Event Structure Semantics for Multiparty Sessions

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joint work with Mariangiola Dezani† and Paola Giannini‡

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Overview (1/4)

Investigate connection between:

Multiparty Session Types

[HYC08,HYC16]

Event Structures

[Win80,NPW81]

Overview (2/4)

ES semantics for a core MPST calculus

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ES interpretation for both sessions and types:

$$\begin{aligned} \text{session} &\longrightarrow \text{Flow ES} \\ \text{global type} &\longrightarrow \text{Prime ES} \end{aligned}$$

Main result: isomorphism of configuration domains.

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Holds for both synchronous and asynchronous communication.

Overview (3/4)

ES semantics for a core synchronous MPST calculus where sessions are:

- described as networks of sequential processes
- specified by global types

ES interpretation for both sessions and types:

$$\text{session} \longrightarrow \text{Flow ES}$$
$$\text{global type} \longrightarrow \text{Prime ES}$$

Main result: isomorphism of configuration domains.

I. Castellani, M. Dezani-Ciancaglini and P. Giannini. Event Structure Semantics for Multiparty Sessions, JLAMP 131, February 2023.

https://doi.org/10.1016/j.jlamp.2022.100844

Overview (4/4)

ES semantics for a core asynchronous MPST calculus where sessions are:

- described as networks of sequential processes with queues
- specified by asynchronous types

ES interpretation for both sessions and types:

$$\text{asynchronous session} \longrightarrow \text{Flow ES}$$
$$\text{asynchronous type} \longrightarrow \text{Prime ES}$$

Main result: isomorphism of configuration domains.

I. Castellani, M. Dezani-Ciancaglini and P. Giannini. Global Types and Event Structure Semantics for Asynchronous Multiparty Sessions.

`https://arxiv.org/abs/2102.00865`

A quick tour on Event Structures

Event Structures



- [NPW79,NPW81] M. Nielsen, G. Plotkin and G. Winskel. Petri Nets, Event Structures and Domains. LNCS 70, 1979, and TCS 13(1), 1981.
- [Win80] G. Winskel. Events in Computation. PhD Thesis, University of Edinburgh, 1980.

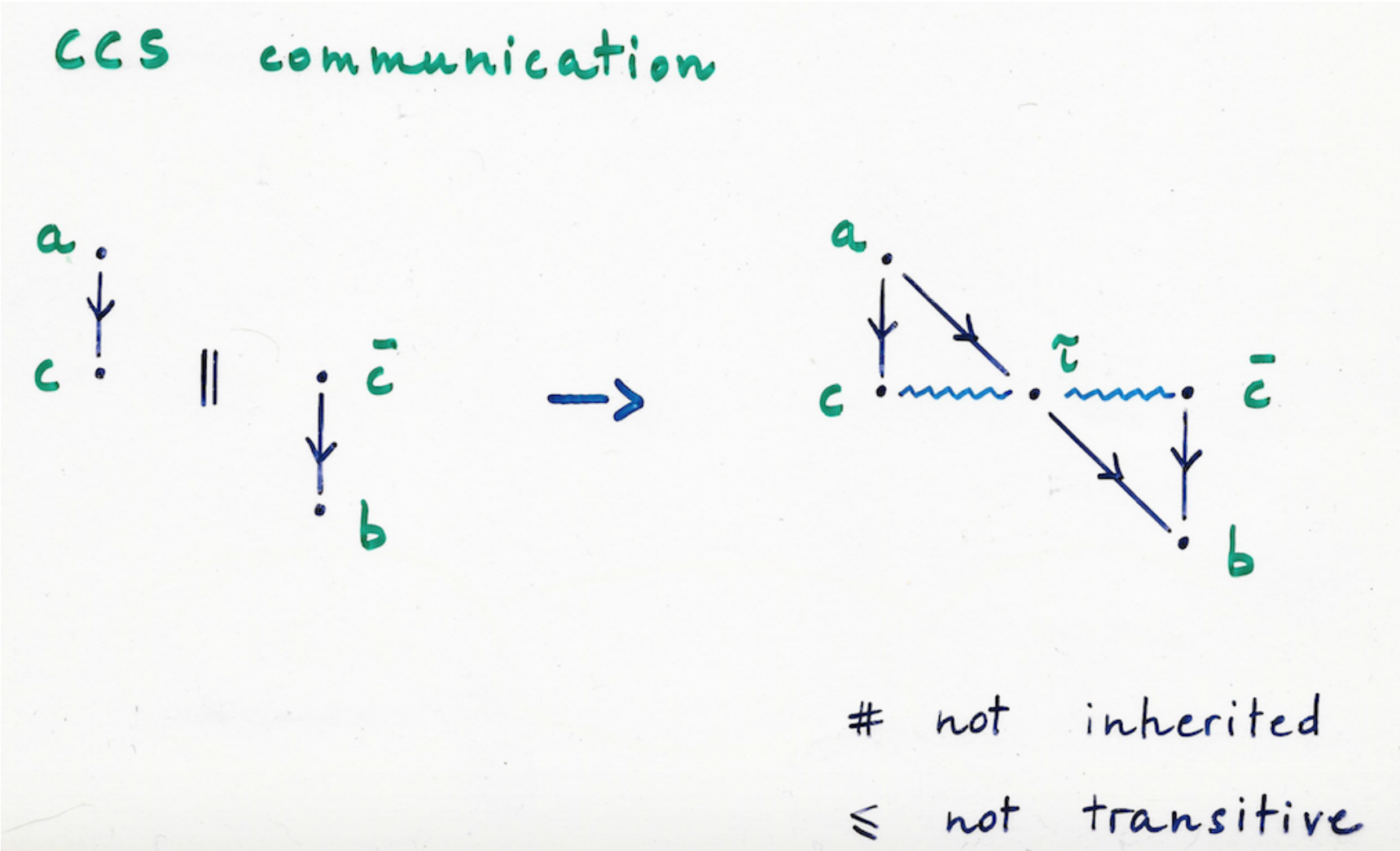
A canonical model for (true) concurrency, strongly connected with particular classes of Petri nets and domains.

Prime event structures

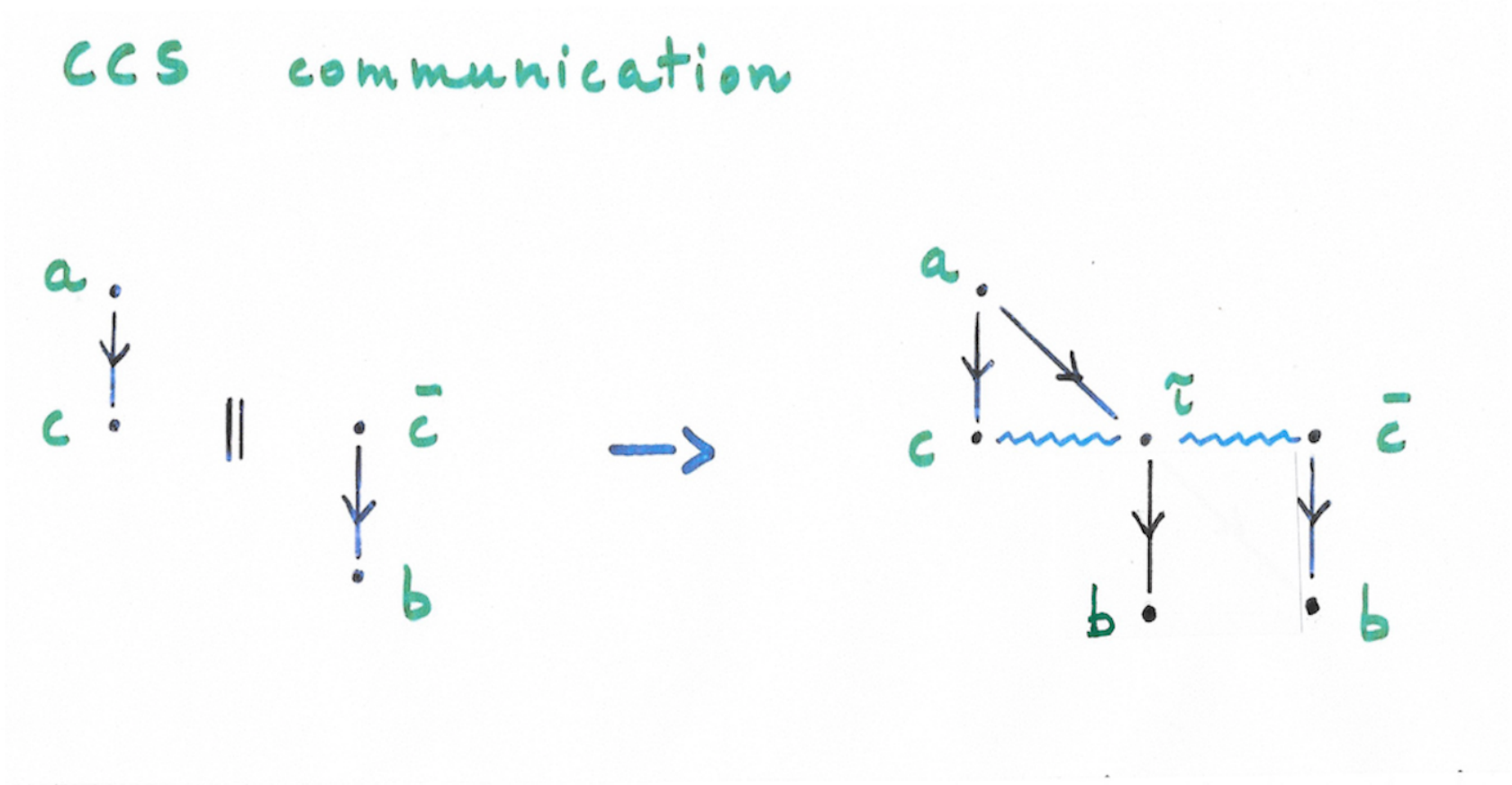
A prime event structure [NPW79] is a tuple $S = (E, \leq, \#)$ where:

1. E is a denumerable set of events;
2. the causality relation $\leq \subseteq (E \times E)$ is a partial order;
3. the conflict relation $\# \subseteq (E \times E)$ is irreflexive, symmetric and satisfies conflict hereditariness: $e \# e' \leq e'' \Rightarrow e \# e''$

Prime ES semantics for process calculi?



Prime ES semantics for process calculi



Drawback: duplication of events after communication.

Stable ES semantics for process calculi

Stable ESs: more general ESs allowing for conflicting causes, while ensuring causal non-ambiguity within configurations.
Used by Winskel to define the first ES semantics for CCS:

- [Win82] Glynn Winskel. Event structure semantics for CCS and related languages, ICALP 1982.

Flow ESs: subclass of Stable ESs, graphical representation.
Used by Boudol and C. to give semantics to CCS and SCCS:

- [BC88] G. Boudol and I. Castellani. Permutation of transitions: an event structure semantics for CCS and SCCS, REX School/Workshop, 1988.
- [BC94] G. Boudol and I. Castellani. Flow models of distributed computations: three equivalent semantics for CCS, Inf. and Comp., 1994.

Relations among some classes of ESs

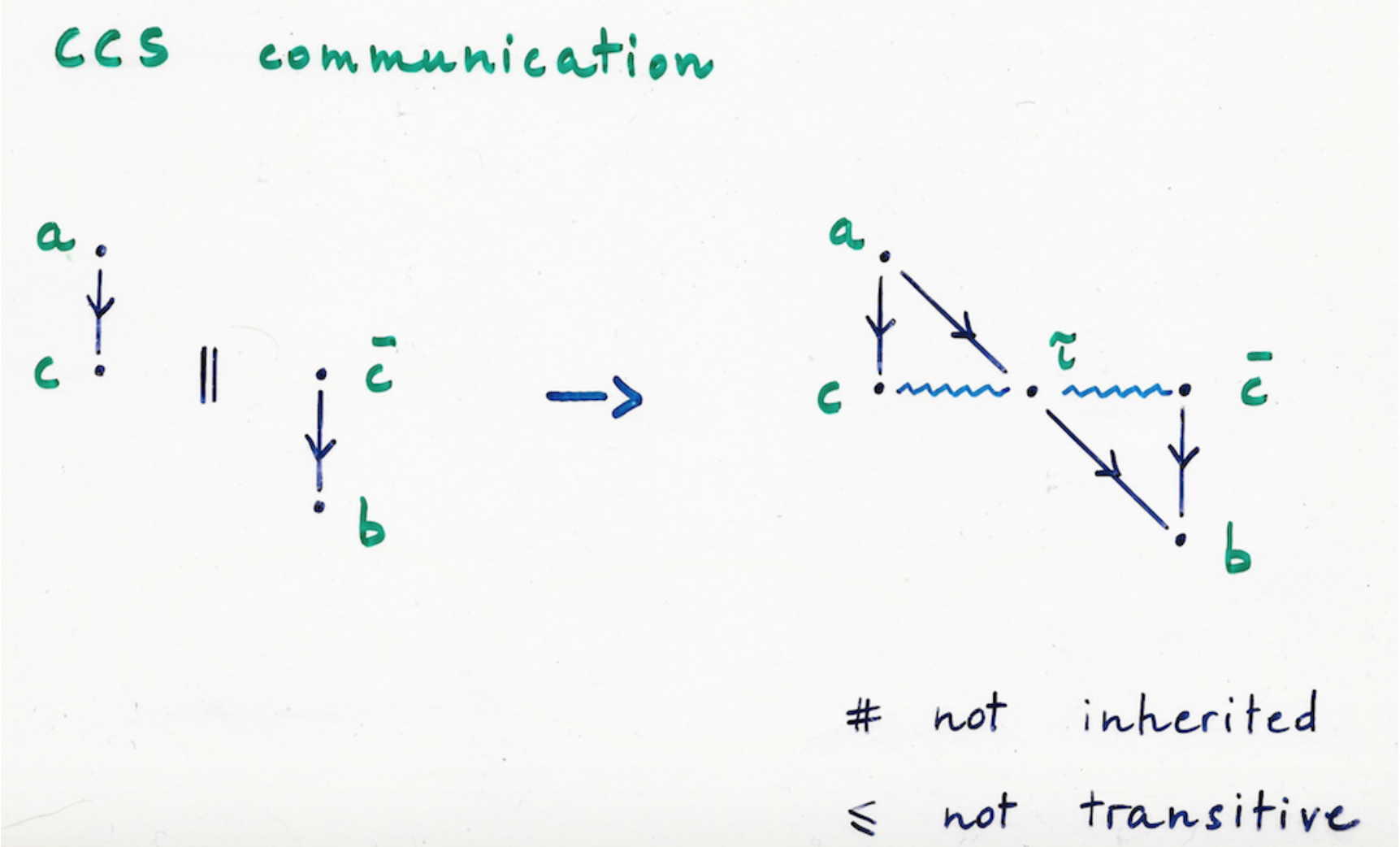
Bundle ESs: subclass of Stable ESs, using bundles (sets of pairwise conflicting causes of an event) to express causality. Used by Langerak and Katoen to give semantics to LOTOS:

- [Lan93] R. Langerak. Bundle Event Structures: a Non-Interleaving Semantics for LOTOS. Formal Description Techniques for Distributed Systems and Communication Protocols, North-Holland, 1993.
- [Kat96] J-P. Katoen. Quantitative and qualitative extensions of event structures. PhD Thesis, University of Twente, 1996.

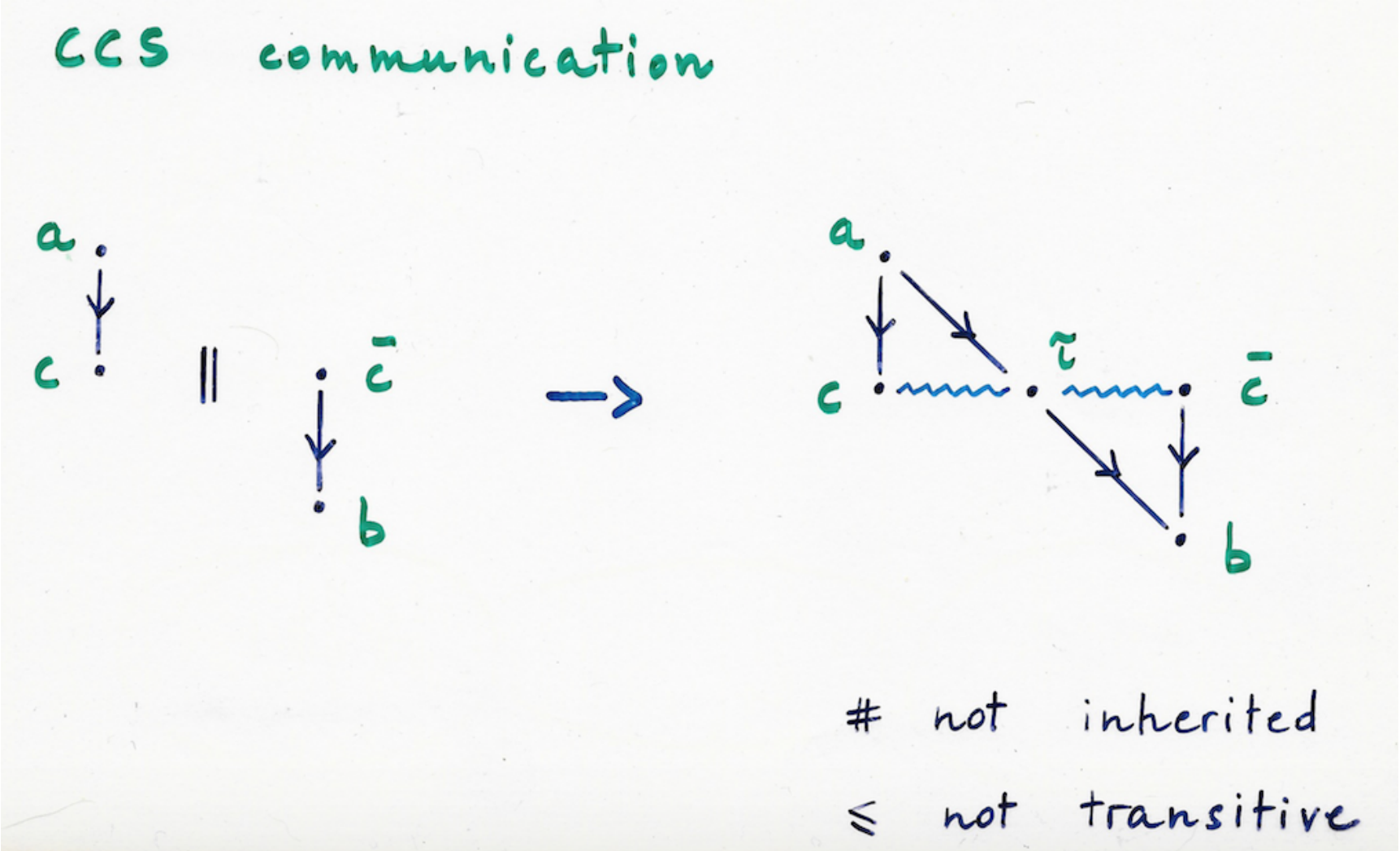
$$\textit{Prime ESs} \subset \textit{Bundle ESs} \subset \textit{Flow ESs} \subset \textit{Stable ESs} \subset \textit{General ESs}$$

- [BC91] Flow models of distributed computations: event structures and nets, G. Boudol and I. Castellani. INRIA Research Report n. 1482 (1991).

Flow ES semantics for process calculi



Flow ES semantics for process calculi



Idea: relax $\#$ and replace $\leq$ by $\prec$, immediate causality.

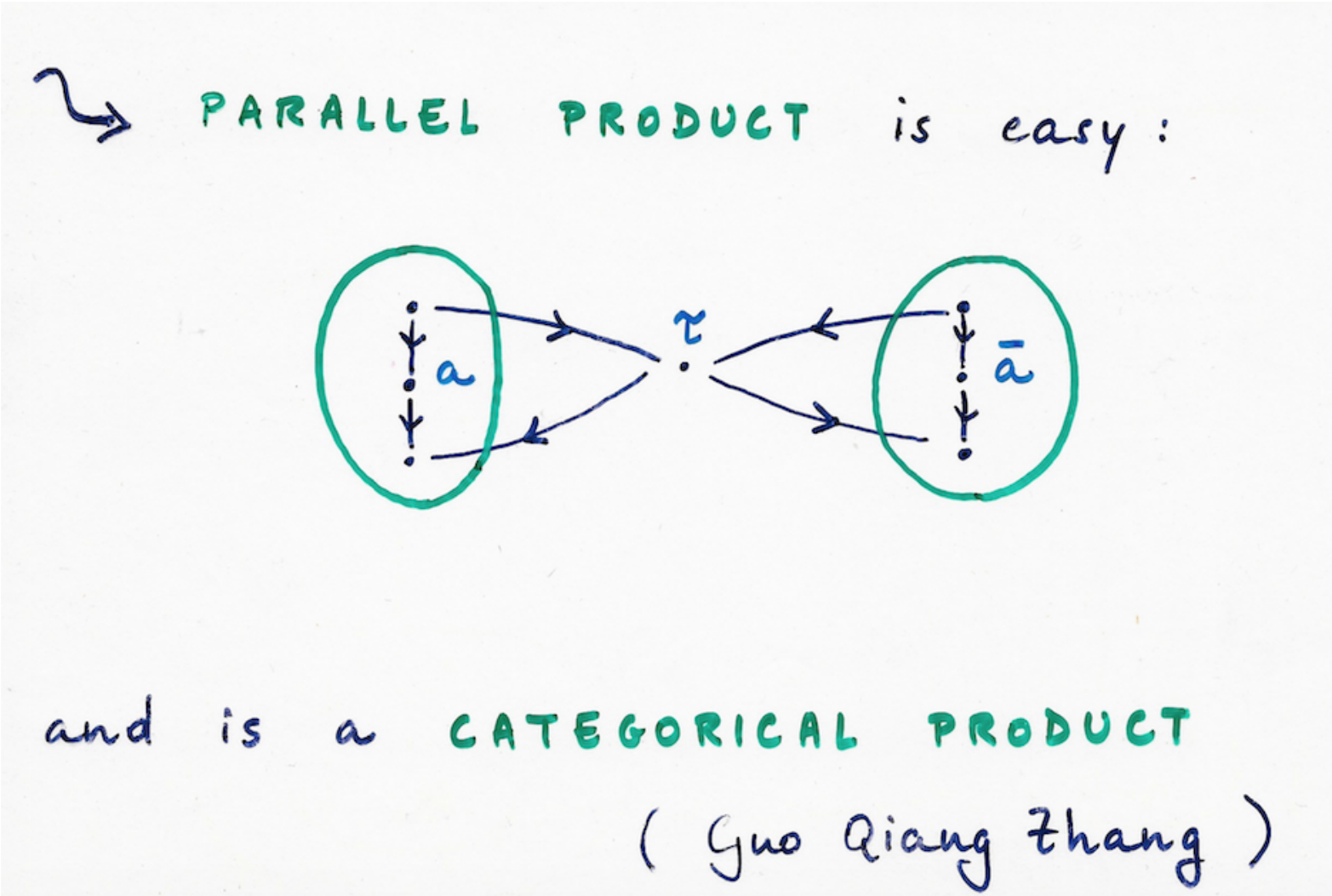
Flow event structures

A flow event structure [BC88] is a tuple $S = (E, \prec, \#)$ where:

1. E is a denumerable set of events;
2. the flow relation $\prec \subseteq (E \times E)$ is irreflexive;
3. the conflict relation $\# \subseteq (E \times E)$ is symmetric.

New feature: self-conflicts (to model CCS restriction).

Flow ES product



– [CZ97] I. Castellani and G. Q. Zhang. Parallel product of event structures. TCS 179, 1997.

Prime ES Configurations

Configuration: set of events having occurred at some stage of execution.

Let $S = (E, \leq, \#)$ be a prime event structure. A configuration of S is a finite subset $\mathcal{X}$ of E such that:

1. $\mathcal{X}$ is left-closed: $e' \leq e \in \mathcal{X} \Rightarrow e' \in \mathcal{X}$;
2. $\mathcal{X}$ is conflict-free: $\forall e, e' \in \mathcal{X}, \neg(e \# e')$.

Flow ES Configurations

Let $S = (E, \prec, \#)$ be a flow event structure. A configuration of S is a finite subset $\mathcal{X}$ of E such that:

1. $\mathcal{X}$ is left-closed up to conflicts:
   $e' \prec e \in \mathcal{X},\ e' \notin \mathcal{X} \ \Rightarrow\ \exists\, e'' \in \mathcal{X}.\ e' \# e'' \prec e$;
2. $\mathcal{X}$ is conflict-free: $\forall e, e' \in \mathcal{X}, \neg(e \# e')$;
3. $\mathcal{X}$ has no causality cycles: the relation $\prec^*_{\mathcal{X}}$ is a partial order.

Domain of configurations

Let S be a PES or a FES with set of configurations $\mathcal{C}(S)$.

The domain of configurations of S is the partially ordered set

$$S =_{\text{def}} (\mathcal{C}(S), \subseteq)$$

Prime algebraic coherent domains, whose prime elements are in 1-1 correspondence with the events of S (if S is a PES).

Cf picture from [NPW79] on next slide

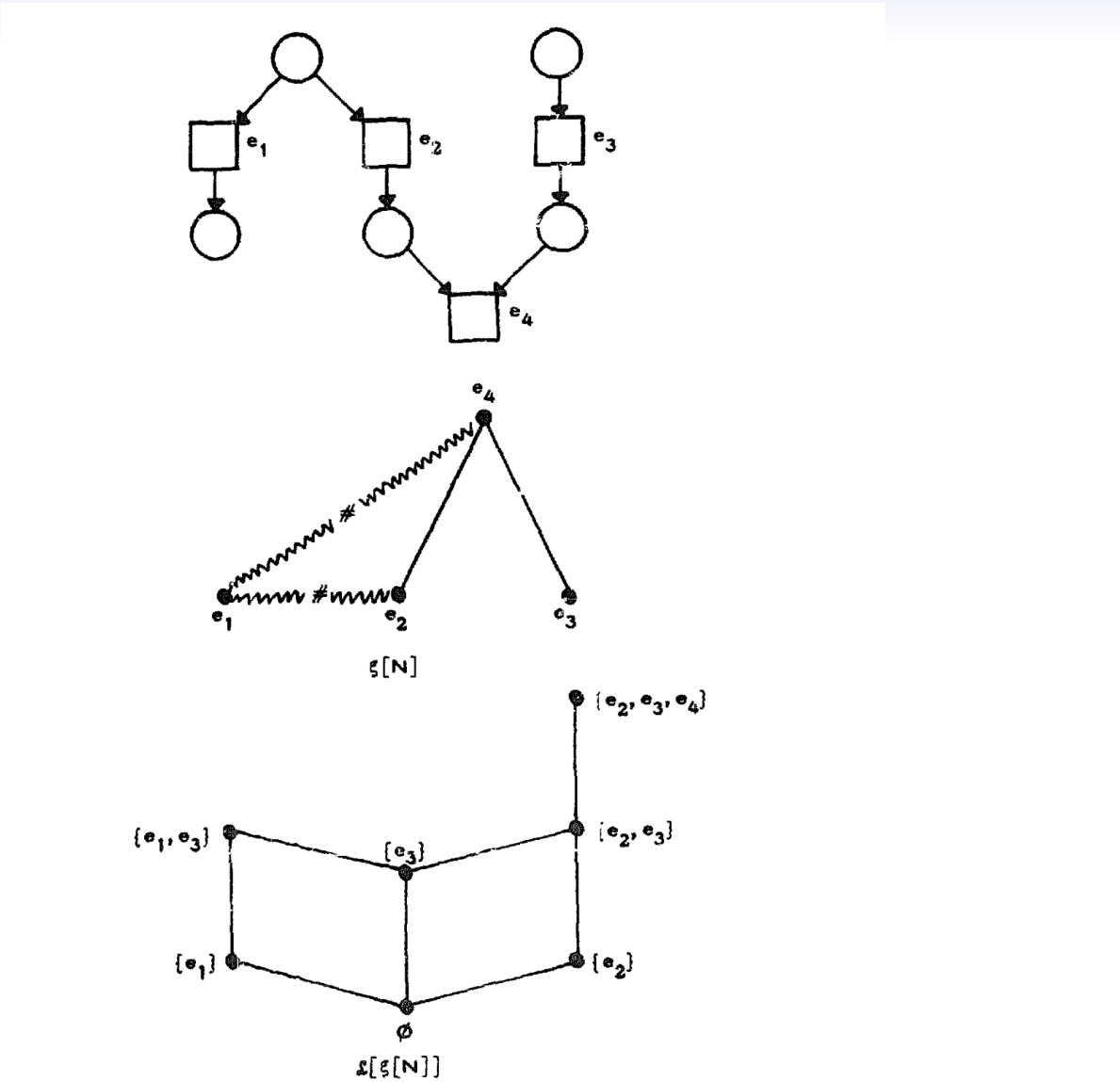


Fig. 7.

Session Types in one slide

- Types for interaction: STs specify process interaction within units called sessions, regulated by a protocol.
- First introduced for binary sessions, then extended to multiparty sessions.

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- K. Honda, V. Vasconcelos and M. Kubo. Language Primitives and Type Disciplines for Structured Communication-based Programming. ESOP 1998.
- K. Honda, N. Yoshida and M. Carbone. Multiparty asynchronous session types. POPL 2008. Most Influential POPL Paper Award 2018.




Menu

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- Multiparty sessions

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- Global types

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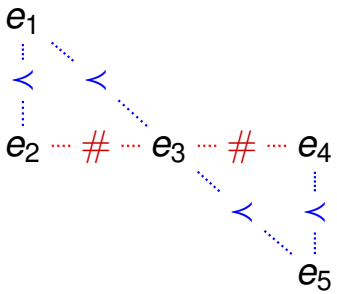
- Event Structure semantics



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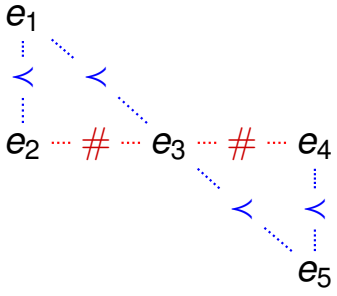

- Isomorphism result

A core synchronous MPST calculus

Processes

$$P \quad ::=^{coind} \quad \bigoplus_{i \in I} \mathsf{p}!\lambda_i; P_i \quad | \quad \sum_{i \in I} \mathsf{p}?\lambda_i; P_i \quad | \quad \mathbf{0}$$

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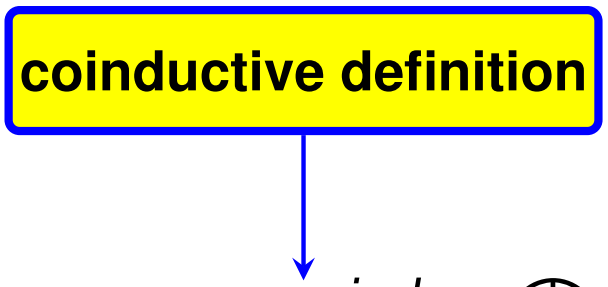
$$\lambda_i \neq \lambda_j \text{ for any } i \neq j$$

Processes

coinductive definition

$$P \quad ::=^{coind} \quad \bigoplus_{i \in I} \mathsf{p}!\lambda_i; P_i \quad | \quad \sum_{i \in I} \mathsf{p}?\lambda_i; P_i \quad | \quad \mathbf{0}$$

Processes



$$P \quad ::=^{coind} \quad \bigoplus_{i \in I} \mathsf{p}!\lambda_i; P_i \quad | \quad \sum_{i \in I} \mathsf{p}?\lambda_i; P_i \quad | \quad \mathbf{0}$$

We focus on regular processes.

Trees with leaves decorated by **0**, internal nodes by p! or p?, and edges by labels λ.

Multiparty Sessions

$$\mathsf{p}_1[\![\, P_1 \,]\!] \,||\, \cdots \,||\, \mathsf{p}_n[\![\, P_n \,]\!]$$

$$\mathsf{p}_i \neq \mathsf{p}_j \;\text{ for any }\; i \neq j$$

LTS for Multiparty Sessions

$$\mathsf{p}[\![\bigoplus_{i\in I} \mathsf{q}!\lambda_i; P_i]\!] \parallel \mathsf{q}[\![\Sigma_{j\in J} \mathsf{p}?\lambda_j; Q_j]\!] \parallel \mathsf{N}$$
$$\xrightarrow{\mathsf{pq}\lambda_k} \mathsf{p}[\![P_k]\!] \parallel \mathsf{q}[\![Q_k]\!] \parallel \mathsf{N}$$
where $k \in I \cap J$

Transitions are labelled by communications.

$\mathsf{pq}\lambda$: communication of label λ from p to q.

LTS for sessions: classical deadlock

Concurrent mutual sends:

$$\mathsf{p}[\![\, \mathsf{q}!\lambda; \mathsf{q}?\lambda' \,]\!] \parallel \mathsf{q}[\![\, \mathsf{p}!\lambda'; \mathsf{p}?\lambda \,]\!]$$

Deadlock due to mismatching expectations.

LTS for sessions: 3-philosopher deadlock

Deadlock due to circular dependencies

$$\mathsf{p}[\![\, \mathsf{r}?\lambda; \mathsf{q}!\lambda' \,]\!] \parallel \mathsf{q}[\![\, \mathsf{p}?\lambda'; \mathsf{r}!\lambda'' \,]\!] \parallel \mathsf{r}[\![\, \mathsf{q}?\lambda''; \mathsf{p}!\lambda \,]\!]$$

NB Here participants have matching expectations.

LTS for sessions: starvation

Starvation due to a denying choice

$$\mathsf{N} = \mathsf{p}[\![P]\!] \,\|\, \mathsf{q}[\![Q]\!] \,\|\, \mathsf{r}[\![R]\!]$$

$$\begin{aligned} P &= \mathsf{q}!\lambda_1 && \oplus && \mathsf{q}!\lambda_2 ; P \\ Q &= \mathsf{p}?\lambda_1 ; \mathsf{r}!\lambda && + && \mathsf{p}?\lambda_2 ; Q \\ R &= \mathsf{q}?\lambda \end{aligned}$$

Eluding path $\mathsf{N} \xrightarrow{\mathsf{pq}\lambda_2} \cdot \xrightarrow{\mathsf{pq}\lambda_2} \cdots$

Global Types: syntax

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$$\mathsf{G} \ ::=^{coind} \ \mathsf{p} \to \mathsf{q} : \boxplus_{i \in I} \lambda_i ; \mathsf{G}_i \ \mid \ \mathsf{End}$$

$$\lambda_i \neq \lambda_j \ \text{ for any } \ i \neq j$$

Again, we focus on regular terms.

Trees with internal nodes decorated by channels pq, leaves by End, and edges by labels λ.

LTS for Global Types

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$$\mathsf{p} \to \mathsf{q} : \boxplus_{i \in I} \lambda_i ; \mathsf{G}_i \xrightarrow{\mathsf{pq}\lambda_k} \mathsf{G}_k \qquad \text{where} \quad k \in I$$

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Also internal communications are allowed:

$$\frac{\mathsf{G}_i \xrightarrow{\alpha} \mathsf{G}_i' \quad \text{for all } i \in I \quad \mathrm{part}(\alpha) \cap \{\mathsf{p}, \mathsf{q}\} = \emptyset}{\mathsf{p} \to \mathsf{q} : \boxplus_{i \in I} \lambda_i ; \mathsf{G}_i \xrightarrow{\alpha} \mathsf{p} \to \mathsf{q} : \boxplus_{i \in I} \lambda_i ; \mathsf{G}_i'}$$

Projection

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Global types are projected on participants, yielding processes.

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$$\mathsf{G} \upharpoonright \mathsf{r} = \mathbf{0} \text{ if } \mathsf{r} \notin \mathrm{part}(\mathsf{G})$$

$$(\mathsf{p} \to \mathsf{q} : \boxplus_{i \in I} \lambda_i; \mathsf{G}_i) \upharpoonright \mathsf{r} = \begin{cases} \bigoplus_{i \in I} \mathsf{q}!\lambda_i; \mathsf{G}_i \upharpoonright \mathsf{r} & \text{if } \mathsf{r} = \mathsf{p}, \\ \sum_{i \in I} \mathsf{p}?\lambda_i; \mathsf{G}_i \upharpoonright \mathsf{r} & \text{if } \mathsf{r} = \mathsf{q}, \\ \mathsf{G}_1 \upharpoonright \mathsf{r} & \text{if } \mathsf{r} \notin \{\mathsf{p}, \mathsf{q}\} \wedge \mathsf{r} \in \mathrm{part}(\mathsf{G}_1) \wedge \\ & \mathsf{G}_i \upharpoonright \mathsf{r} = \mathsf{G}_1 \upharpoonright \mathsf{r} \text{ for all } i \in I \end{cases}$$

Boundedness

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$$\mathsf{G} = \mathsf{p} \to \mathsf{q} : (\lambda_1; \mathsf{q} \xrightarrow{\lambda} \mathsf{r} \boxplus \lambda_2; \mathsf{G})$$

The type G is projectable but not bounded.

Boundedness ensures that each participant can move.

Type system

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Well formedness: boundedness + projectability

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$$\frac{P_i \leq Q_i \quad \text{for all } i \in I}{\sum_{i \in I \cup J} \mathsf{p}?\lambda_i; P_i \leq \sum_{i \in I} \mathsf{p}?\lambda_i; Q_i}$$

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$$\frac{P_i \leq \mathtt{G} \upharpoonright \mathtt{p}_i \quad \text{for all } i \in I \qquad \mathrm{part}(\mathtt{G}) \subseteq \{\mathtt{p}_i \mid i \in I\}}{\vdash \prod_{i \in I} \mathtt{p}_i [\![P_i]\!] : \mathtt{G}}$$

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Properties

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Theorem (Subject Reduction)

If $\vdash \mathsf{N} : \mathsf{G}$ *and* $\mathsf{N} \xrightarrow{\alpha} \mathsf{N}'$, *then* $\mathsf{G} \xrightarrow{\alpha} \mathsf{G}'$ *and* $\vdash \mathsf{N}' : \mathsf{G}'$.

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Theorem (Session Fidelity)

If $\vdash \mathsf{N} : \mathsf{G}$ *and* $\mathsf{G} \xrightarrow{\alpha} \mathsf{G}'$, *then* $\mathsf{N} \xrightarrow{\alpha} \mathsf{N}'$ *and* $\vdash \mathsf{N}' : \mathsf{G}'$.

Properties

Progress: every participant whose process is not terminated will eventually move.

$\mathsf{p}[\![P]\!] \in \mathsf{N}$: means $P \neq \mathbf{0}$ and $\mathsf{N} \equiv \mathsf{p}[\![P]\!] \parallel \mathsf{N}'$ for some N'.

Under some fairness assumption:

Theorem (Progress)
If $\vdash \mathsf{N} : \mathsf{G}$ *and* $\mathsf{p}[\![P]\!] \in \mathsf{N}$, *then* $\mathsf{N} \xrightarrow{\sigma} \mathsf{N}' \xrightarrow{\alpha}$ *and* $\mathsf{p} \in \mathsf{part}(\alpha)$.

Event Structure Semantics

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Both networks and global types are modelled as ESs:

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First step to model networks:

$$\text{process} \longrightarrow \text{Prime ES}$$

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Prime ES Semantics of Processes

Process event (p-event): path in the syntactic tree of a process:

$\eta = \zeta \cdot \pi$ represents a specific occurrence of the action π preceded by its local history ζ.

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Flow ES Semantics of Sessions: events

Located event p :: η: p-event η pertaining to a participant p.

$\eta \twoheadrightarrow$ p : projection of p-event η on participant p.

Duality of located events:

$$\mathsf{p} :: \eta \ \bowtie \ \mathsf{q} :: \eta' \text{ if}$$
$$\eta \twoheadrightarrow \mathsf{q} \ \bowtie \ \eta' \twoheadrightarrow \mathsf{p} \text{ and } \mathrm{part}(\mathrm{act}(\eta)) = \mathsf{q} \text{ and } \mathrm{part}(\mathrm{act}(\eta')) = \mathsf{p}$$

Network event (n-event): unordered pair of dual located events:

$$\{\mathsf{p} :: \zeta \cdot \pi, \mathsf{q} :: \zeta' \cdot \pi'\}$$

with matching actions π and π' and matching histories ζ and ζ'.

Flow ES Semantics of Sessions: relations

Network event (n-event): $\nu = \{\mathsf{p} :: \eta, \mathsf{q} :: \eta'\}$

Flow relation $\nu \prec \nu'$ if $\mathsf{p} :: \eta \in \nu$ & $\mathsf{p} :: \eta' \in \nu'$ & $\eta < \eta'$

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or 2) ($\mathsf{p} :: \eta \in \nu$ & $\mathsf{q} :: \eta' \in \nu'$ & $\mathsf{p} \neq \mathsf{q}$ &
$|\eta \upharpoonright \mathsf{q}| = |\eta' \upharpoonright \mathsf{p}|$ & $\neg(\eta \upharpoonright \mathsf{q} \bowtie \eta' \upharpoonright \mathsf{p})$)

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Conflict may hold between n-events with disjoint participants.

$\nu = \{\mathsf{p} :: \mathsf{q}!\lambda_1 \cdot \mathsf{r}!\lambda, \mathsf{r} :: \mathsf{p}?\lambda\} \quad \# \quad \nu' = \{\mathsf{q} :: \mathsf{p}?\lambda_2 \cdot \mathsf{s}!\lambda, \mathsf{s} :: \mathsf{q}?\lambda\}$

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Pruning non causally-founded n-events

Some n-events $\nu = \{\mathsf{p} :: \zeta \cdot \pi, \mathsf{q} :: \zeta' \cdot \pi'\}$ are not admissible.

The network

$$\mathsf{N} = \mathsf{p}[\![\, \mathsf{q}?\lambda \cdot \mathsf{r}!\lambda' \,]\!] \parallel \mathsf{r}[\![\, \mathsf{p}?\lambda' \,]\!]$$

has the unique n-event $\nu = \{\mathsf{p} :: \mathsf{q}?\lambda \cdot \mathsf{r}!\lambda', \mathsf{r} :: \mathsf{p}?\lambda'\}$.

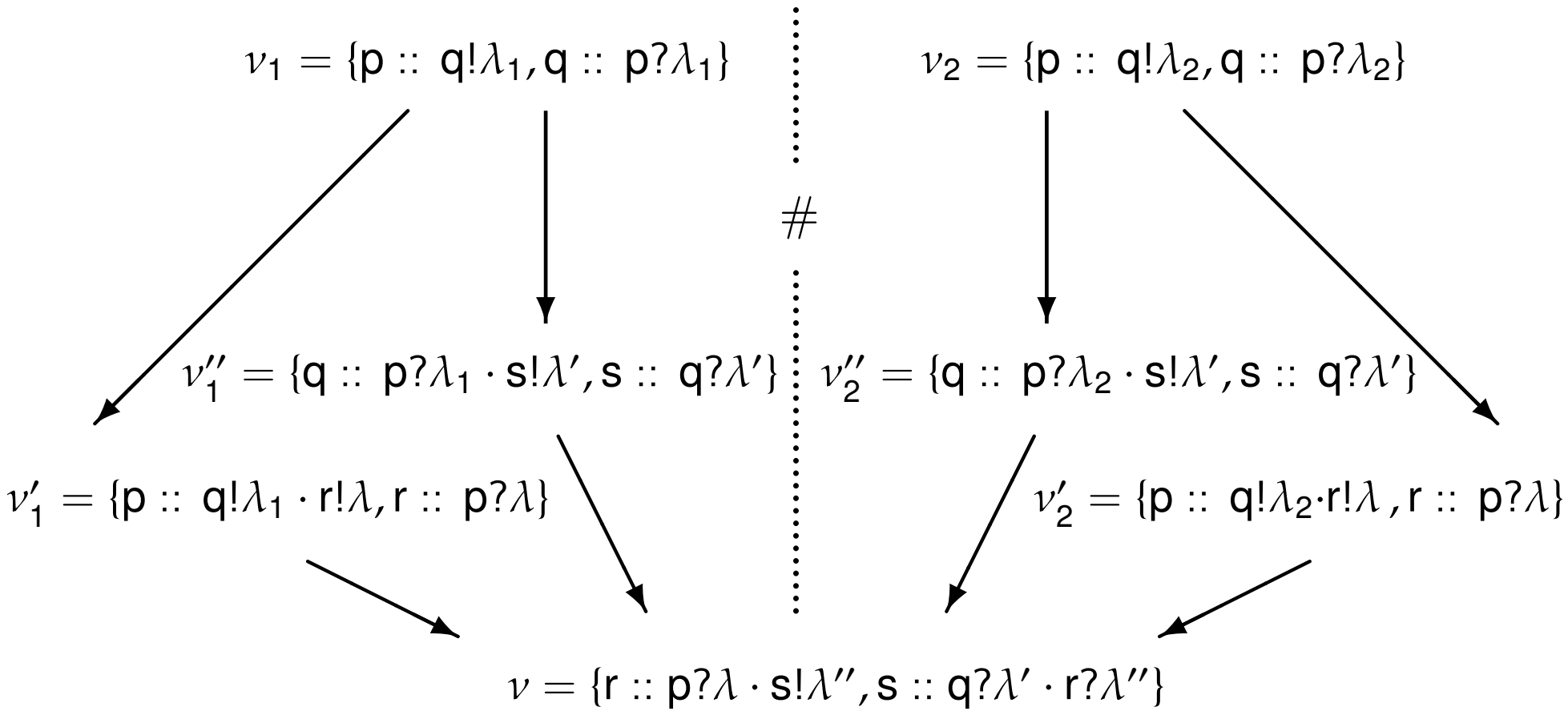
Since ν is minimal, then $\{\nu\}$ would be a configuration of $\mathcal{S}^{\mathcal{N}}(\mathsf{N})$.

WRONG! $\Longrightarrow$ need for causal well-foundedness

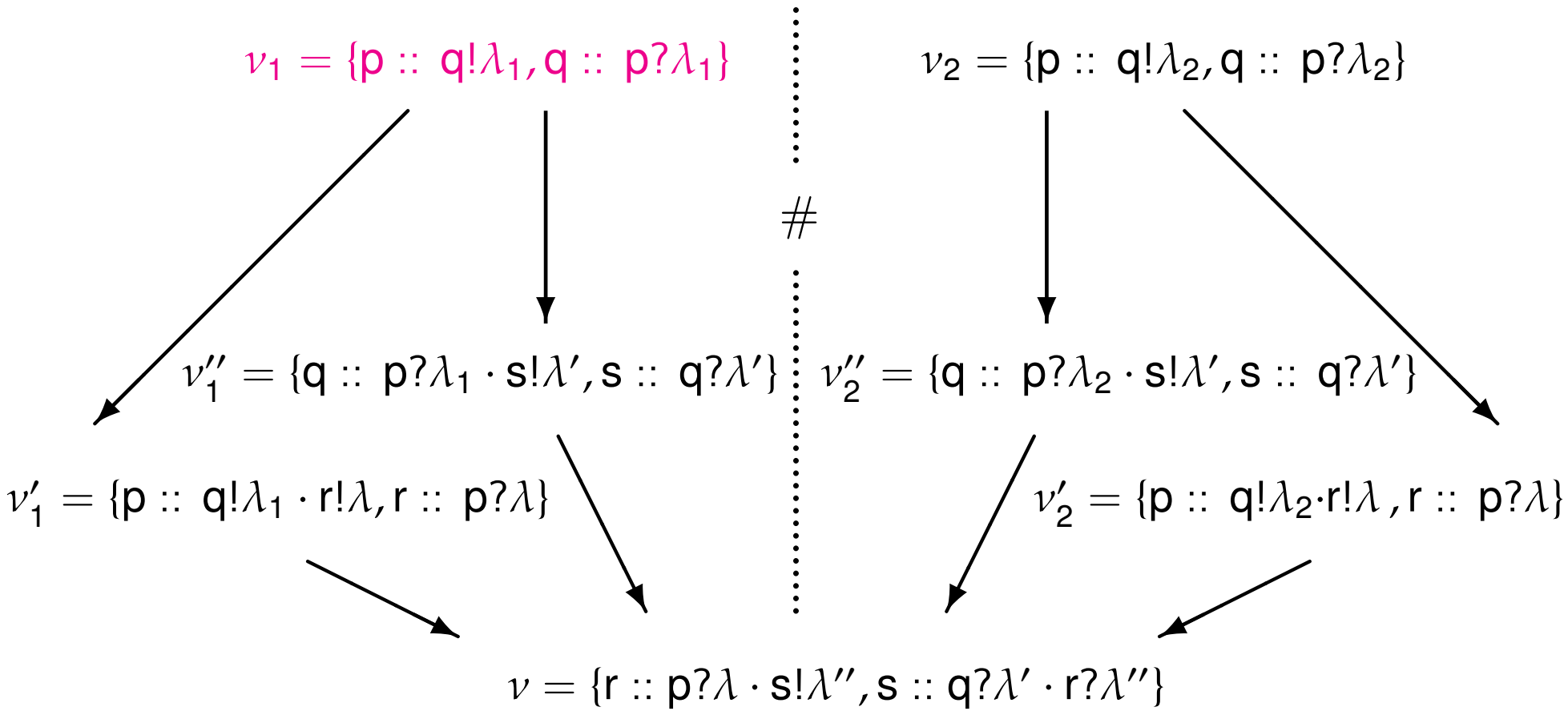
$\Rightarrow$ Pruning operation ("narrowing") on the set of events:

removes non causally well-founded n-events.

$$\mathsf{N} = \mathsf{p}[\![\, \mathsf{q}!\lambda_1; \mathsf{r}!\lambda \oplus \mathsf{q}!\lambda_2; \mathsf{r}!\lambda \,]\!] \parallel \mathsf{q}[\![\, \mathsf{p}?\lambda_1; \mathsf{s}!\lambda' + \mathsf{p}?\lambda_2; \mathsf{s}!\lambda' \,]\!] \parallel \mathsf{r}[\![\, \mathsf{p}?\lambda; \mathsf{s}!\lambda'' \,]\!] \parallel \mathsf{s}[\![\, \mathsf{q}?\lambda'; \mathsf{r}?\lambda'' \,]\!]$$



$$\mathsf{N} = \mathsf{p}[\![\, \mathsf{q}!\lambda_1; \mathsf{r}!\lambda \oplus \mathsf{q}!\lambda_2; \mathsf{r}!\lambda \,]\!] \;\|\; \mathsf{q}[\![\, \mathsf{p}?\lambda_1; \mathsf{s}!\lambda' + \mathsf{p}?\lambda_2; \mathsf{s}!\lambda' \,]\!] \;\|\; \mathsf{r}[\![\, \mathsf{p}?\lambda; \mathsf{s}!\lambda'' \,]\!] \;\|\; \mathsf{s}[\![\, \mathsf{q}?\lambda'; \mathsf{r}?\lambda'' \,]\!]$$



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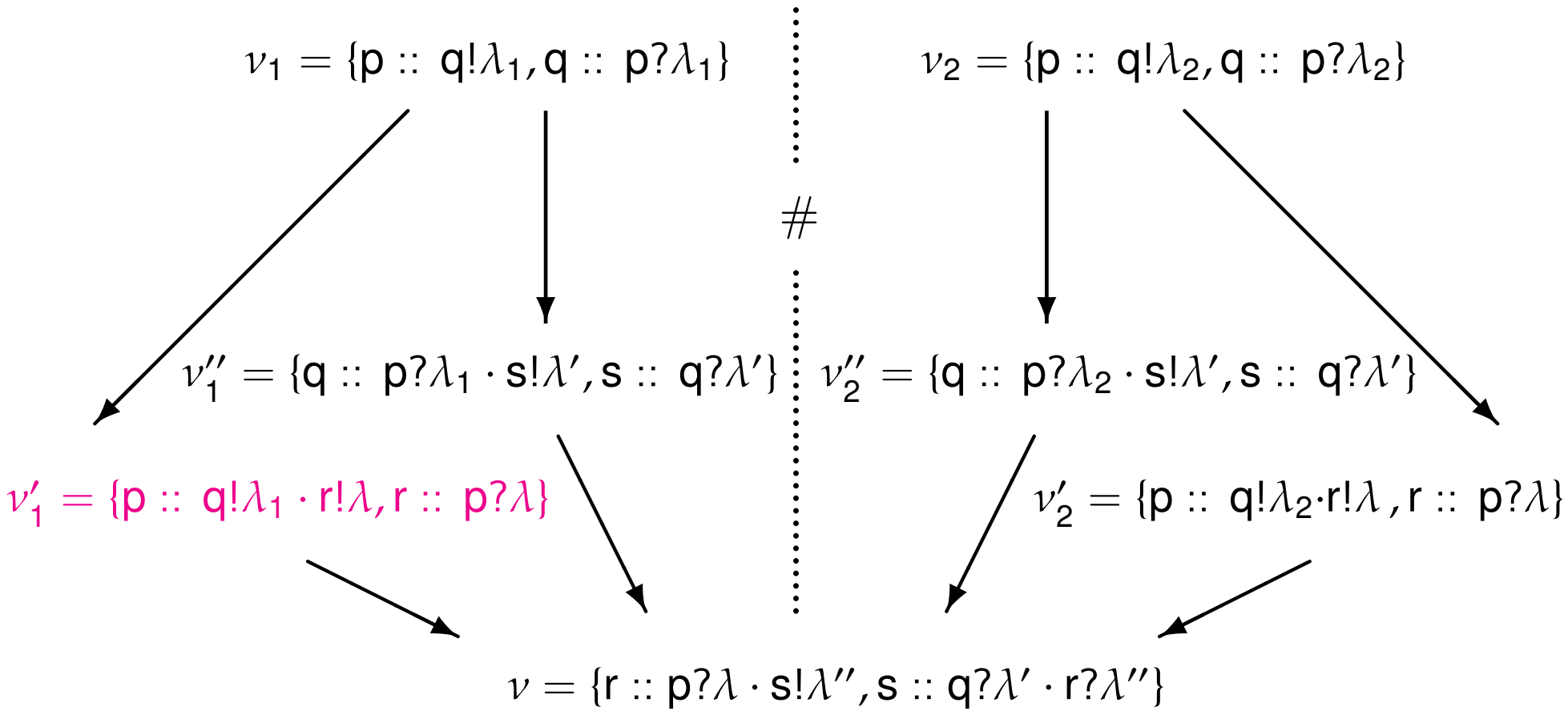

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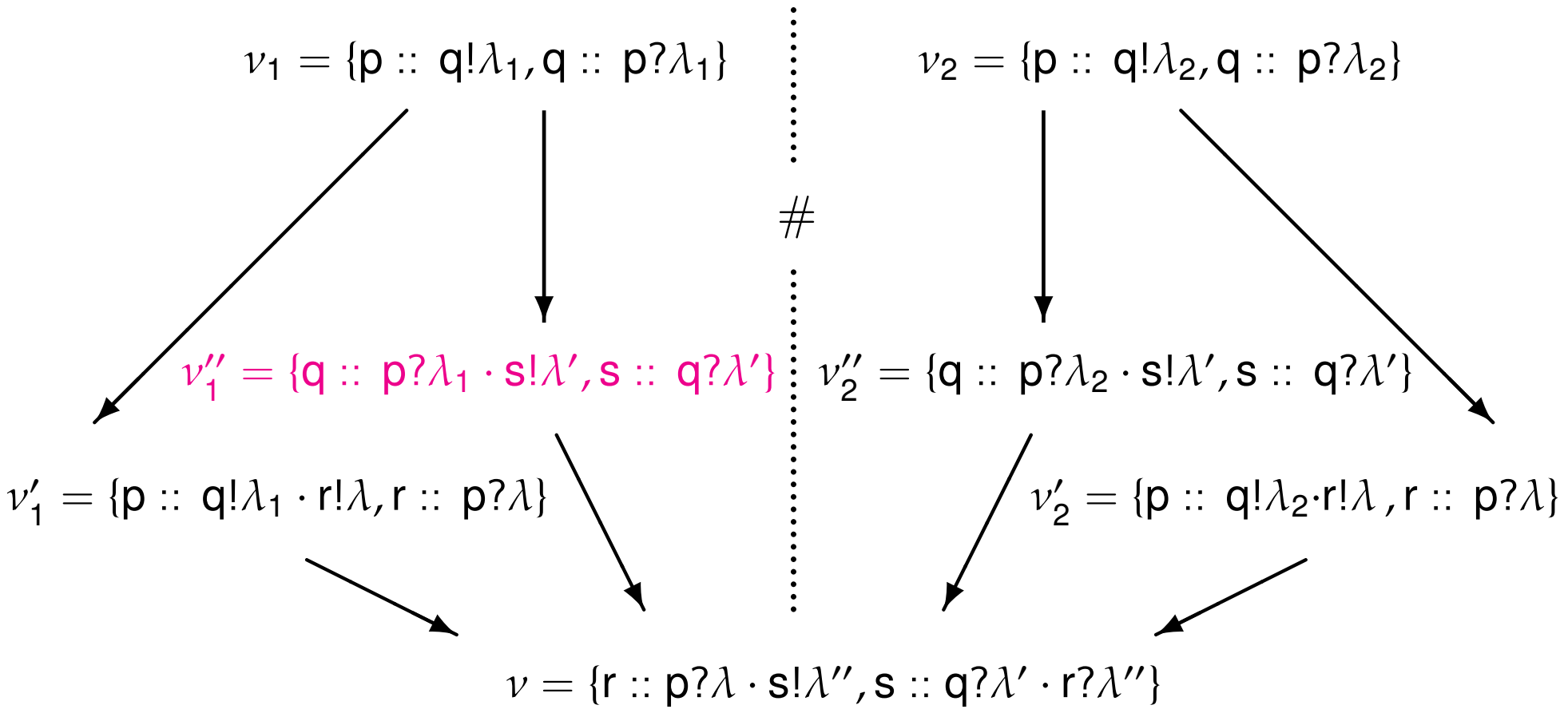

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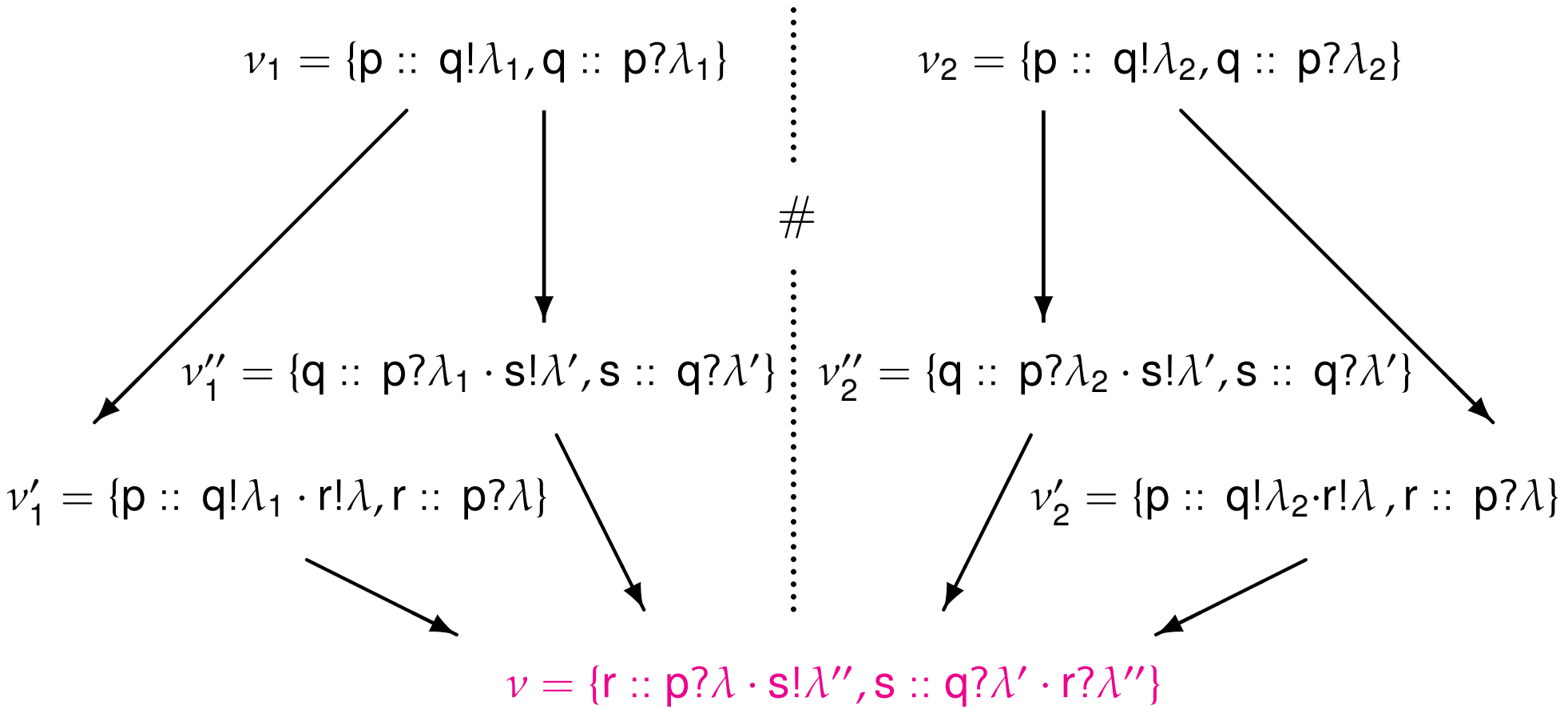

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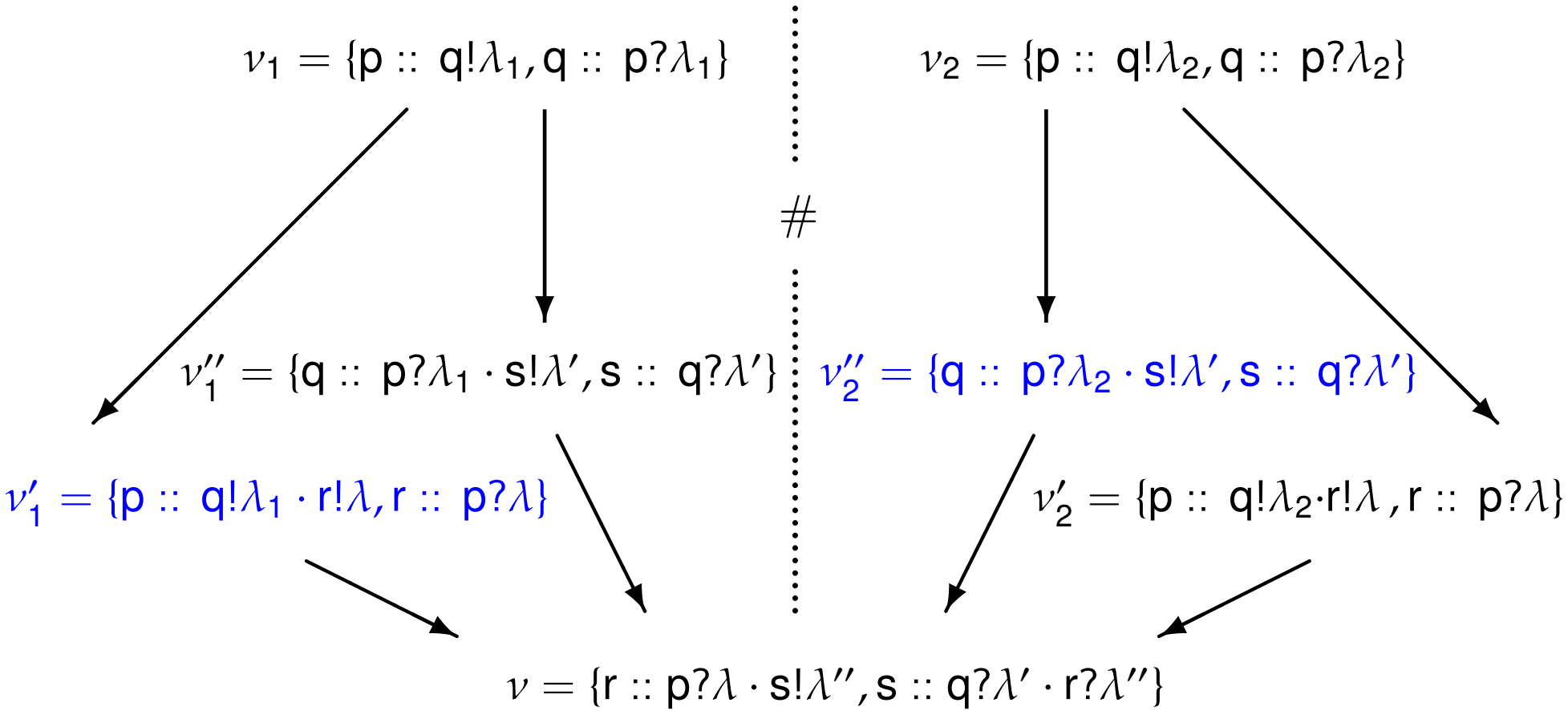

Conflict between n-events with disjoint participants.

Prime ES Semantics of Global Types: events

Trace σ: finite sequence of communications.

Permutation equivalence $\sim$ on traces:

$$\sigma \cdot \alpha \cdot \alpha' \cdot \sigma' \sim \sigma \cdot \alpha' \cdot \alpha \cdot \sigma' \quad \text{if} \quad \mathsf{part}(\alpha) \cap \mathsf{part}(\alpha') = \emptyset$$

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A trace $\sigma = \alpha_1 \cdots \alpha_n$ is pointed if

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Global type event (g-event): $\gamma = [\sigma]_\sim$ where σ is pointed.

$\gamma = [\sigma \cdot \alpha]_\sim$ represents an occurrence of communication α preceded by its global history σ (considered modulo $\sim$).

Prime ES Semantics of Global Types: relations

Global type event (g-event):

$$\gamma = [\sigma]_\sim \text{ where } \sigma \text{ is pointed.}$$

Causality relation

$\gamma \leq \gamma'$ if $\gamma = [\sigma]_\sim$ & $\gamma' = [\sigma \cdot \sigma']_\sim$ for some σ, σ'

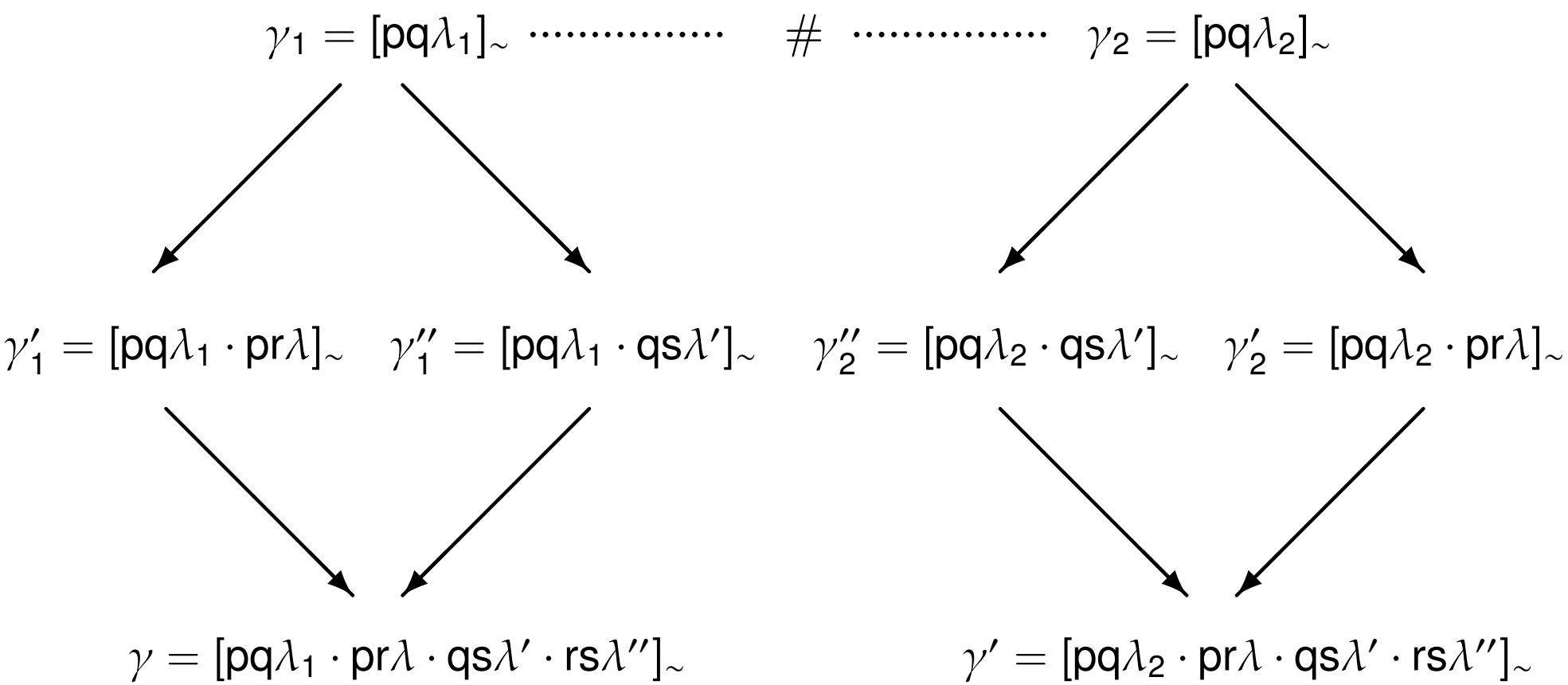
Conflict relation

$[\sigma]_\sim \# [\sigma']_\sim$ if $\sigma@p \ \# \ \sigma'@p$ for some p

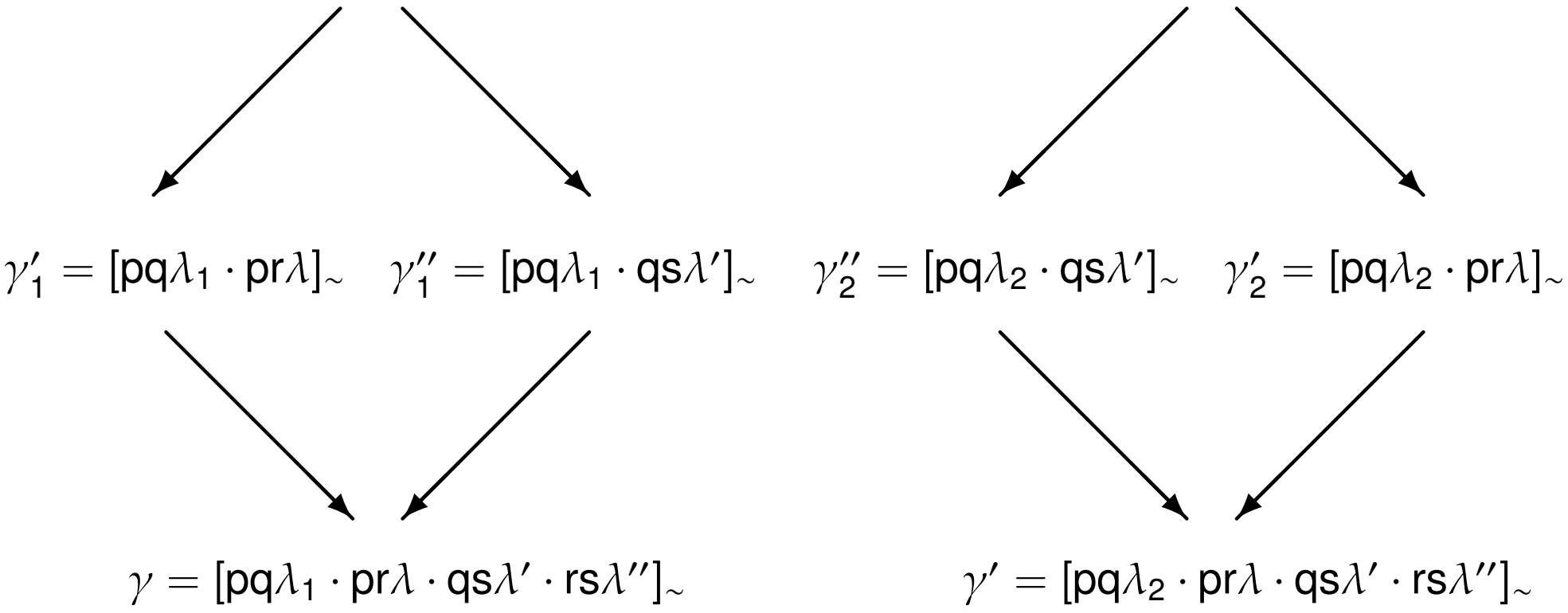
$$[pq\lambda \cdot rp\lambda_1 \cdot qp\lambda']_\sim \ \# \ [pq\lambda \cdot rp\lambda_2]_\sim \quad \text{because}$$

$$(pq\lambda \cdot rp\lambda_1 \cdot qp\lambda')@p = q!\lambda \cdot r?\lambda_1 \cdot q?\lambda' \ \# \ q!\lambda \cdot r?\lambda_2 = (pq\lambda \cdot rp\lambda_2)@p$$

$$\mathsf{G} = \mathsf{p} \to \mathsf{q} : (\lambda_1; \mathsf{p} \xrightarrow{\lambda} \mathsf{r}; \mathsf{q} \xrightarrow{\lambda'} \mathsf{s}; \mathsf{r} \xrightarrow{\lambda''} \mathsf{s} \boxplus \lambda_2; \mathsf{p} \xrightarrow{\lambda} \mathsf{r}; \mathsf{q} \xrightarrow{\lambda'} \mathsf{s}; \mathsf{r} \xrightarrow{\lambda''} \mathsf{s})$$



$$G = p \to q : (\lambda_1; p \xrightarrow{\lambda} r; q \xrightarrow{\lambda'} s; r \xrightarrow{\lambda''} s \boxplus \lambda_2; p \xrightarrow{\lambda} r; q \xrightarrow{\lambda'} s; r \xrightarrow{\lambda''} s)$$



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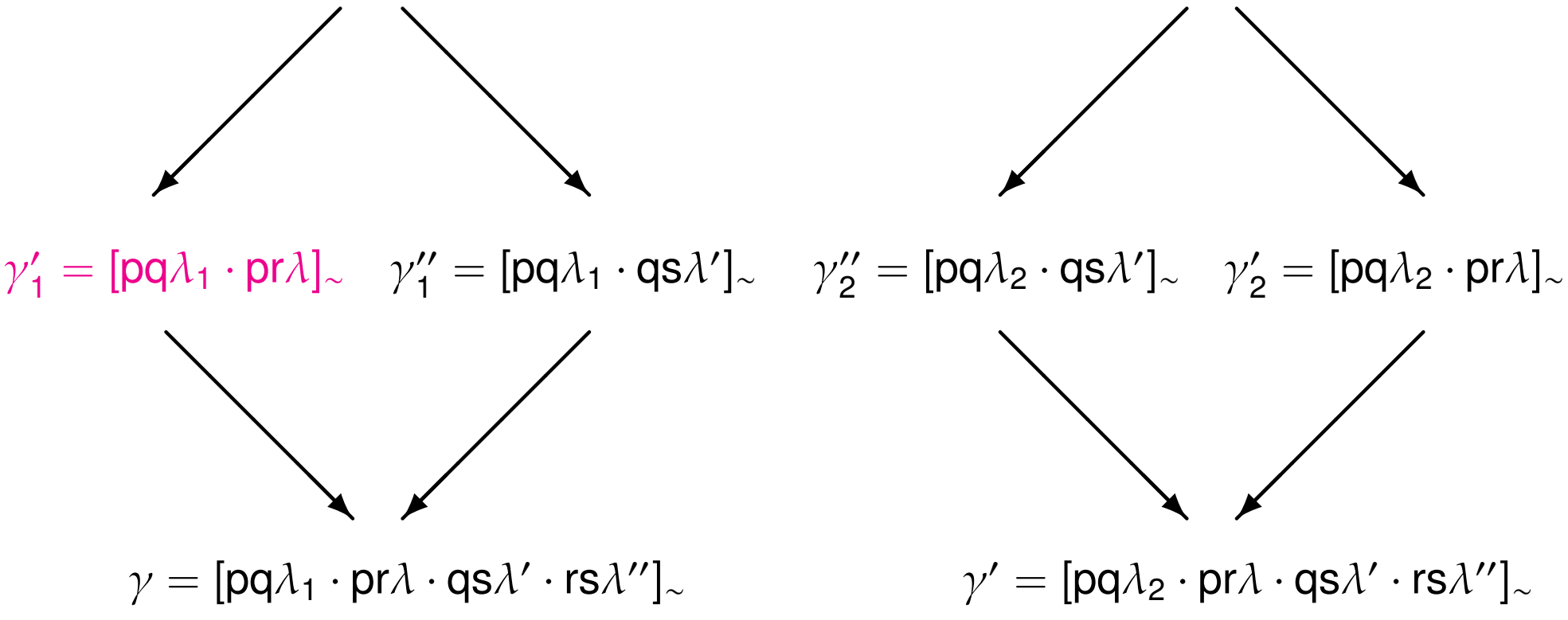

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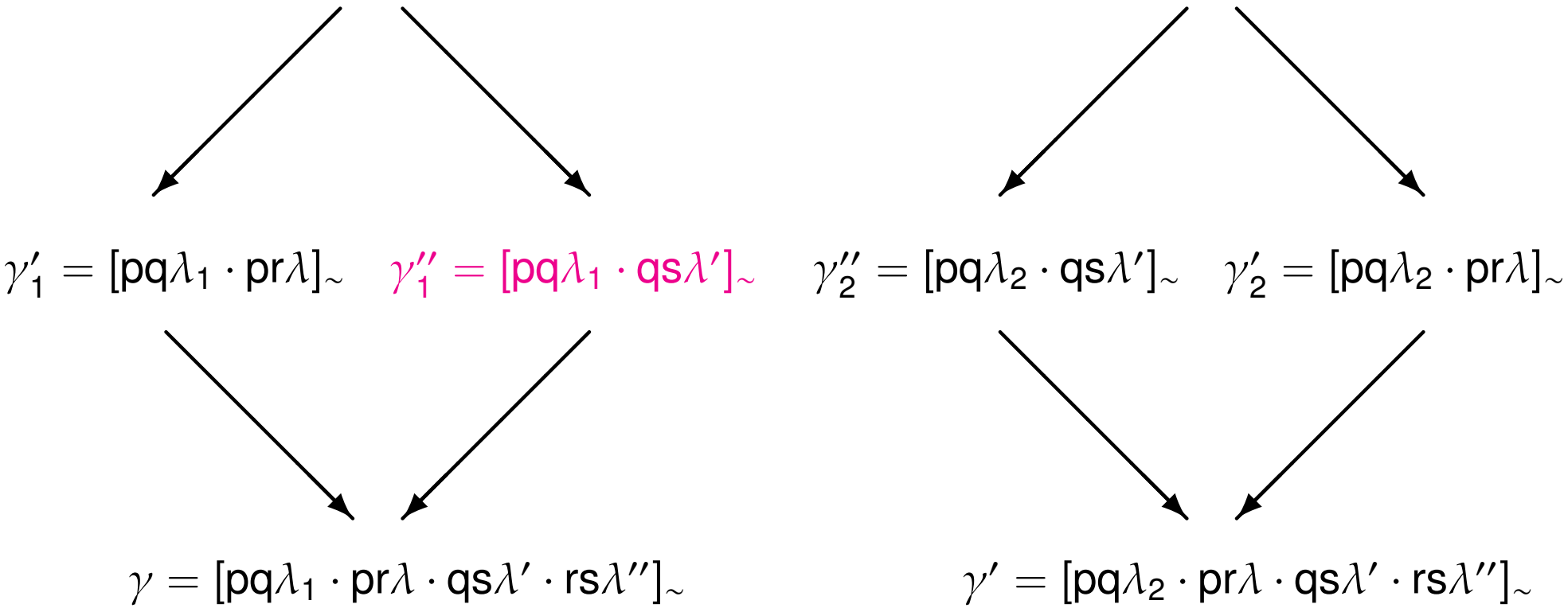

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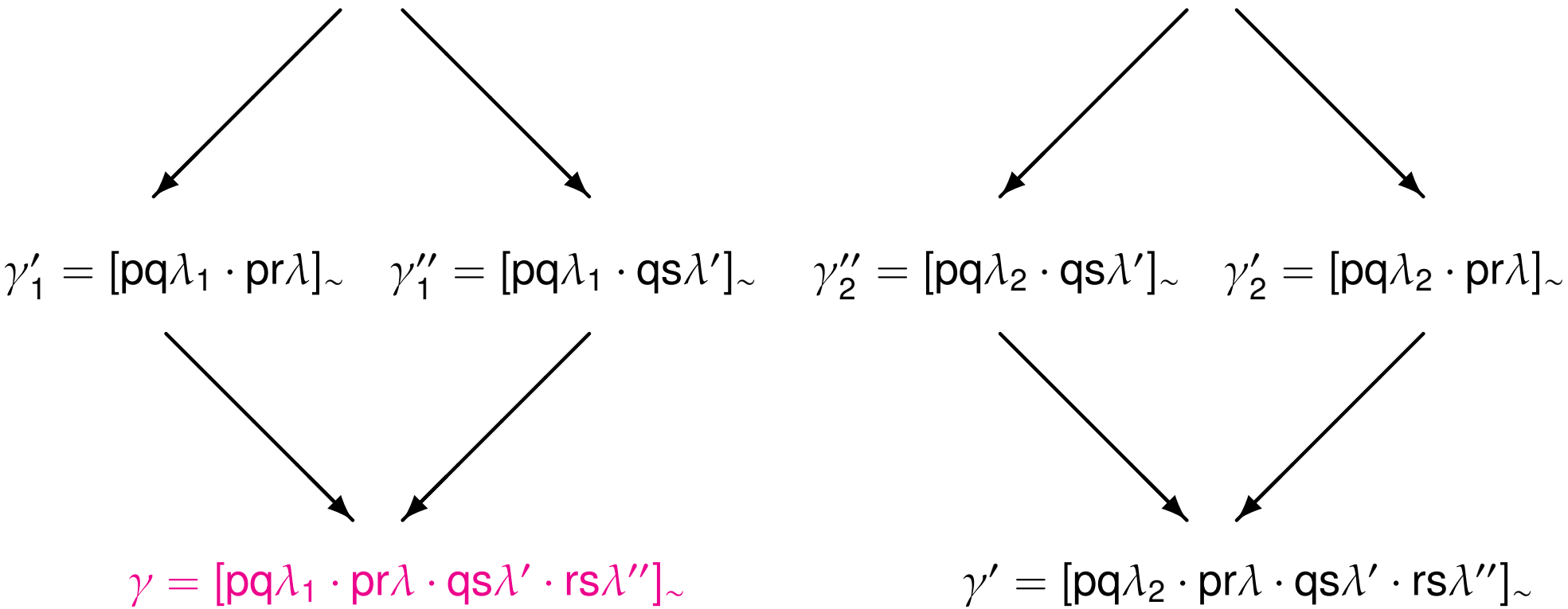

Relating a session FES with its type PES

Relating a session FES with its type PES

The FES of a session is not isomorphic to the PES of its type:

$$\nu = \{\mathsf{r} :: \mathsf{p}?\lambda \cdot \mathsf{s}!\lambda'', \mathsf{s} :: \mathsf{q}?\lambda' \cdot \mathsf{r}?\lambda''\}$$

The n-event ν has a single local history but two global histories.

Hence, there are two g-events corresponding to it:

$$\gamma = [\mathsf{pq}\lambda_1 \cdot \mathsf{pr}\lambda \cdot \mathsf{qs}\lambda' \cdot \mathsf{rs}\lambda'']_\sim \qquad \gamma' = [\mathsf{pq}\lambda_2 \cdot \mathsf{pr}\lambda \cdot \mathsf{qs}\lambda' \cdot \mathsf{rs}\lambda'']_\sim$$

However:

The FES of a session $\mathcal{S}^{\mathcal{N}}(\mathsf{N})$ and the PES of its type $\mathcal{S}^{\mathcal{G}}(\mathsf{G})$ yield isomorphic domains of configurations.

Configurations and proving sequences

Proving sequence of a FES $S = (E, \prec, \#)$: sequence $e_1; \cdots; e_n$ of distinct non-conflicting events such that

$$\forall i \leq n \; \forall e \in E : \; e \prec e_i \;\Rightarrow\; \exists j < i \,.\, \text{either } e = e_j \text{ or } e \# e_j \prec e_i$$

Proving sequence of a PES $S = (E, \leq, \#)$: sequence $e_1; \cdots; e_n$ of distinct non-conflicting events such that

$$\forall i \leq n \; \forall e \in E : \; e < e_i \;\Rightarrow\; \exists j < i \,.\, e = e_j$$

Every configuration of a FES or a PES can be enumerated (possibly in several different ways) as a proving sequence.

Auxiliary functions

The function comm(·) extracts the communication from n-events and g-events:

$$\begin{aligned} \mathsf{comm}(\nu) &= \mathsf{pq}\lambda \text{ if } \nu = \{\mathsf{p} :: \zeta \cdot \mathsf{q}!\lambda, \mathsf{q} :: \zeta' \cdot \mathsf{p}?\lambda\} \\ \mathsf{comm}([\sigma]_\sim) &= \alpha \text{ if } \sigma = \sigma' \cdot \alpha \end{aligned}$$

The function nec(·) reconstructs a sequence of n-events from a sequence of communications

$$\mathsf{nec}(\alpha_1 \cdots \alpha_n) = \nu_1; \ldots; \nu_n \text{ where} \ldots$$

The function gec(·) reconstructs a sequence of g-events from a sequence of communications

$$\mathsf{gec}(\alpha_1 \cdots \alpha_n) = \gamma_1; \ldots; \gamma_n \text{ where} \ldots$$

Correspondence between network transition sequences and FES proving sequences

Transition sequences of N $\iff$ proving sequences of $\mathcal{S}^{\mathcal{N}}(\mathrm{N})$

If $\mathrm{N} \xrightarrow{\sigma} \mathrm{N}'$, then $\mathrm{nec}(\sigma)$ is a proving sequence in $\mathcal{S}^{\mathcal{N}}(\mathrm{N})$.

If $\nu_1; \cdots; \nu_n$ is a proving sequence in $\mathcal{S}^{\mathcal{N}}(\mathrm{N})$, then $\mathrm{N} \xrightarrow{\sigma} \mathrm{N}'$, where $\sigma = \mathrm{comm}(\nu_1) \cdots \mathrm{comm}(\nu_n)$.

Transition sequences of G $\iff$ proving sequences of $\mathcal{S}^{\mathcal{G}}(\mathrm{G})$

If $\mathrm{G} \xrightarrow{\sigma} \mathrm{G}'$, then $\mathrm{gec}(\sigma)$ is a proving sequence in $\mathcal{S}^{\mathcal{G}}(\mathrm{G})$.

If $\gamma_1; \cdots; \gamma_n$ is a proving sequence in $\mathcal{S}^{\mathcal{G}}(\mathrm{G})$, then $\mathrm{G} \xrightarrow{\sigma} \mathrm{G}'$, where $\sigma = \mathrm{comm}(\gamma_1) \cdot \cdots \cdot \mathrm{comm}(\gamma_n)$.

Isomorphism of configuration domains

Relies on Subject Reduction (SR) and Session Fidelity (SF).

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$\Downarrow$ *SF*

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- For typable sessions: equivalence of the two semantics (isomorphism of configuration domains).
- Future work: semantic counterpart for well-formedness conditions $\Rightarrow$ would enable synthesis of the FES of a network from the PES of its global type.
- Related work:
  - [TG18] E. Tuosto, R. Guanciale. Semantics of global view of choreographies. JLAMP 95, 2018.
  - [LMT22] U. de' Liguoro, Hernán C. Melgratti, E. Tuosto: Towards refinable choreographies. JLAMP. 127, 2022.

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4. Characterise the FESs/PESs obtained by interpreting sessions/global types (as done previously for fragments of CCS), or at least identify some of their structural properties.

Thank you