
- $m \longmapsto n$  means either 1 or 2:
  1.  $m \rightarrow n$
  2.  $m \Rightarrow^+ n$  where  
term( $m$ ) negative and  
term( $n$ ) positive
- Path  $p$  through  $\mathcal{C}$ : sequence  
 $p_1 \longmapsto p_2 \longmapsto \cdots \longmapsto p_k$ 
  - Typically assume  $p_1$  positive node,  
 $p_k$  negative node
  - Notation:  
 $|p| = k, \quad \ell(p) = p_k$
- Penetrator path:  $p_j$  penetrator node,  
except possibly  $j = 1$  or  $j = k$

MITRE

# A Penetrator Path



MITRE

# Construction and Destruction

- $A \Rightarrow +$ -edge between penetrator nodes is
  - Constructive if part of a E or C strand
  - Destructive if part of a D or S strand
  - Initial if part of a K or M strand
- Constructive edge followed by a destructive edge  
Possible forms:
  - Node on  $E_{h,K}$  immediately followed by node on  $D_{h,K}$   
(for some  $h, K$ )
  - Node on  $C_{g,h}$  immediately followed by node on  $S_{g,h}$   
(for some  $g, h$ )
- This uses freeness of term algebra

**MITRE**

# Normality

- Bundle  $\mathcal{C}$  normal iff
  - No penetrator path  $p$  has constructive  $\Rightarrow$  edge before destructive  $\Rightarrow$  edge
- Any bundle is equivalent to a normal one
  - Eliminate redundancies
  - No other constructive/destructive pairs by freeness

**MITRE**

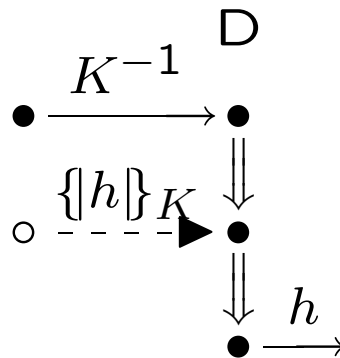
# Rising and Falling Paths

- Definitions: ( $p$  a penetrator path)

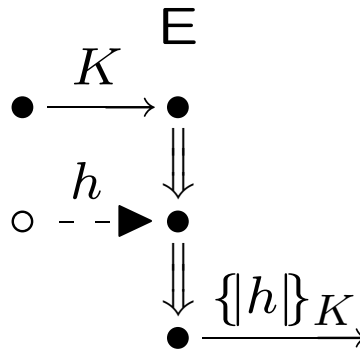
**Rising**  $\text{term}(p_i) \sqsubset \text{term}(p_{i+1})$

**Falling**  $\text{term}(p_{i+1}) \sqsubset \text{term}(p_i)$

- Destructive paths may not be falling:

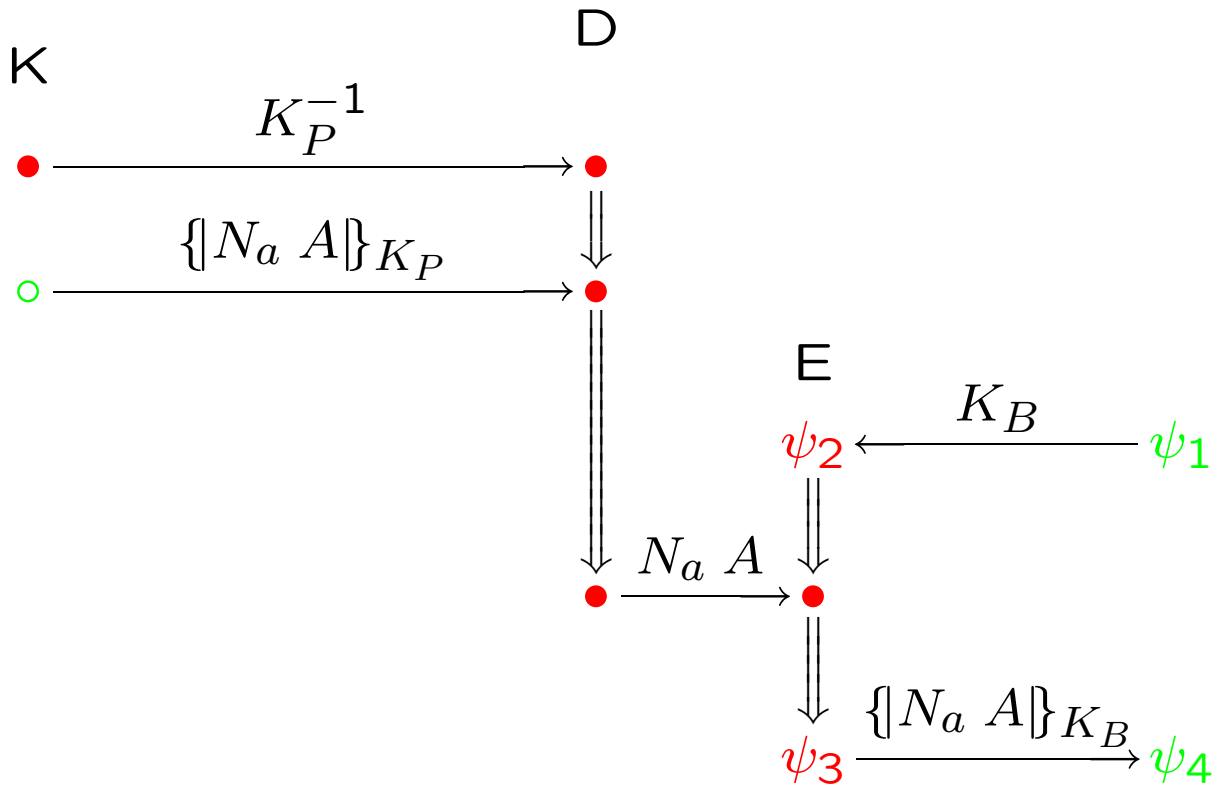


Constructive paths may not be rising:



**MITRE**

# Another Penetrator Path



**MITRE**



# Paths that Avoid Key Edges

- If  $p$  is destructive and  
 $p$  never traverses D-key edge  
then  $p$  is falling  
$$\text{term}(\ell(p)) \sqsubset \text{term}(p_1)$$
- If  $p$  is constructive and  
 $p$  never traverses E-key edge  
then  $p$  is rising  
$$\text{term}(p_1) \sqsubset \text{term}(\ell(p))$$
- If bundle normal and  $p$  avoids key edges  
$$p = q \rightarrow q'$$
$$q \text{ falling}$$
$$q' \text{ rising}$$
- $\text{term}(\ell(q)) = \text{term}(q'_1) = \text{pbt}(p)$   
called “path bridge term”  
$$\text{pbt}(p) \sqsubset p_1$$
$$\text{pbt}(p) \sqsubset \ell(p)$$

**MITRE**

# Classifying Penetrator Paths

- Let  $p$  penetrator path; traverse backward.  
It may either:
  - Reach an initial penetrator node  $(M, K)$
  - or Reach a non-initial E- or D-key edge
  - or  $p_1$  is regular
- If penetrator path  $p$  is useful, then either:
  - $\ell(p)$  is regular
  - or  $\ell(p)$  is a key edge
- All penetrator activity divides into paths  $p$   
where  $p$  never traverses key edge
  - $p_1, \ell(p)$  both regular
  - $p_1$  initial,  $\ell(p)$  reg.      \*  $\text{term}(p_1) \sqsubset \text{term}(\ell(p))$
  - $p_1$  regular       $K = \text{term}(\ell(p))$
  - $p_1$  a K-node      \*  $p = p_1 \rightarrow p_2$
- \* If bundle  $\mathcal{C}$  normal

**MITRE**

# Falling Penetrator Paths

- Suppose  $p_i$  negative with  $1 < i < |p|$   
 Then  $\text{term}(p_i)$  not atomic and  
     either  $\text{term}(p_i) = \{h\}_K$  and  $p_i$  on  $D$   
     or  $\text{term}(p_i) = g\ h$  and  $p_i$  on  $S$
- If  $p_i$  positive,  $\text{term}(p_i) = \text{term}(p_{i+1})$
- Suppose  $p$  traverses  $D$  with key edge  $K^{-1}$   
 only if  $K \in \mathcal{K}$   
 Then  $\text{term}(\ell(p)) \sqsubseteq_{\mathcal{K}} \text{term}(p_1)$
- Definition:  $t_0 \sqsubseteq_{\mathcal{K}} t$  iff  
 $t$  can be built from  $t_0$  using only
  - concatenation (with anything)
  - encryption using  $K \in \mathcal{K}$
$$\dots \{ \dots t_0 \dots \}_K \dots$$

**MITRE**

# Well-Behaved Bundles

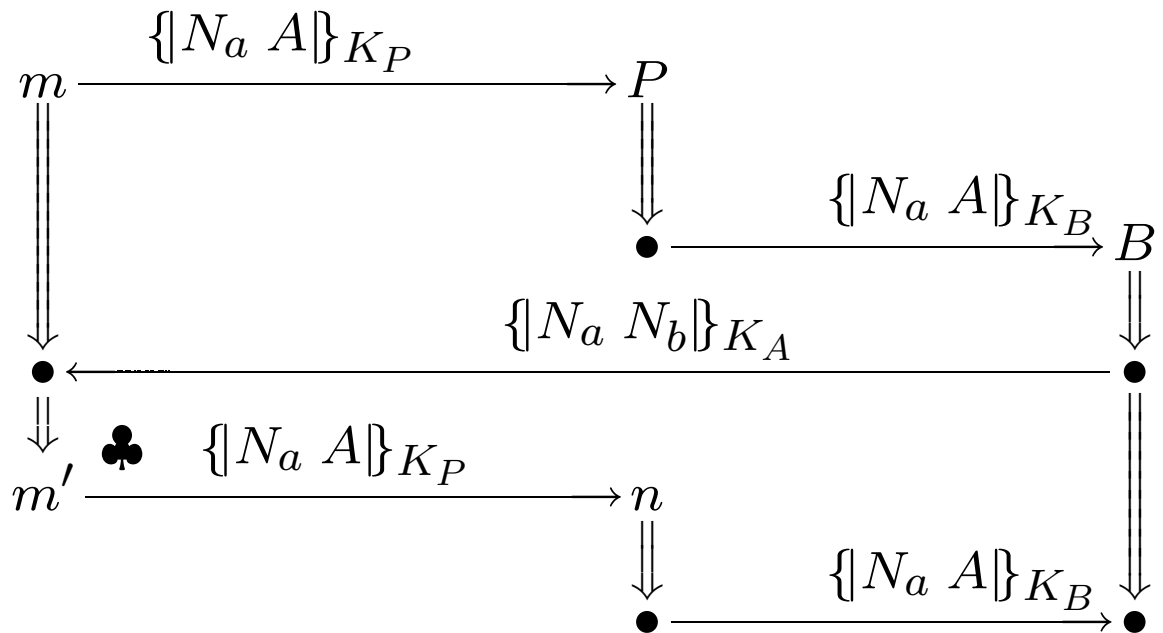
**MITRE**

# Well-Behaved: Definition

- A bundle is well-behaved if
  - Normal
  - Efficient
  - Has simple bridges
- Will define “efficient,” “simple bridges”
- Every bundle is equivalent to a well-behaved bundle

**MITRE**

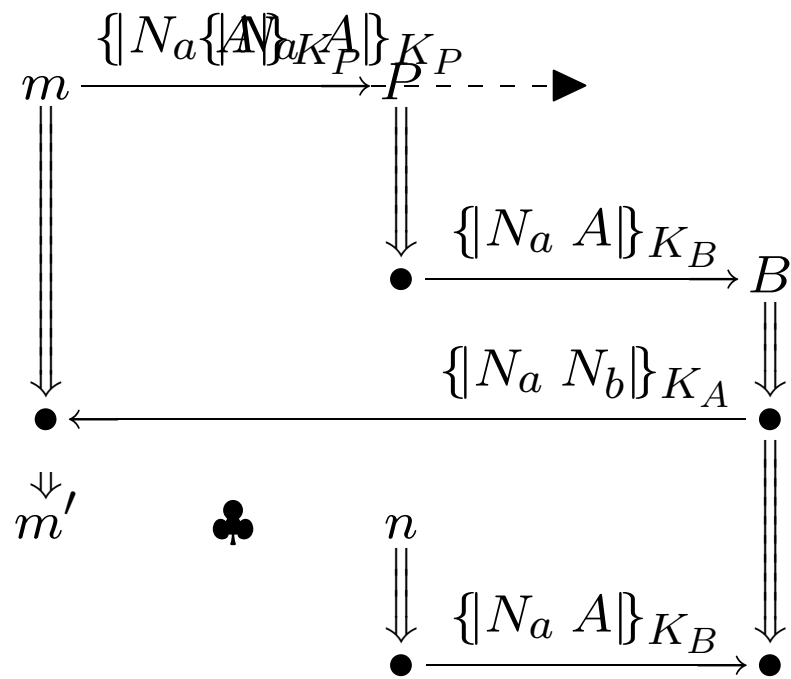
# An Inefficient Bundle



● Note: This protocol is fictitious!

**MITRE**

# An Efficient Bundle



# Efficient Bundles

- In efficient bundle,  
penetrator avoids unnecessary regular nodes
- $\mathcal{C}$  is an efficient bundle iff:  
If  $m, n$  are nodes  
     $n$  negative penetrator node  
    every component of  $n$  is a component of  $m$

Then there are no regular nodes  $m'$  such that

$$m \prec m' \prec n$$

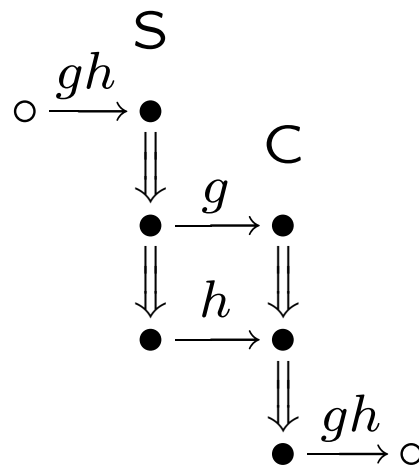
- For all  $\mathcal{C}$ , there exists  $\mathcal{C}'$  where  
     $\mathcal{C} \equiv \mathcal{C}'$   
     $\mathcal{C}'$  efficient, normal

**MITRE**



# Simple Bridges

- Simple term is either
  - An atomic value  $K$ ,  $N_a$ , etc.
  - An encryption  $\{h\}_K$
  - Anything but a concatenation
- $\mathcal{C}$  has simple bridges iff
  - whenever  $p$  a penetrator path
  - $\text{pbt}(p)$  is simple
- Every  $\mathcal{C}$  has an equivalent  $\mathcal{C}'$  with simple bridges



MITRE

# Transforming Edges, Transformation Paths, Pedigree

**MITRE**

# Transformed and Transforming Edges

$n_1 \Rightarrow^+ n_2$  is a *transformed edge* for  $a \in A$  if  
[ $n_1 \Rightarrow^+ n_2$  is a *transforming edge* for  $a \in A$  if]

1.  $n_1$  is positive      [ $n_1$  is negative]
2.  $n_2$  is negative      [ $n_2$  is positive]
3.  $a \sqsubset \text{term}(n_1)$
4. There is a new component  $t_2$  of  $n_2$   
such that  $a \sqsubset t_2$

- Penetrator transforming edge:  
either D or E

**MITRE**

# Transformation Paths

- Path  $p$  with  $p_i$  labelled by component  $\mathcal{L}_i$  of  $p_i$   
where  $\mathcal{L}_i = \mathcal{L}_{i+1}$   
unless  $p_i \Rightarrow^+ p_{i+1}$  and  
 $\mathcal{L}_{i+1}$  is new at  $p_{i+1}$
- $\mathcal{L}_i$  is the “component of interest”  
at node  $p_i$
- Example:

$$\langle (\pi_1, \{N_a A\}_{K_P}), (\pi_2, \{N_a A\}_{K_P}), \\ (\pi_3, N_a), (\pi_4, N_a), (\pi_5, \{N_a A\}_{K_B}), \\ (\pi_6, \{N_a A\}_{K_B}) \rangle$$

# Separated Transformation Paths

Theorem:

- Suppose  $\mathcal{C}$  well-behaved and  
 $(p, \mathcal{L}), (p', \mathcal{L}')$  transformation paths  
 $\ell(p) \prec m \prec p'_1$   $m$  regular  
 $p_1, \ell(p')$  simple  
(not concatenated)
- Then  $\mathcal{L}_i \neq \mathcal{L}'_j$   
for all  $i, j$  where  $1 \leq i \leq |p|, 1 \leq j \leq |p'|$
- That means:  
Never repeat component of interest  
when penetrator paths  
separated by regular nodes

**MITRE**

# Pedigree Path for $a$

- Transformation path  $p, \mathcal{L}$  where
  - $a$  originates at  $p_1$
  - $a \sqsubseteq \mathcal{L}_i$  for all  $i$
  - $p$  does not traverse any D, E key edges
- Whenever  $a \sqsubseteq \text{term}(n)$ ,  
exists pedigree path with  $\ell(p) = n$ 
  - Proof idea:  
Keep tracing backward,  
selecting components containing  $a$
- Authentication test proof idea:  
Check which steps on pedigree path  
must occur on *regular* edges

# **Key Safety, Authentication Tests Justified**

**MITRE**

# Penetrable Keys, Safe Keys

$$P_0 = K_{\mathcal{P}}$$

$$K \in P_{i+1} \text{ iff}$$

exists  $n$  regular, positive, with

$$K \sqsubseteq_{\mathcal{K}} \boxed{t}^{\text{new}} \sqsubseteq \text{term}(n), \text{ and}$$

$$\text{for all } K_0 \in \mathcal{K}, \quad K_0^{-1} \in P_i$$

$$P = \bigcup_i P_i$$

- Let  $\mathcal{C}$  be a bundle

    If  $n \in \mathcal{C}$

    and  $\text{term}(n) = K$

    then  $K \in P$

- Proof idea:

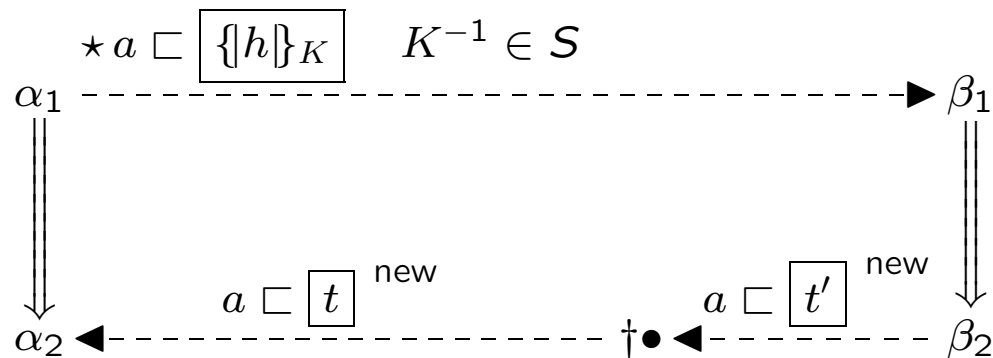
- Use “Classifying Penetrator Paths”
- Use “Falling Penetrator Paths”

- $S \cap P = \emptyset$

**MITRE**



# Outgoing test Authentication



The test shows the  $\beta$  regular nodes exist. Proof:

- Exists pedigree path  $p, \mathcal{L}$  where

$$p_1 = \alpha_1 \quad (a \text{ uniquely originates})$$

$$\{h\}_K = \mathcal{L}_1 \neq \ell(\mathcal{L}) = t$$

- Must be first transforming edge  $n_1 \Rightarrow^+ n_2$ 
  - Penetrator D impossible
  - Penetrator E contradicts normalcy
  - Otherwise regular strand, QED

**MITRE**

# Summary of this Lecture

- Authentication tests:  
Methods to establish
  - Secrecy (especially of keys)
  - Authentication
- Justifying the authentication tests
  - Bundles, equivalence
    - Normal and well-behaved bundles
  - Paths through bundles
    - Transformation paths
    - Pedigree paths
- Tomorrow:
  - Protocol independence through disjoint encryption
  - Authentication tests as a design strategy

**MITRE**