Probabilities in Higher-Order Languages

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Higher-order languages

Variables may be instantiated with terms of the language itself
(e.g. terms can be copied)

- functions (λ-calculus)

$$(\lambda x.M)N \longrightarrow M\{^{N}\!/\!_{x}\}$$

- higher-order communication (Higher-Order π-calculus)

$$a(x).M \mid \overline{a}\langle N\rangle.R \longrightarrow M\{^{N}\!/\!_{x}\} \mid R$$

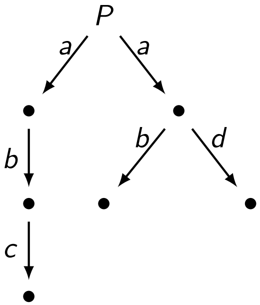
- action on locations (kells), as with passivation

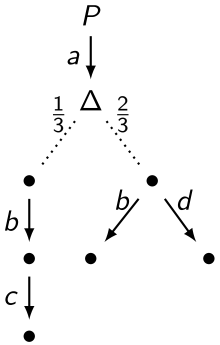
$$[\![M]\!]_l \mid \mathtt{pass}_l(x).N \longrightarrow N\{^{M}\!/\!_{x}\} \qquad M \text{ is a running term}$$

[Schmitt,Stefani GC'04, Lenglet et al. Inf.Comp.'11,
Piérard,Sumii FOSSACS'12, Koutavas,Hennessy CONCUR'13]

Probabilistic processes

nondeterministic (LTSs) Vs. reactive probabilistic (RPLTSs)





external & internal nondeterminism

only external nondeterminism

[Larsen,Skou POPL'89, van Breugel et al. Theor.Comp.Sci.'05]

A contextual/testing approach

Discriminating power of higher-order languages

and

interplay between probabilities, concurrency, higher-order operators

Plan of the talk

1. main ingredients
2. results in the nondeterministic setting
3. results in the probabilistic setting

[Bernardo,Sangiorgi,Vignudelli CSL-LICS'14]

The discriminating power of a language

A testing language $\mathcal{L}$, a set of tested terms $P, Q \dots$

$$P \simeq_{\mathcal{L}} Q \quad \triangleq \quad C[P] \text{ and } C[Q] \text{ `equally successful'}, \forall \text{ contexts } C \text{ of } \mathcal{L}$$

Otherwise: P, Q are discriminated by $\mathcal{L}$

Two classes of first-order (CCS-like) processes as tested terms P, Q:

- nondeterministic
- reactive probabilistic

Comparison of testing languages $\mathcal{L}$ with different constructs:

- sequential higher-order (λ-calculus)
- higher-order communication (HOπ)
- ordinary first-order concurrency (CCS)
- passivation
- refusal

no probabilities in $\mathcal{L}$

Higher-order sequential languages: Kell λ-calculus ($\mathtt{K}\Lambda$)

M is evaluated in kell N (of process type)

$$\langle N\,;\,M\rangle \longrightarrow \langle N'\,;\,M'\rangle$$

Contexts interact with the kell by:

- testing actions

$$\begin{array}{lll} \langle P\,;\,a?\rangle & \longrightarrow \langle P'\,;\,\mathsf{true}\rangle & \text{if } P \xrightarrow{a} P' \\ \langle P\,;\,a?\rangle & \longrightarrow \langle P\,;\,\mathsf{false}\rangle & \text{if } P \stackrel{a}{\not\longrightarrow} \end{array}$$

- reading (passivating) and rewriting the kell

$$\langle P\,;\,\mathrm{pass}\rangle \longrightarrow \langle P\,;\,P\rangle \qquad \langle P\,;\,\langle P'\,;\,M\rangle\rangle \longrightarrow \langle P'\,;\,M\rangle$$

Variants: - call by name ($\mathtt{K}\Lambda_{\mathrm{N}}$) and call by value ($\mathtt{K}\Lambda_{\mathrm{V}}$)

- refusal free ($\mathtt{K}\Lambda_{\mathrm{N-ref}}, \mathtt{K}\Lambda_{\mathrm{V-ref}}$)

Concurrent languages

Interactions between context and process:

- action synchronization (CCS)

$$P \mid \overline{a}.M \quad \longrightarrow \quad P' \mid M \qquad\qquad \text{if } P \xrightarrow{a} P'$$

- higher-order communication (HOπ)

$$a(x).M \mid \overline{a}\langle P\rangle.N \longrightarrow M\{^{P}\!/\!_{x}\} \mid N$$

- refusal on kells ($\mathrm{CCS}_{\mathrm{ref}}$, $\mathrm{HO}\pi_{\mathrm{ref}}$)

$$[\![P]\!]_l \mid \widetilde{a}_l.M \quad \longrightarrow \quad [\![P]\!]_l \mid M \qquad\qquad \text{if } P \overset{a}{\not\longrightarrow}$$

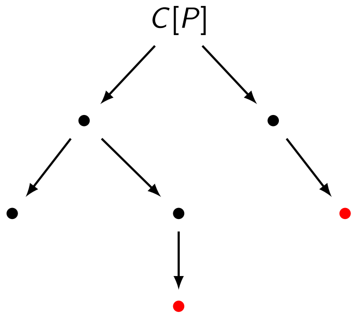
- passivation of kells ($\mathrm{HO}\pi_{\mathtt{pass}}$, $\mathrm{HO}\pi_{\mathtt{pass},\mathrm{ref}}$)

$$[\![P]\!]_l \mid \mathtt{pass}_l(x).M \longrightarrow M\{^{P}\!/\!_{x}\}$$

Testing nondeterministic processes

'Success' on nondeterministic processes

$P \simeq_{\mathcal{L}} Q \quad \triangleq \quad C[P]$ and $C[Q]$ 'equally successful', $\forall$ contexts C of $\mathcal{L}$



P is an LTS, $C[P] \Downarrow$

Success states $\color{red}\bullet$ are:

$M \xrightarrow{\omega}$ in CCS, HOπ

$\langle N \,;\, \text{true} \rangle$ in $\mathtt{K}\Lambda$

Success is $\Downarrow$ ($=$ a success state $\color{red}\bullet$ is reachable) [may success]

The Spectrum on LTSs

$$
\begin{tikzcd}[column sep=tiny]
 & \substack{\text{ready simulation equivalence} \\ \mathtt{K}\Lambda_{\mathrm{V}} = \mathsf{HO}\pi_{\mathrm{ref},\mathbf{pass}}} \arrow[dl] \arrow[dr] & \\
\substack{\text{simulation equivalence} \\ \mathtt{K}\Lambda_{\mathrm{V-ref}} = \mathsf{HO}\pi_{\mathbf{pass}}} \arrow[dr] & & \substack{\text{failure trace equivalence} \\ \mathtt{K}\Lambda_{\mathrm{N}} = \mathsf{HO}\pi_{\mathrm{ref}} = \mathsf{CCS}_{\mathrm{ref}}} \arrow[dl] \\
 & \substack{\mathtt{K}\Lambda_{\mathrm{N-ref}} = \mathsf{HO}\pi = \mathsf{CCS} \\ \text{trace equivalence}} &
\end{tikzcd}
$$

- CBV $=$ passivation in HOπ
- sequential $=$ concurrent
- first-order communication $=$ higher-order communication

Call By Name & Call By Value

P does a

$$\langle P\,;\,a?\rangle \Downarrow \quad \textit{iff} \quad P \xrightarrow{a} \qquad\qquad \mathrm{T}_a \triangleq a?$$

P refuses a

$$\langle P\,;\,\mathrm{T}_{\neg a}\rangle \Downarrow \quad \textit{iff} \quad P \stackrel{a}{\not\longrightarrow} \qquad\qquad \mathrm{T}_{\neg a} \triangleq \texttt{if}\ \ a?\ \ \texttt{then}\ \ \mathsf{false}\ \ \texttt{else}\ \ \mathsf{true}$$

P passes T_1 and T_2 in sequence

$$\langle P\,;\,\underline{\mathrm{Seq}}\,(\lambda.T_1)(\lambda.T_2)\rangle \Downarrow \quad \textit{iff} \quad \langle P\,;\,T_1\rangle \Longrightarrow \langle P'\,;\,\mathsf{true}\rangle \ \wedge\ \langle P'\,;\,T_2\rangle \Downarrow$$

for $\underline{\mathrm{Seq}} \triangleq \lambda x.\lambda y.\,\texttt{if}\ \ x\star\ \ \texttt{then}\ \ y\star\ \ \texttt{else}\ \ \mathsf{false}$

$\lambda.M$	thunking
$M\star$	unthunking

Example: discriminating LTSs

$$
\begin{tikzcd}
 & & P \arrow[dl, "a"'] \arrow[dr, "a"] & & \\
 & P_1 \arrow[dl, "b"'] \arrow[dr, "c"] & & P_2 \arrow[dr, "b"] & \\
P_3 & & P_4 & & P_5
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
 & Q \arrow[d, "a"] & \\
 & Q_1 \arrow[dl, "b"'] \arrow[dr, "c"] & \\
Q_2 & & Q_3
\end{tikzcd}
$$

$$C[P] \Downarrow \qquad\qquad\qquad C[Q] \not\Downarrow$$

$$C = \langle \, \cdot \; ; \; \underline{\mathrm{Seq}} \; \mathrm{T}_a \; \mathrm{T}_{\neg c} \, \rangle$$

completed trace/simulation equivalent but not failure equivalent processes

[see van Glabbeek '90 spectrum]

Call By Value

P passes both T_1 and T_2

$$\langle P\,;\ \underline{\mathrm{And}}\,(\lambda.T_1)(\lambda.T_2)\rangle \Downarrow \quad \textit{iff} \quad \langle P\,;\ T_1\rangle \Downarrow \wedge \langle P\,;\ T_2\rangle \Downarrow$$

for $\underline{\mathrm{And}} \ \triangleq\ \lambda x.\lambda y.((\lambda z.\ \mathtt{if}\ x\star\ \mathtt{then}\ \langle z\,;\, y\star\rangle\ \mathtt{else}\ \mathsf{false})\mathsf{pass})$

$$C[P] \Downarrow \quad \textit{iff} \quad P \xrightarrow{a} \wedge P \xrightarrow{b}$$

$$\begin{aligned}
&\langle P\,;\,(\lambda x.\ \mathtt{if}\ a?\ \mathtt{then}\ \langle x\,;\, b?\rangle\ \mathtt{else}\ \mathsf{false})\mathsf{pass}\rangle \\
&\qquad\longrightarrow \langle P\,;\,(\lambda x.\,\mathtt{if}\ a?\ \mathtt{then}\ \langle x\,;\, b?\rangle\ \mathtt{else}\ \mathsf{false})P\rangle \\
&\qquad\longrightarrow \langle P\,;\ \mathtt{if}\ a?\ \mathtt{then}\ \langle P\,;\, b?\rangle\ \mathtt{else}\ \mathsf{false}\rangle \\
&\qquad\longrightarrow \langle P'\,;\ \mathtt{if}\ \mathsf{true}\ \mathtt{then}\ \langle P\,;\, b?\rangle\ \mathtt{else}\ \mathsf{false}\rangle \\
&\qquad\longrightarrow \langle P'\,;\,\langle P\,;\, b?\rangle\rangle \\
&\qquad\longrightarrow \langle P\,;\, b?\rangle \Downarrow
\end{aligned}$$

Call By Value

P passes both T_1 and T_2

$$\langle P\,;\ \underline{\mathrm{And}}\,(\lambda.T_1)(\lambda.T_2)\rangle \Downarrow \quad \textit{iff} \quad \langle P\,;\ T_1\rangle \Downarrow \wedge \langle P\,;\ T_2\rangle \Downarrow$$

for $\underline{\mathrm{And}} \triangleq \lambda x.\lambda y.((\lambda z.\ \mathtt{if}\ x\star\ \mathtt{then}\ \langle z\,;\,y\star\rangle\ \mathtt{else}\ \mathsf{false})\mathtt{pass})$

$$C[P]\Downarrow \quad \textit{iff} \quad P \xrightarrow{a} \wedge P \xrightarrow{b}$$

$$\begin{aligned}
&\langle {\color{red}P}\,;(\lambda x.\ \mathtt{if}\ a?\ \mathtt{then}\ \langle x\,;\,b?\rangle\ \mathtt{else}\ \mathsf{false}){\color{red}\mathtt{pass}}\rangle\\
&\qquad\longrightarrow \langle P\,;(\lambda x.\,\mathtt{if}\ a?\ \mathtt{then}\ \langle x\,;\,b?\rangle\ \mathtt{else}\ \mathsf{false})P\rangle\\
&\qquad\longrightarrow \langle P\,;\ \mathtt{if}\ a?\ \mathtt{then}\ \langle P\,;\,b?\rangle\ \mathtt{else}\ \mathsf{false}\rangle\\
&\qquad\longrightarrow \langle P'\,;\ \mathtt{if}\ \mathsf{true}\ \mathtt{then}\ \langle P\,;\,b?\rangle\ \mathtt{else}\ \mathsf{false}\rangle\\
&\qquad\longrightarrow \langle P'\,;\langle P\,;\,b?\rangle\rangle\\
&\qquad\longrightarrow \langle P\,;\,b?\rangle\Downarrow
\end{aligned}$$

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P passes both T_1 and T_2

$$\langle P\,;\ \underline{\mathrm{And}}\,(\lambda.T_1)(\lambda.T_2)\rangle \Downarrow \quad \textit{iff} \quad \langle P\,;\ T_1\rangle \Downarrow \wedge \langle P\,;\ T_2\rangle \Downarrow$$

for $\underline{\mathrm{And}} \triangleq \lambda x.\lambda y.((\lambda z.\ \texttt{if}\ x\star\ \texttt{then}\ \langle z\,;\,y\star\rangle\ \texttt{else}\ \mathsf{false})\mathsf{pass})$

$$C[P]\Downarrow \quad \textit{iff} \quad P \xrightarrow{a} \wedge P \xrightarrow{b}$$

$$\begin{aligned}
&\langle P\,;\,(\lambda x.\ \texttt{if}\ a?\ \texttt{then}\ \langle x\,;\,b?\rangle\ \texttt{else}\ \mathsf{false})\mathsf{pass}\rangle \\
&\qquad \longrightarrow \langle P\,;\,({\color{red}\lambda x}.\,\texttt{if}\ a?\ \texttt{then}\ \langle {\color{red}x}\,;\,b?\rangle\ \texttt{else}\ \mathsf{false}){\color{red}P}\rangle \\
&\qquad \longrightarrow \langle P\,;\ \texttt{if}\ a?\ \texttt{then}\ \langle P\,;\,b?\rangle\ \texttt{else}\ \mathsf{false}\rangle \\
&\qquad \longrightarrow \langle P'\,;\ \texttt{if}\ \mathsf{true}\ \texttt{then}\ \langle P\,;\,b?\rangle\ \texttt{else}\ \mathsf{false}\rangle \\
&\qquad \longrightarrow \langle P'\,;\,\langle P\,;\,b?\rangle\rangle \\
&\qquad \longrightarrow \langle P\,;\,b?\rangle \Downarrow
\end{aligned}$$

Call By Value

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for $\underline{\mathrm{And}} \triangleq \lambda x.\lambda y.((\lambda z.\ \mathtt{if}\ x\star\ \mathtt{then}\ \langle z\,;\,y\star\rangle\ \mathtt{else}\ \mathsf{false})\mathtt{pass})$

$$C[P]\Downarrow \quad \textit{iff} \quad P \xrightarrow{a} \wedge P \xrightarrow{b}$$

$$\begin{aligned}
&\langle P\,;(\lambda x.\ \mathtt{if}\ a?\ \mathtt{then}\ \langle x\,;\,b?\rangle\ \mathtt{else}\ \mathsf{false})\mathtt{pass}\rangle\\
&\qquad\longrightarrow \langle P\,;(\lambda x.\,\mathtt{if}\ a?\ \mathtt{then}\ \langle x\,;\,b?\rangle\ \mathtt{else}\ \mathsf{false})P\rangle\\
&\qquad\longrightarrow \langle P\,;\ \mathtt{if}\ a?\ \mathtt{then}\ \langle P\,;\,b?\rangle\ \mathtt{else}\ \mathsf{false}\rangle\\
&\qquad\longrightarrow \langle P'\,;\ \mathtt{if}\ \mathsf{true}\ \mathtt{then}\ \langle P\,;\,b?\rangle\ \mathtt{else}\ \mathsf{false}\rangle\\
&\qquad\longrightarrow \langle P'\,;\,\langle P\,;\,b?\rangle\rangle\\
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\end{aligned}$$

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for $\underline{\mathrm{And}} \triangleq \lambda x.\lambda y.((\lambda z.\ \texttt{if}\ x\star\ \texttt{then}\ \langle z\,;\ y\star\rangle\ \texttt{else}\ \mathsf{false})\mathtt{pass})$

$$C[P]\Downarrow \quad \textit{iff} \quad P \xrightarrow{a} \wedge P \xrightarrow{b}$$

$$\begin{aligned}
&\langle P\,;(\lambda x.\ \texttt{if}\ a?\ \texttt{then}\ \langle x\,;\ b?\rangle\ \texttt{else}\ \mathsf{false})\mathtt{pass}\rangle\\
&\qquad\longrightarrow \langle P\,;(\lambda x.\,\texttt{if}\ a?\ \texttt{then}\ \langle x\,;\ b?\rangle\ \texttt{else}\ \mathsf{false})P\rangle\\
&\qquad\longrightarrow \langle P\,;\ \texttt{if}\ a?\ \texttt{then}\ \langle P\,;\ b?\rangle\ \texttt{else}\ \mathsf{false}\rangle\\
&\qquad\longrightarrow \langle P'\,;\ \texttt{if}\ \mathsf{true}\ \texttt{then}\ \langle P\,;\ b?\rangle\ \texttt{else}\ \mathsf{false}\rangle\\
&\qquad\longrightarrow \langle P'\,;\langle P\,;\ b?\rangle\rangle\\
&\qquad\longrightarrow \langle P\,;\ b?\rangle\Downarrow
\end{aligned}$$

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P passes both T_1 and T_2

$$\langle P\,;\ \underline{\mathrm{And}}\,(\lambda.T_1)(\lambda.T_2)\rangle \Downarrow \quad \textit{iff} \quad \langle P\,;\ T_1\rangle \Downarrow \wedge \langle P\,;\ T_2\rangle \Downarrow$$

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$$C[P]\Downarrow \quad \textit{iff} \quad P \xrightarrow{a} \wedge P \xrightarrow{b}$$

$$\begin{aligned}
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&\qquad\longrightarrow \langle P\,;\ \texttt{if}\ a?\ \texttt{then}\ \langle P\,;\,b?\rangle\ \texttt{else}\ \mathsf{false}\rangle\\
&\qquad\longrightarrow \langle P'\,;\ \texttt{if}\ \mathsf{true}\ \texttt{then}\ \langle P\,;\,b?\rangle\ \texttt{else}\ \mathsf{false}\rangle\\
&\qquad\longrightarrow \langle P'\,;\,\langle P\,;\,b?\rangle\rangle\\
&\qquad\longrightarrow \langle P\,;\,b?\rangle\Downarrow
\end{aligned}$$

Example: discriminating LTSs

$$\begin{tikzcd}
& P \arrow[dl, "a"'] \arrow[dr, "a"] & \\
P_1 \arrow[d, "b"'] & & P_2 \arrow[d, "b"] \\
P_3 \arrow[d, "c"'] & & P_4 \arrow[d, "d"] \\
P_5 & & P_6
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
& Q \arrow[d, "a"'] & \\
& Q_1 \arrow[dl, "b"'] \arrow[dr, "b"] & \\
Q_2 \arrow[d, "c"'] & & Q_3 \arrow[d, "d"] \\
Q_4 & & Q_5
\end{tikzcd}$$

$C[P] \not\Downarrow$ $\qquad\qquad$ $C[Q] \Downarrow$

$$C = \langle \cdot \; ; \; \underline{\mathrm{Seq}}\ \mathrm{T}_a\, (\,\underline{\mathrm{And}}\,(\,\underline{\mathrm{Seq}}\ \mathrm{T}_b\ \mathrm{T}_c\,)(\,\underline{\mathrm{Seq}}\ \mathrm{T}_b\ \mathrm{T}_d\,))\rangle$$

ready trace equivalent but not simulation equivalent processes

Passivation in HOπ

$$\forall P, C[P] \Downarrow \quad \textit{iff} \quad P \xrightarrow{a} P' \text{ s.t. } \; P' \xrightarrow{b} \; \wedge P' \xrightarrow{c}$$

$$P \xrightarrow{a} P' \wedge P' \xrightarrow{b} P'' \wedge P' \xrightarrow{c} P'''$$

$$[\![\cdot]\!]_I \mid \overline{a}.\mathtt{pass}_I(x).([\![x]\!]_I \mid \overline{b}.\mathtt{pass}_I(y).([\![x]\!]_I \mid \overline{c}.\omega))$$

$$\begin{aligned}
&[\![P]\!]_I \mid \overline{a}.\, \mathtt{pass}_I(x).([\![x]\!]_I \mid \overline{b}.\mathtt{pass}_I(y).([\![x]\!]_I \mid \overline{c}.\omega)) \\
&\qquad \xrightarrow{\tau} [\![P']\!]_I \mid \mathtt{pass}_I(x).([\![x]\!]_I \mid \overline{b}.\mathtt{pass}_I(y).([\![x]\!]_I \mid \overline{c}.\omega)) \\
&\qquad \xrightarrow{\tau} \mathbf{0} \mid [\![P']\!]_I \mid \overline{b}.\mathtt{pass}_I(y).([\![P']\!]_I \mid \overline{c}.\omega) \\
&\qquad \xrightarrow{\tau} \mathbf{0} \mid [\![P'']\!]_I \mid \mathtt{pass}_I(y).([\![P']\!]_I \mid \overline{c}.\omega) \\
&\qquad \xrightarrow{\tau} \mathbf{0} \mid \mathbf{0} \mid [\![P']\!]_I \mid \overline{c}.\omega \\
&\qquad \xrightarrow{\tau} \mathbf{0} \mid \mathbf{0} \mid [\![P''']\!]_I \mid \omega
\end{aligned}$$

impossible to mimic in HOπ without passivation

Passivation in HOπ

$$\forall P, C[P] \Downarrow \quad \textit{iff} \quad P \xrightarrow{a} P' \text{ s.t. } \quad P' \xrightarrow{b} \wedge P' \xrightarrow{c}$$

$$P \xrightarrow{a} P' \wedge P' \xrightarrow{b} P'' \wedge P' \xrightarrow{c} P'''$$

$$[\![\,\cdot\,]\!]_l \mid \overline{a}.\mathtt{pass}_l(x).([\![x]\!]_l \mid \overline{b}.\mathtt{pass}_l(y).([\![x]\!]_l \mid \overline{c}.\omega))$$

$$\begin{aligned}
{\color{red}[\![P]\!]_l} \mid {\color{red}\overline{a}}.\ & \mathtt{pass}_l(x).([\![x]\!]_l \mid \overline{b}.\mathtt{pass}_l(y).([\![x]\!]_l \mid \overline{c}.\omega)) \\
& \xrightarrow{\tau} [\![P']\!]_l \mid \mathtt{pass}_l(x).([\![x]\!]_l \mid \overline{b}.\mathtt{pass}_l(y).([\![x]\!]_l \mid \overline{c}.\omega)) \\
& \xrightarrow{\tau} \mathbf{0} \mid [\![P']\!]_l \mid \overline{b}.\mathtt{pass}_l(y).([\![P']\!]_l \mid \overline{c}.\omega) \\
& \xrightarrow{\tau} \mathbf{0} \mid [\![P'']\!]_l \mid \mathtt{pass}_l(y).([\![P']\!]_l \mid \overline{c}.\omega) \\
& \xrightarrow{\tau} \mathbf{0} \mid \mathbf{0} \mid [\![P']\!]_l \mid \overline{c}.\omega \\
& \xrightarrow{\tau} \mathbf{0} \mid \mathbf{0} \mid [\![P''']\!]_l \mid \omega
\end{aligned}$$

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Passivation in HOπ

$$\forall P, C[P] \Downarrow \quad \textit{iff} \quad P \xrightarrow{a} P' \text{ s.t. } \; P' \xrightarrow{b} \wedge P' \xrightarrow{c}$$

$$P \xrightarrow{a} P' \wedge P' \xrightarrow{b} P'' \wedge P' \xrightarrow{c} P'''$$

$$[\![\cdot]\!]_I \mid \overline{a}.\mathtt{pass}_I(x).([\![x]\!]_I \mid \overline{b}.\mathtt{pass}_I(y).([\![x]\!]_I \mid \overline{c}.\omega))$$

$$\begin{aligned} & [\![P]\!]_I \mid \overline{a}.\, \mathtt{pass}_I(x).([\![x]\!]_I \mid \overline{b}.\mathtt{pass}_I(y).([\![x]\!]_I \mid \overline{c}.\omega)) \\ & \xrightarrow{\tau} {\color{red}[\![P']\!]_I} \mid {\color{red}\mathtt{pass}_I(x)}.([\![{\color{red}x}]\!]_I \mid \overline{b}.\mathtt{pass}_I(y).([\![{\color{red}x}]\!]_I \mid \overline{c}.\omega)) \\ & \xrightarrow{\tau} \mathbf{0} \mid [\![P']\!]_I \mid \overline{b}.\mathtt{pass}_I(y).([\![P']\!]_I \mid \overline{c}.\omega) \\ & \xrightarrow{\tau} \mathbf{0} \mid [\![P'']\!]_I \mid \mathtt{pass}_I(y).([\![P']\!]_I \mid \overline{c}.\omega) \\ & \xrightarrow{\tau} \mathbf{0} \mid \mathbf{0} \mid [\![P']\!]_I \mid \overline{c}.\omega \\ & \xrightarrow{\tau} \mathbf{0} \mid \mathbf{0} \mid [\![P''']\!]_I \mid \omega \end{aligned}$$

impossible to mimic in HOπ without passivation

Passivation in HOπ

$$\forall P, C[P] \Downarrow \quad \textit{iff} \quad P \xrightarrow{a} P' \text{ s.t. } \quad P' \xrightarrow{b} \wedge P' \xrightarrow{c}$$

$P \xrightarrow{a} P' \wedge P' \xrightarrow{b} P'' \wedge P' \xrightarrow{c} P'''$

$[\![\cdot]\!]_I \mid \overline{a}.\mathrm{pass}_I(x).([\![x]\!]_I \mid \overline{b}.\mathrm{pass}_I(y).([\![x]\!]_I \mid \overline{c}.\omega))$

$$\begin{aligned} & [\![P]\!]_I \mid \overline{a}.\,\mathrm{pass}_I(x).([\![x]\!]_I \mid \overline{b}.\mathrm{pass}_I(y).([\![x]\!]_I \mid \overline{c}.\omega)) \\ & \quad \xrightarrow{\tau} [\![P']\!]_I \mid \mathrm{pass}_I(x).([\![x]\!]_I \mid \overline{b}.\mathrm{pass}_I(y).([\![x]\!]_I \mid \overline{c}.\omega)) \\ & \quad \xrightarrow{\tau} \mathbf{0} \mid [\![P']\!]_I \mid \overline{b}.\mathrm{pass}_I(y).([\![P']\!]_I \mid \overline{c}.\omega) \\ & \quad \xrightarrow{\tau} \mathbf{0} \mid [\![P'']\!]_I \mid \mathrm{pass}_I(y).([\![P']\!]_I \mid \overline{c}.\omega) \\ & \quad \xrightarrow{\tau} \mathbf{0} \mid \mathbf{0} \mid [\![P']\!]_I \mid \overline{c}.\omega \\ & \quad \xrightarrow{\tau} \mathbf{0} \mid \mathbf{0} \mid [\![P''']\!]_I \mid \omega \end{aligned}$$

impossible to mimic in HOπ without passivation

Passivation in HOπ

$$\forall P, C[P] \Downarrow \quad \textit{iff} \quad P \xrightarrow{a} P' \text{ s.t. } \; P' \xrightarrow{b} \wedge P' \xrightarrow{c}$$

$P \xrightarrow{a} P' \wedge P' \xrightarrow{b} P'' \wedge P' \xrightarrow{c} P'''$

$[\![\cdot]\!]_l \mid \overline{a}.\mathtt{pass}_l(x).([\![x]\!]_l \mid \overline{b}.\mathtt{pass}_l(y).([\![x]\!]_l \mid \overline{c}.\omega))$

$$\begin{aligned}
&[\![P]\!]_l \mid \overline{a}.\ \mathtt{pass}_l(x).([\![x]\!]_l \mid \overline{b}.\mathtt{pass}_l(y).([\![x]\!]_l \mid \overline{c}.\omega)) \\
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&\quad \xrightarrow{\tau} \mathbf{0} \mid [\![P']\!]_l \mid \overline{b}.\mathtt{pass}_l(y).([\![P']\!]_l \mid \overline{c}.\omega) \\
&\quad \xrightarrow{\tau} \mathbf{0} \mid [\![P'']\!]_l \mid \mathtt{pass}_l(y).([\![P']\!]_l \mid \overline{c}.\omega) \\
&\quad \xrightarrow{\tau} \mathbf{0} \mid \mathbf{0} \mid [\![P']\!]_l \mid \overline{c}.\omega \\
&\quad \xrightarrow{\tau} \mathbf{0} \mid \mathbf{0} \mid [\![P''']\!]_l \mid \omega
\end{aligned}$$

impossible to mimic in HOπ without passivation

Passivation in HOπ

$$\forall P, C[P] \Downarrow \quad \textit{iff} \quad P \xrightarrow{a} P' \text{ s.t. } \; P' \xrightarrow{b} \; \wedge P' \xrightarrow{c}$$

$P \xrightarrow{a} P' \wedge P' \xrightarrow{b} P'' \wedge P' \xrightarrow{c} P'''$

$[\![\cdot]\!]_I \mid \overline{a}.\mathtt{pass}_I(x).([\![x]\!]_I \mid \overline{b}.\mathtt{pass}_I(y).([\![x]\!]_I \mid \overline{c}.\omega))$

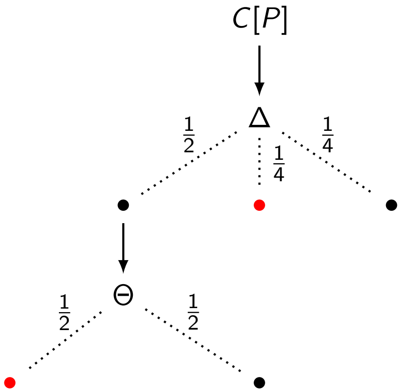
$$\begin{aligned}
&[\![P]\!]_I \mid \overline{a}.\ \mathtt{pass}_I(x).([\![x]\!]_I \mid \overline{b}.\mathtt{pass}_I(y).([\![x]\!]_I \mid \overline{c}.\omega)) \\
&\quad \xrightarrow{\tau} [\![P']\!]_I \mid \mathtt{pass}_I(x).([\![x]\!]_I \mid \overline{b}.\mathtt{pass}_I(y).([\![x]\!]_I \mid \overline{c}.\omega)) \\
&\quad \xrightarrow{\tau} \mathbf{0} \mid [\![P']\!]_I \mid \overline{b}.\mathtt{pass}_I(y).([\![P']\!]_I \mid \overline{c}.\omega) \\
&\quad \xrightarrow{\tau} \mathbf{0} \mid [\![P'']\!]_I \mid \mathtt{pass}_I(y).([\![P']\!]_I \mid \overline{c}.\omega) \\
&\quad \xrightarrow{\tau} \mathbf{0} \mid \mathbf{0} \mid [\![P']\!]_I \mid \overline{c}.\omega \\
&\quad \xrightarrow{\tau} \mathbf{0} \mid \mathbf{0} \mid [\![P''']\!]_I \mid \omega
\end{aligned}$$

impossible to mimic in HOπ without passivation

Testing reactive probabilistic processes

'Success' on reactive probabilistic processes

$P \simeq_{\mathcal{L}} Q \quad \triangleq \quad C[P]$ and $C[Q]$ 'equally successful', $\forall$ contexts C of $\mathcal{L}$



P is an RPLTS, $C[P] \Downarrow_{\frac{1}{2}}$

Success states • are:

$M \xrightarrow{\omega}$ in CCS, HOπ

$\langle N\, ;\, \mathrm{true} \rangle$ in $\mathtt{K}\Lambda$

Success is $\Downarrow_p$, where $p =$ sum of the probabilities of all success paths

The Spectrum on RPLTSs

$$
\begin{tikzcd}[column sep=small]
 & \text{prob. bisimilarity } \mathtt{K}\Lambda_{\mathrm{V-ref}} = \mathtt{K}\Lambda_{\mathrm{V}} = \mathsf{HO}\pi_{\mathtt{pass}} = \mathsf{HO}\pi_{\mathrm{ref},\mathtt{pass}} \arrow[d] & \\
 & \mathsf{HO}\pi_{\mathrm{ref}} \arrow[dl] \arrow[dr] & \\
\mathsf{HO}\pi \arrow[dr] & & \mathsf{CCS}_{\mathrm{ref}} \arrow[dl] \\
 & \mathsf{CCS} \arrow[d] & \\
 & \text{prob. failure trace equivalence } \mathtt{K}\Lambda_{\mathrm{N}} \arrow[d] & \\
 & \text{prob. trace equivalence } \mathtt{K}\Lambda_{\mathrm{N-ref}} &
\end{tikzcd}
$$

- CBV = passivation in HOπ $\pm$ref
- sequential $\neq$ concurrent
- first-order communication $\neq$ higher-order communication

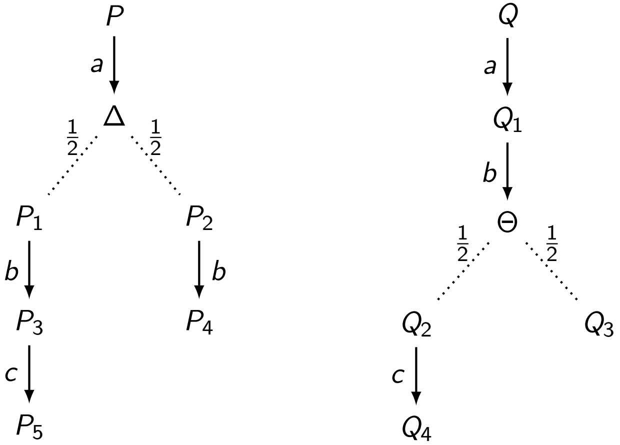
Testing LTSs: refusal and conjunction

$$
\begin{tikzcd}
 & P \arrow[dl, "a"'] \arrow[dr, "a"] & \\
P_1 \arrow[d, "b"'] & & P_2 \arrow[d, "b"] \\
P_3 \arrow[d, "c"'] & & P_4 \\
P_5 & &
\end{tikzcd}
\qquad\qquad
\begin{tikzcd}
 & Q \arrow[d, "a"'] & \\
 & Q_1 \arrow[dl, "b"'] \arrow[dr, "b"] & \\
Q_2 \arrow[d, "c"'] & & Q_3 \\
Q_4 & &
\end{tikzcd}
$$

$C[P] \not\Downarrow$ $\qquad\qquad$ $C[Q] \Downarrow$

$$C = \langle \cdot \; ; \; \underline{\mathrm{Seq}}\ \mathrm{T}_a\,(\,\underline{\mathrm{And}}\,(\,\underline{\mathrm{Seq}}\ \mathrm{T}_b\ \mathrm{T}_c\,)(\,\underline{\mathrm{Seq}}\ \mathrm{T}_b\ \mathrm{T}_{\neg c}\,))\rangle$$

Testing RPLTSs: refusal and conjunction

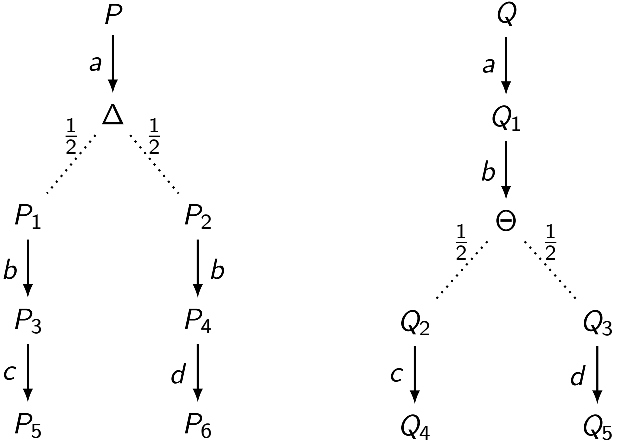


$C[P] \Downarrow_{\frac{1}{2}}$ $C[Q] \Downarrow_{\frac{1}{4}}$

$$C = \langle \cdot \; ; \; \underline{\mathrm{Seq}}\ \mathrm{T}_a \,(\, \underline{\mathrm{And}}\,(\, \underline{\mathrm{Seq}}\ \mathrm{T}_b\ \mathrm{T}_c \,)(\, \underline{\mathrm{Seq}}\ \mathrm{T}_b\ \mathrm{T}_c \,))\rangle$$

Testing RPLTSs: sequential vs. concurrent tests

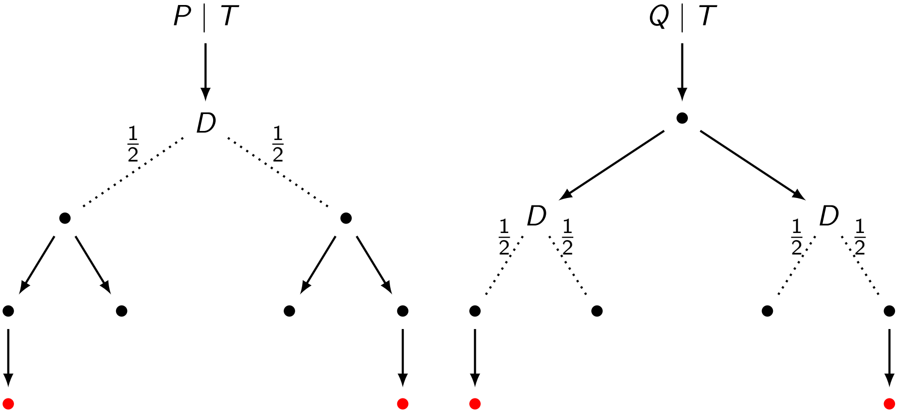
Internal nondeterminism of the tests affects the discriminating power (P and Q are equivalent in call-by-name $\mathtt{K}\Lambda$)



$$T = \overline{a}.(\overline{b}.\overline{c}.\omega + \overline{b}.\overline{d}.\omega)$$

Testing RPLTSs: sequential vs. concurrent tests

Internal nondeterminism of the tests affects the discriminating power
(P and Q are equivalent in call-by-name $\mathtt{K}\Lambda$)

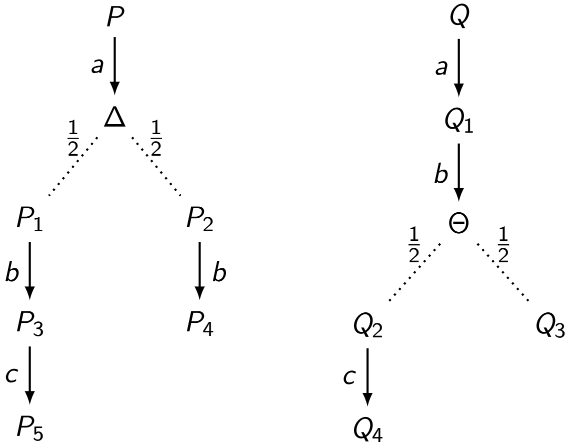


$P \mid T \Downarrow_1$ $Q \mid T \Downarrow_{\frac{1}{2}}$

Testing RPLTSs: CCS vs. HOπ

Probabilistic choices in the test affect the discriminating power

Tests in Hoπ can copy $P \Rightarrow$ tests can mimic probabilistic choices



$C = \overline{h}\langle \cdot \rangle \mid h(x).(x \mid \overline{d}.\overline{a}.(\overline{b}.\overline{c}.\omega \mid \overline{b}.(x \mid \overline{e}) \mid \overline{f}.\omega))$

Testing RPLTSs: CCS vs. HOπ

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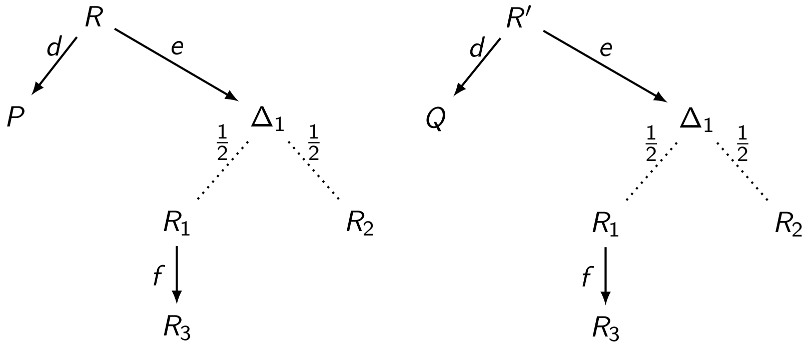

$C = \overline{h}\langle \cdot \rangle \mid h(x).(x \mid \overline{d}.\overline{a}.(\overline{b}.\overline{c}.\omega \mid \overline{b}.(x \mid \overline{e}) \mid \overline{f}.\omega))$

Related works

- rule formats for LTSs
  - coarsest congruences contained in trace equivalence
  - GSOS, tyft/tyxt
- rule formats for probabilistic processes
  - emphasis on ensuring congruence properties for bisimilarity
- [Larsen and Skou 1989]
  [van Breugel, Mislove, Ouaknine and Worrell 2005]
  - testers allowed to make copies so as to recover bisimilarity
  - our characterizations of bisimilarity exploit these results
- [Deng, van Glabbeek, Hennessy, Morgan and Zhang 2007]
  [Deng, van Glabbeek, Hennessy and Morgan 2008,2009]
  - tests and tested processes have both probabilities and nondeterminism
  - simulation-like equivalences

Future work

- tested f.o. processes with both probabilities and nondeterminism
- missing direct characterization in the spectrum of RPLTSs
- tests with probabilistic constructs

Future work

- tested f.o. processes with both probabilities and nondeterminism
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Thank you!

Backup Slides

Ready simulation equivalence

A binary relation $\mathcal{R}$ on processes is a ready simulation if:

$$
\begin{tikzcd} P \arrow[d, "a"] & \mathcal{R} & Q \\ P' & & \end{tikzcd}
\Rightarrow
\begin{tikzcd} P \arrow[d, "a"] & \mathcal{R} & Q \arrow[d, "a"] \\ P' & \mathcal{R} & Q' \end{tikzcd}
\quad \& \quad
\begin{tikzcd} P \arrow[d, "/" marking, "a"] & \mathcal{R} & Q \\ {} & & \end{tikzcd}
\Rightarrow
\begin{tikzcd} P \arrow[d, "/" marking, "a"] & \mathcal{R} & Q \arrow[d, "/" marking, "a"] \\ {} & & {} \end{tikzcd}
$$

Probabilistic bisimilarity

A binary relation $\mathcal{R}$ on probabilistic processes is a probabilistic bisimulation if:

$$\begin{array}{ccc} P & \mathcal{R} & Q \\ \big\downarrow a & & \\ \Delta & & \end{array} \Rightarrow \begin{array}{ccc} P & \mathcal{R} & Q \\ \big\downarrow a & & \big\downarrow a \\ \Delta & \overline{\mathcal{R}} & \Theta \end{array} \quad \& \quad \begin{array}{ccc} P & \mathcal{R} & Q \\ & & \big\downarrow a \\ & & \Theta \end{array} \Rightarrow \begin{array}{ccc} P & \mathcal{R} & Q \\ \big\downarrow a & & \big\downarrow a \\ \Delta & \overline{\mathcal{R}} & \Theta \end{array}$$

$\Delta \overline{\mathcal{R}} \Theta$ whenever there is a finite index set I such that:

- $\Delta = \sum_{i \in I} p_i \cdot \overline{P_i}$ and $\sum_{i \in I} p_i = 1$,
- for every $i \in I$ there is a probabilistic process Q_i such that $P_i \ \mathcal{R} \ Q_i$,
- $\Theta = \sum_{i \in I} p_i \cdot \overline{Q_i}$.

Testing Languages

$r \;::=\; a \mid \overline{a}$ (input/output channels)

$$
\begin{array}{llll}
M ::= & x & \text{(variables)} & \mid\ \texttt{if}\ M_1\ \texttt{then}\ M_2\ \texttt{else}\ M_3 \quad \text{(if-then-else)} \\
\mid & \lambda x.M & \text{(functions)} & \mid\ r? \quad \text{(action test)} \\
\mid & M_1 M_2 & \text{(applications)} & \mid\ \texttt{pass} \quad \text{(passivation)} \\
\mid & c & \text{(constants)} & \mid\ \langle M_1 \,;\, M_2 \rangle \quad \text{(kell creation)} \\
 & & & \mid\ P \quad \text{(f.o. processes)}
\end{array}
$$

$$
\begin{array}{lll}
\alpha & ::= & \omega \;\Big|\; r \;\Big|\; \widetilde{r}_l \qquad \text{(prefixes)} \\
M & ::= & \mathbf{0} \;\Big|\; \alpha.M \;\Big|\; M + M \\
 & & \;\Big|\; M \mid M \;\Big|\; (\nu a)M \;\Big|\; [\![M]\!]_l \;\Big|\; P \qquad \text{(processes)}
\end{array}
$$

$$
\begin{array}{lll}
\alpha & ::= & a(x) \;\Big|\; \overline{a}\langle M\rangle \;\Big|\; \omega \;\Big|\; \mathtt{pass}_l(x) \;\Big|\; \widetilde{r}_l \qquad \text{(prefixes)} \\
M & ::= & \mathbf{0} \;\Big|\; x \;\Big|\; \star \;\Big|\; \alpha.M \\
 & & \;\Big|\; M \mid M \;\Big|\; (\nu a)M \;\Big|\; [\![M]\!]_l \;\Big|\; P \qquad \text{(processes and values)}
\end{array}
$$

Global vs. Local communication

Global communication: $[\![a]\!]_{l_1} \mid \overline{a} \mid [\![[\![a]\!]_{l_1}]\!]_{l_2}$

Local communication: $[\![a]\!]_{l_1} \mid \overline{a}^{l_1} \mid [\![[\![a]\!]_{l_1}]\!]_{l_2}$

- local communication can encode disjunction on LTSs

$$[\![P]\!]_l \mid \mathtt{pass}_l(x).([\![x]\!]_{l_1} \mid \overline{a}^{l_1}.\omega \mid [\![x]\!]_{l_2} \mid \overline{b}^{l_2}.\omega)$$

- the spectrum does not change

May vs. Must equivalences

On LTSs:

- in $\mathrm{K}\Lambda_{\mathrm{V}}$ and $\mathrm{HO}\pi_{\mathrm{ref},\mathtt{pass}}$, may = ready simulation equivalence = must

On RPLTS:

- in λ-calculi, may = must
- in $\mathrm{HO}\pi_{\mathtt{pass}}$ and $\mathrm{HO}\pi_{\mathrm{ref},\mathtt{pass}}$, may = bisimilarity = must