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Parallel Pushdown Automata and Commutative Context-Free Grammars in Bisimulation Semantics

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OPCT, Bertinoro, 27 June 2023

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Draft paper

- Parallel Pushdown Automata and Commutative Context-Free Grammars in Bisimulation Semantics
- with Bas Luttik (Eindhoven University of Technology)
- to be submitted to EXPRESS/SOS 2023, Antwerp

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What is a computation?

- Church-Turing Thesis
- Given by a Turing machine: input at begin, deterministic steps, output at end
- A computation is a function
- Models a deaf, dumb and blind computer (before advent of terminal)
- Far removed from a modern-day computer as students see all around them

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Reactive Systems

“A Turing machine cannot drive a car, but a real computer can!”

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Interaction

User interaction: not just initial, final word on the tape.

Make interaction between control and memory explicit.

Integration of automata theory and process theory.

Aim to develop course on foundations of computer science for all first-year computer science students.
Syllabus: Models of Computation based on Automata: Formal Languages and Communicating Processes

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Reactive Turing Machine

Defined in I&C 2013 with Bas Luttik and Paul van Tilburg

Executability instead of computability, but not more expressive

Robustness: π-calculus (Robin Milner) is the λ-calculus with interaction (with Bas Luttik and Fei Yang)

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Pushdown Automata and Context-Free Grammars in Bisimulation Semantics

- with Cesare Carissimo and Bas Luttik
- arXiv:2203.01713
- LMCS 2023

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Well-known theorem

A language is defined by a pushdown automaton iff it is defined by a context-free grammar.

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A process is defined by a pushdown automaton iff it is defined by a finite guarded recursive specification over a process algebra with actions, choice and sequencing including sequential value passing.

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Well-known theorem

A language is defined by a pushdown automaton iff it is defined by a context-free grammar.

A process is defined by a pushdown automaton iff it is defined by a finite guarded recursive specification over a process algebra with actions, choice and sequencing including sequential value passing.

What if we replace the stack of the pda by a bag, and value passing sequencing by parallel composition with value passing communication?

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Process

Labelled transition system $(\mathcal{S}, \mathcal{A}, \rightarrow, \downarrow)$

1. $\mathcal{S}$ is a set of *states*;
2. $\mathcal{A}$ is a set of *actions*, $\tau \notin \mathcal{A}$ is the *unobservable* or *silent* action;
3. $\rightarrow \subseteq \mathcal{S} \times \mathcal{A} \cup \{\tau\} \times \mathcal{S}$ is a *transition relation*
4. $\downarrow \subseteq \mathcal{S}$ is the set of *accepting* states

A *process graph* is a labelled transition system with a root state $\uparrow$.
Strong bisimulation $\leftrightarroweq$, branching bisimulation $\leftrightarroweq_{\mathsf{b}}$,
divergence-preserving branching bisimulation $\leftrightarroweq_{\mathsf{b}}^{\Delta}$.
A *process* is a $\leftrightarroweq_{\mathsf{b}}^{\Delta}$-equivalence class of process graphs.

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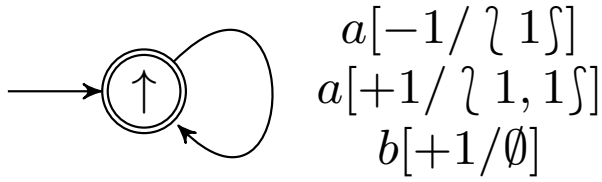
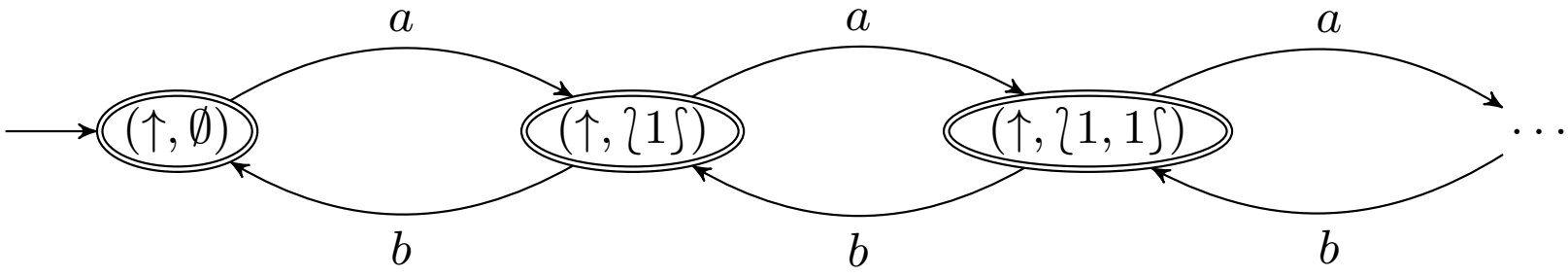
Parallel Pushdown Automaton





Insert and remove. Bounded branching under $\leftrightarrows$. Acceptance by final state can express more than acceptance by empty bag.

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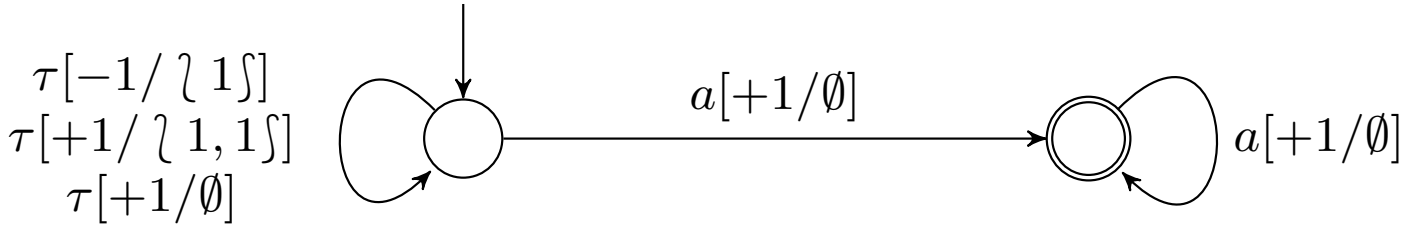
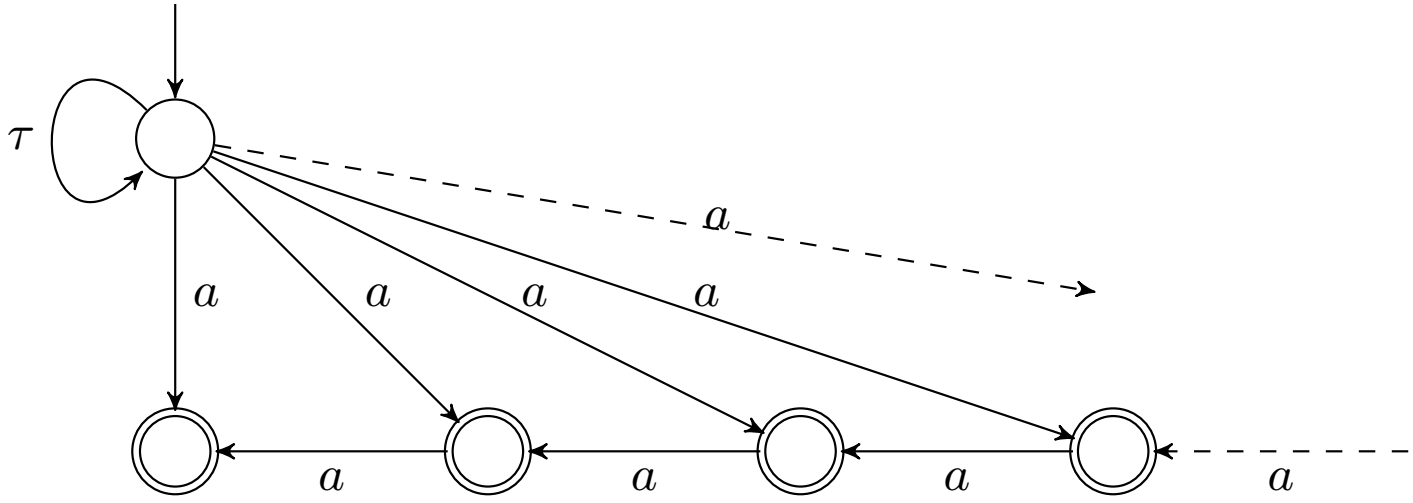
Bounded branching?





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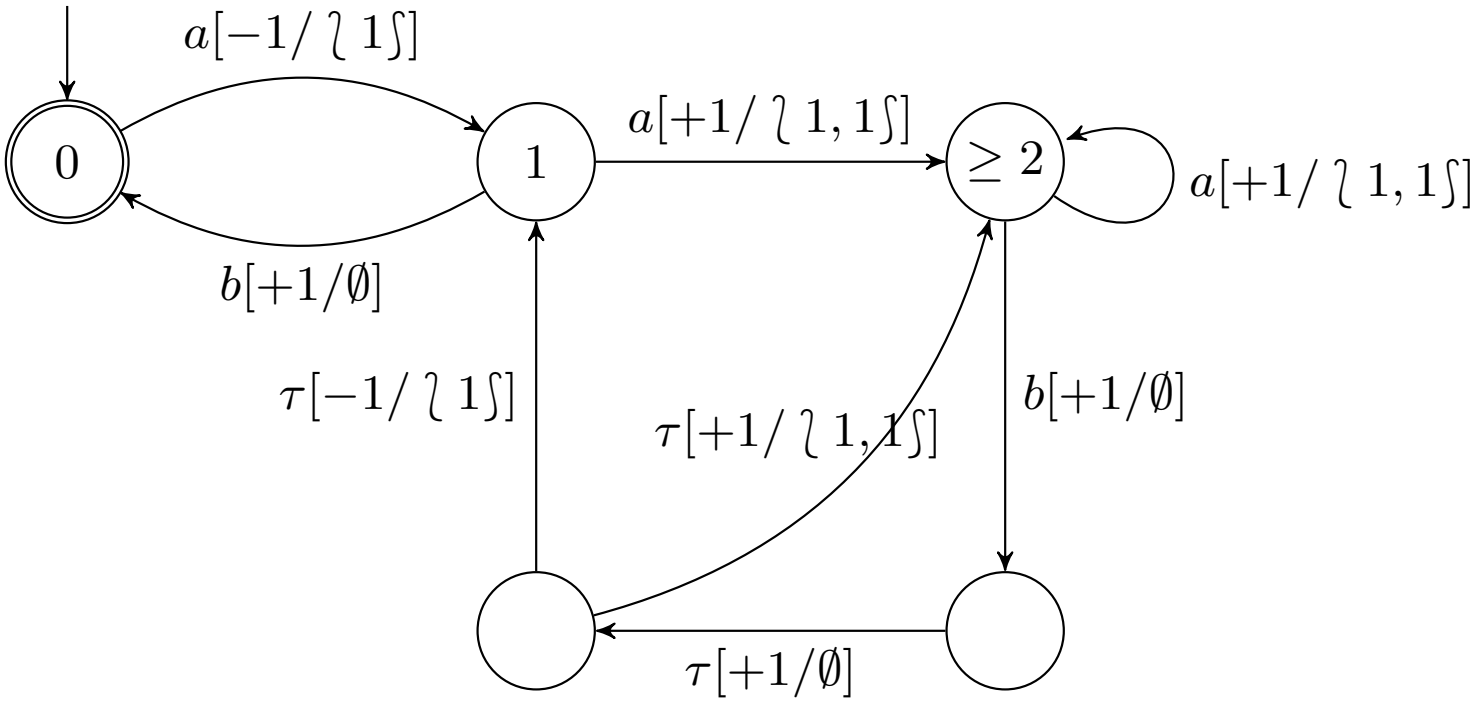
Acceptance by empty bag.



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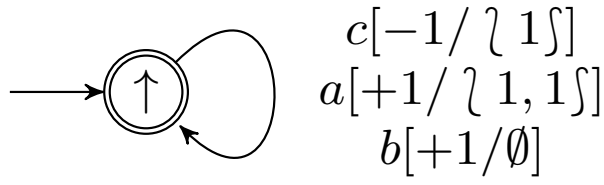
Counter slightly changed.



The c can be executed only when no b can be executed any longer.
Executing a b takes priority to executing a c.

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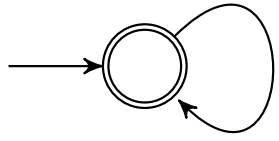
Bag always accepting.



$ins(d)[-d/\wr\, d\int]$
$ins(d)[+d/\wr\, d, d\int]$
$rem(d)[+d/\emptyset]$
$show(^{-}d)[-d/\emptyset]$

Again, a $[-d/x]$ transition can only occur when a $[+d/y]$ transition can no longer occur. The remove transition should take priority over the show absence transition.

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Basic Parallel Processes

Use SOS to give automata for syntax $\mathbf{0}, \mathbf{1}, a., +, \|$

$$\frac{}{\mathbf{1}\downarrow} \qquad \frac{}{a.p \xrightarrow{a} p}$$

$$\frac{p\downarrow}{(p+q)\downarrow} \qquad \frac{q\downarrow}{(p+q)\downarrow} \qquad \frac{p \xrightarrow{a} p'}{p+q \xrightarrow{a} p'} \qquad \frac{q \xrightarrow{a} q'}{p+q \xrightarrow{a} q'}$$

$$\frac{p\downarrow \quad q\downarrow}{p \parallel q\downarrow} \qquad \frac{p \xrightarrow{a} p'}{p \parallel q \xrightarrow{a} p' \parallel q} \qquad \frac{q \xrightarrow{a} q'}{p \parallel q \xrightarrow{a} p \parallel q'}$$

$$\frac{p \xrightarrow{a} p' \quad X \stackrel{\text{def}}{=} p}{X \xrightarrow{a} p'} \qquad \frac{p\downarrow \quad X \stackrel{\text{def}}{=} p}{X\downarrow}$$

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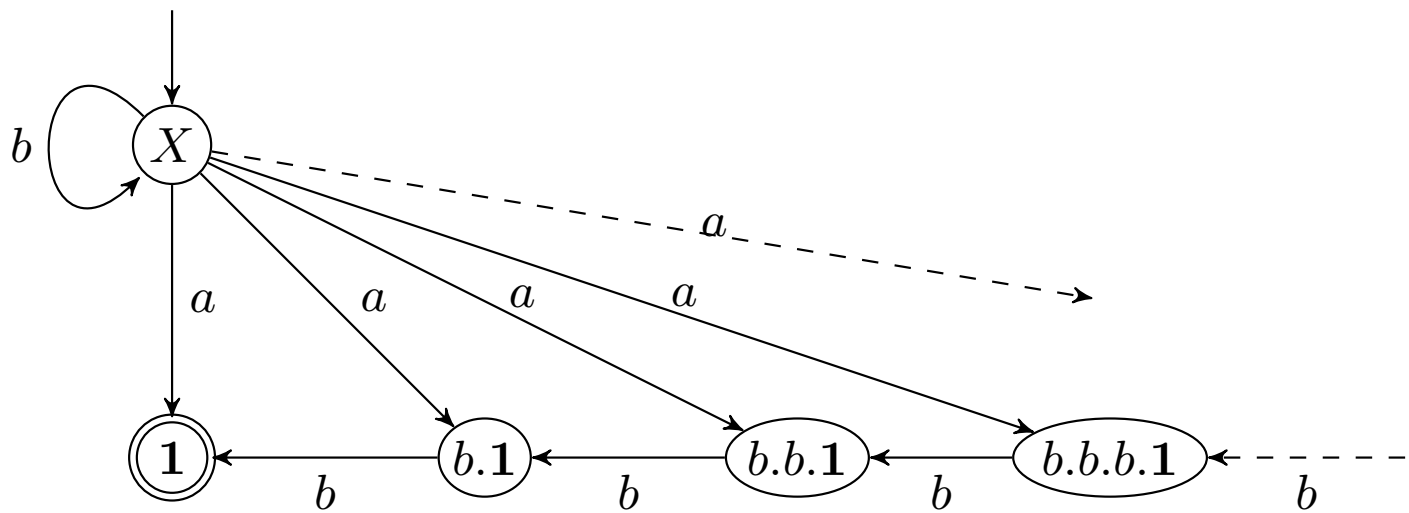
Guardedness necessary

$$X \stackrel{\text{def}}{=} a.\mathbf{1} + X \parallel b.\mathbf{1} \ .$$



Not divergence-preserving branching bisimilar to the process graph of a parallel pushdown automaton

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BPP to CCFG

Theorem. Every weakly guarded recursive specification has a process graph that is divergence-preserving branching bisimilar to the process graph of a parallel pushdown automaton.

Proof. Every process expression p can be brought into basic parallel normal form $p \leftrightarrows (\mathbf{1}+) \sum_{i=1}^{n} a_i.p_i$, using

$$(\sum_{i=1}^{n} a_i.p_i) \parallel (\sum_{j=1}^{m} b_j.q_j) \leftrightarrows \sum_{i=1}^{n} a_i.(p_i \parallel q) + \sum_{j=1}^{m} b_j.(p \parallel q_j)$$

A weakly guarded specification can be brought with $\leftrightarrows$ into Greibach Normal Form $X \stackrel{\text{def}}{=} (\mathbf{1}+) \sum_{i=1}^{n} a_i.\xi_i$ (ξ_i a parallel composition of identifiers).

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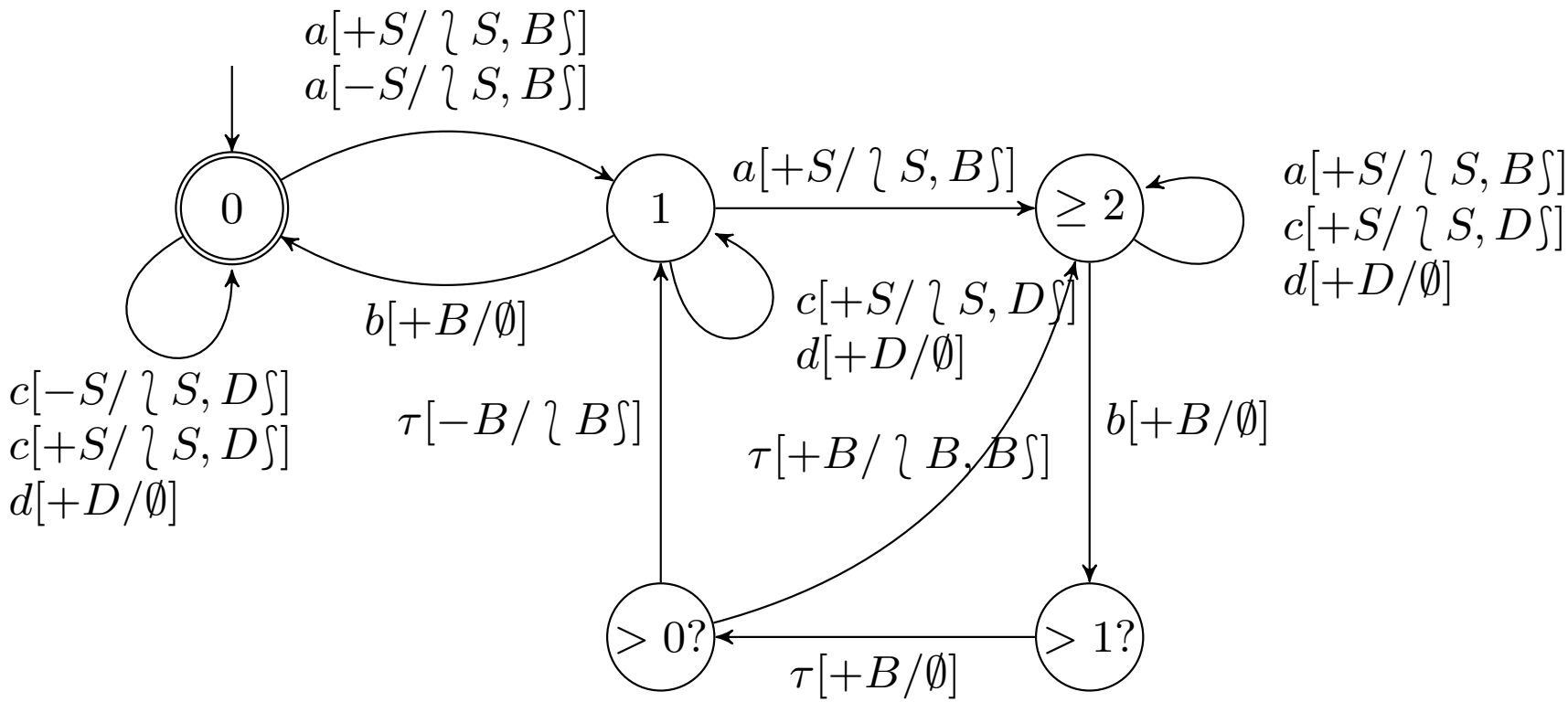
BPP to CCFG

$$S \stackrel{\text{def}}{=} \mathbf{1} + a.(S \parallel B) + c.(S \parallel D) \qquad B \stackrel{\text{def}}{=} b.\mathbf{1} \qquad D \stackrel{\text{def}}{=} \mathbf{1} + d.\mathbf{1}$$



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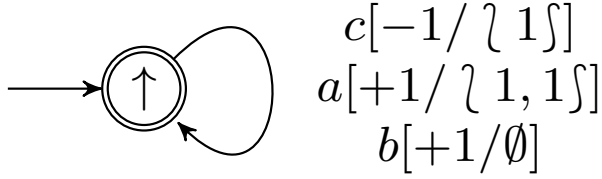
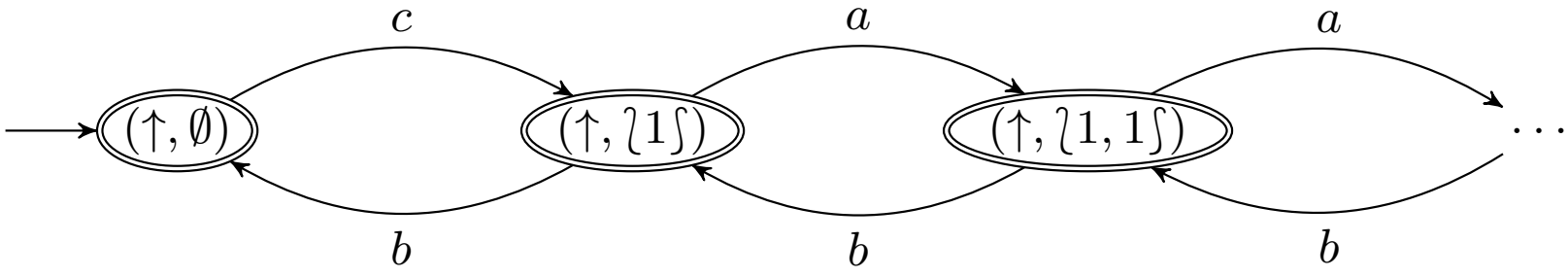
One-state counterexample





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Specification of the bag.

$$AB \stackrel{\text{def}}{=} \mathbf{1} + \sum_{d \in \mathcal{D}} ins(d).(AB \parallel (\mathbf{1} + rem(d).\mathbf{1})) \ .$$

$$ABag \stackrel{\text{def}}{=} \mathbf{1} + \sum_{d \in \mathcal{D}} ins(d).\theta(ABag \parallel (\mathbf{1} + rem(d).\mathbf{1})) +$$

$$+ \sum_{d \in \mathcal{D}} show(^{-}d).ABag \ .$$

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Priorities.

$$\frac{p\downarrow}{\theta(p)\downarrow} \qquad \frac{p \xrightarrow{a} p' \qquad \forall b > a \quad p \not\xrightarrow{b}}{\theta(p) \xrightarrow{a} \theta(p')}$$

$$\frac{p \xrightarrow{a} p'}{\rho_f(p) \xrightarrow{f(a)} \rho_f(p')} \qquad \frac{p\downarrow}{\rho_f(p)\downarrow}$$

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Theorem: For every one-state parallel pushdown automaton there is a finite weakly guarded $\mathrm{BPP}_{\theta}^{\mathbf{01}}$ specification such that their process graphs are divergence-preserving branching bisimilar.

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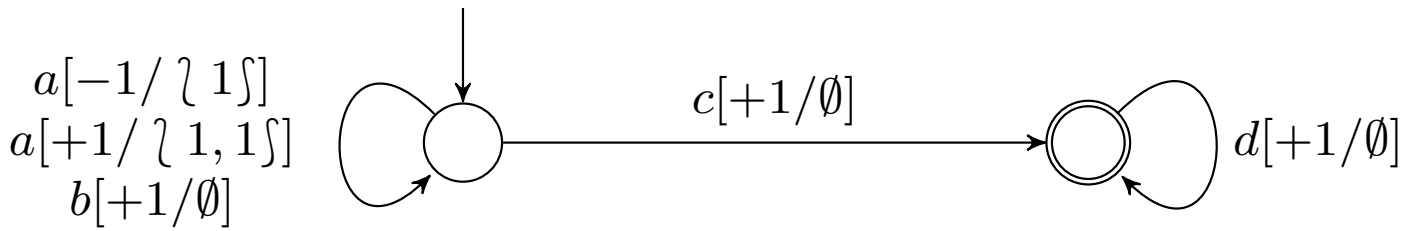
Not for two states.



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Basic Communicating Processes

$$\frac{p \xrightarrow{c!d} p' \qquad q \xrightarrow{c?d} q'}{p \parallel q \xrightarrow{c(d)} p' \parallel q' \quad q \parallel p \xrightarrow{c(d)} q' \parallel p'}$$

$$\frac{p\downarrow}{\partial_C(p)\downarrow} \qquad \frac{p \xrightarrow{a} p' \qquad a \notin COM_C}{\partial_C(p) \xrightarrow{a} \partial_C(p')}$$

$$\frac{p\downarrow}{\tau_C(p)\downarrow} \qquad \frac{p \xrightarrow{c(d)} p' \qquad c \in C}{\tau_C(p) \xrightarrow{\tau} \tau_C(p')} \qquad \frac{p \xrightarrow{a} p' \qquad a \neq c(d) \text{ for } c \in C}{\tau_C(p) \xrightarrow{a} \tau_C(p')}$$

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Communicating Bags

$$ABag^{io} \stackrel{\text{def}}{=} \mathbf{1} + \sum_{d \in \mathcal{D}} i?d.\theta(ABag^{io} \parallel (\mathbf{1} + o!(^{+}d).\mathbf{1})) + \sum_{d \in \mathcal{D}} o!(^{-}d).ABag^{io}$$

$$ABag^{io} \leftrightarrows_{\mathsf{b}}^{\Delta} \tau_{\{\ell\}}(\partial_{\{\ell\}}(ABag^{i\ell} \parallel ABag^{\ell o}))$$

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Correspondence

Theorem. For every parallel pushdown automaton there is a finite weakly guarded BCP-specification such that their process graphs are divergence-preserving branching bisimilar.

Proof. Again, we have parallel head normal form and Greibach normal form under $\leftrightarrows$

$$X \stackrel{\text{def}}{=} (\mathbf{1}+) \sum_{i=1}^{n} a_i.\tau_C(\partial_C(\theta(\xi_i))).$$

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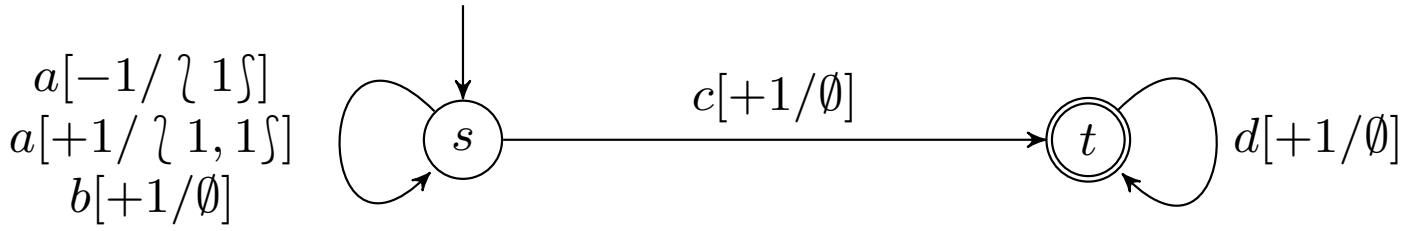
Two-state example



$$S \stackrel{\text{def}}{=} a^{-}.\tau_p(\partial_p(\theta(p!s.\mathbf{1} \parallel X_0 \parallel X_1)))$$

$$X_1 \stackrel{\text{def}}{=} p?s.((a^{+}.p!s.\mathbf{1} \parallel X_1 \parallel X_1)+(b.p!s.\mathbf{1})+(c.p!t.\mathbf{1}))+p?t.(\mathbf{1}+d.p!t.\mathbf{1})$$

$$X_0 \stackrel{\text{def}}{=} p?s.(a^{-}.p!s.\mathbf{1} \parallel X_1) + p?t.\mathbf{1}$$

Can do without priorities, but then we need a countable set of values and countable sum.

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Communication not too powerful

For every finite weakly guarded BCP-specification there is a parallel pushdown automaton such that their process graphs are divergence-preserving rooted branching bisimilar.

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A characterization

A process p is a parallel push-down process, if and only there is a regular process q such that

$$p \leftrightarrows^{\Delta}_{\mathsf{b}} \tau_{\{i,o\}}(\partial_{\{i,o\}}(q \parallel ABag^{io}))$$

Can do without priorities, but then we need *get* communication (asymmetric communication).

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Conclusion

The set of processes given by a parallel pushdown automaton coincides with the set of processes given by a finite weakly guarded recursive specification over a process algebra with actions, choice, priorities and parallel composition with value passing communication.