

UNCERTAINTIES,  
ADAPTABILITY, AND  
VERIFICATION

Valentina Castiglioni,  
Michele Loreti, and  
Simone Tini

OPCT, Bertinoro, 26 June 2023



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aka Project

ULTRON

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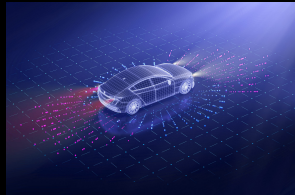
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The name of the game

System: agent + environment

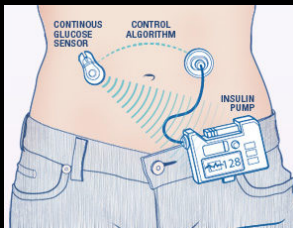
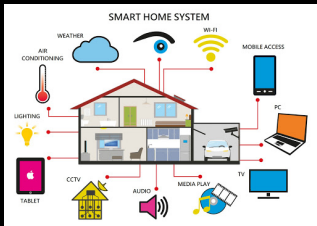
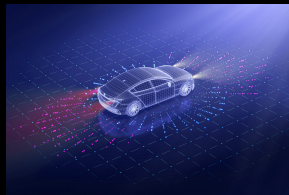
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System: Agent + environment



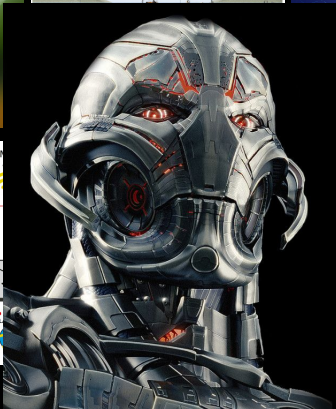
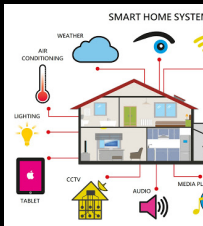
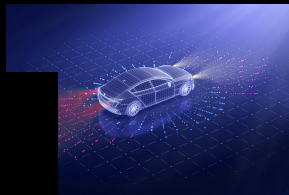
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- must **adapt** to changes in the environment

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    - ▶ attacks

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    - ▶ heterogeneous devices
    - ▶ failures, conflicts, ...



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Problem: **uncertainty**

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Our aim is to provide the tools to model and verify the robustness of cyber-physical systems against uncertainties

## The model

- We model agent and environment **separately**.
- We model their **interaction** in terms of the **changes** they induce on a set of application-relevant **data**.
- Setting: **discrete time**, **continuous space**

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- We model agent and environment **separately**.
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  - Setting: **discrete time**, **continuous space**
- ▶ System behaviour: **how data evolve in time**.

## The model (intuition)

Data space:  $\mathcal{D}$  closed subset of  $\mathbb{R}^n$

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$$\begin{aligned} P & ::= (\bar{e} \rightarrow \bar{x}).P' \mid \text{if } [e] P_1 \text{ else } P_2 \mid \sum_{i \in I} r_i \cdot P_i \mid P_1 \parallel P_2 \mid A \\ e & ::= \text{value} \mid x \mid \text{op}_k(e_1, \dots, e_k) \end{aligned}$$

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**Environment:** function  $\mathcal{E} : \mathcal{D} \rightarrow \Pi(\mathcal{D})$  that maps a *data state* into a *probability measure over data states*

**Configuration:** a particular state of the system  $\mathbf{c} = \langle P, \mathbf{d} \rangle_{\mathcal{E}}$

## System behaviour

$$\text{Agent semantics: } P\text{step}(P, \mathbf{d}) = \sum_{i \in I} q_i \cdot \delta(P_i, \mathbf{d}_i)$$

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$$\text{Agent semantics: } P_{\text{step}}(P, d) = \sum_{i \in I} q_i \cdot \delta(P_i, d_i)$$

Dirac's delta

probability weights

effects on d

process at next step

## System behaviour

$$\text{Agent semantics: } Pstep(P, \mathbf{d}) = \sum_{i \in I} q_i \cdot \delta(P_i, \mathbf{d}_i)$$

One-step configuration semantics:

$$Cstep(\langle P, \mathbf{d} \rangle_{\mathcal{E}})(C) = \sum_{(P', \mathbf{d}')} Pstep(P, \mathbf{d})(P', \mathbf{d}') \cdot \langle \delta(P'), \mathcal{E}(\mathbf{d}') \rangle_{\mathcal{E}}(C)$$

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$$\text{Agent semantics: } P\text{step}(P, \mathbf{d}) = \sum_{i \in I} q_i \cdot \delta(P_i, \mathbf{d}_i)$$

One-step configuration semantics: *discrete distribution induced by P*

$$C\text{step}(\langle P, \mathbf{d} \rangle_{\mathcal{E}})(C) = \sum_{(P', \mathbf{d}')} \boxed{P\text{step}(P, \mathbf{d})(P', \mathbf{d}')} \cdot \boxed{\langle \delta(P'), \mathcal{E}(\mathbf{d}') \rangle_{\mathcal{E}}(C)}$$

*continuous distribution induced by  $\mathcal{E}$*

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Multi-step configuration semantics:

$$M\text{step}_{c,i+1}(C) = \int_C C\text{step}(b)(C) d(M\text{step}_{c,i}(b))$$

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$$\text{Markov process generated by } C\text{step}(c)(C): M\text{step}_{c, \tau}(C)$$

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Multi-step configuration semantics:

Markov process generated by  $C_{\text{step}}(c)(C)$ :  $M_{\text{step}_{c,\tau}}(C)$

System behaviour: **the Evolution Sequence**

$$\mathcal{S}_{c,\tau}(D) = M_{\text{step}_{c,\tau}}(\langle \mathcal{P}, D \rangle_{\mathcal{E}})$$

sequence of  
probability  
measures over  
data states

## *Formalisation of robustness*

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**Robustness:** being able to function correctly even in the presence of uncertainties

Expressed by **measuring** the capability of an agent to tolerate perturbations in the environmental conditions and still fulfill its tasks

- ▶ We need to **measure the differences** between the behaviour of the system and its behaviour under the effect of perturbations, possibly **at different moments in time**

## Formalisation of robustness

Robustness is a temporal  
property of distances between  
system behaviours



## *Robustness Temporal Logic*

**RobTL:** a temporal logic for the specification of **requirements on the evolution of distances** between systems behaviours

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**RobTL expressions**

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*RobTL expressions*

- Compare distances and verify temporal requirements on them

*RobTL formulae*

## RobTL expressions

$\text{exp} ::= <^{\rho} \mid >^{\rho} \mid$

$F^l \text{exp} \mid G^l \text{exp} \mid \text{exp } U^l \text{exp} \mid$

$\min(\text{exp}, \text{exp}) \mid \max(\text{exp}, \text{exp}) \mid \sum_{k \in K} w_k \cdot \text{exp}_k \mid$

$\sigma(\text{exp}, \bowtie \zeta)$

## RobTL expressions

$\text{exp} ::= \langle \rho \mid \rangle \rho \mid$

### Atomic expressions

Evaluate the distance between two distributions at a given time

$F^l \text{exp} \mid G^l \text{exp} \mid \text{exp} U^l \text{exp} \mid$

$\min(\text{exp}, \text{exp}) \mid \max(\text{exp}, \text{exp}) \mid \sum_{k \in K} w_k \cdot \text{exp}_k \mid$

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## RobTL expressions

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Evaluate the distance between two distributions at a given time

**Penalty function**  $\rho: \mathcal{D} \rightarrow [0, 1]$  to quantify flaws in behaviour

Distance over data states  $m_\rho: \mathcal{D} \times \mathcal{D} \rightarrow [0, 1]$ :

$$m_\rho(\mathbf{d}_1, \mathbf{d}_2) = \max\{\rho(\mathbf{d}_2) - \rho(\mathbf{d}_1), 0\}$$

hemimetric expressing how much  $\mathbf{d}_2$  is worse than  $\mathbf{d}_1$  (wrt  $\rho$ )

**Wasserstein lifting** to distributions  $\mathbf{W}(m_\rho): \Pi(\mathcal{D}) \times \Pi(\mathcal{D}) \rightarrow [0, 1]$

$$\mathbf{W}(m_\rho)(\mu, \nu) = \inf_{\mathfrak{w} \in \mathfrak{W}(\mu, \nu)} \int_{\mathcal{D} \times \mathcal{D}} m_\rho(\mathbf{d}_1, \mathbf{d}_2) d\mathfrak{w}(\mathbf{d}_1, \mathbf{d}_2)$$

the inf of the expected values of the distance over the couplings



## RobTL expressions

$\text{exp} ::= <^{\rho} \mid >^{\rho} \mid$

### temporal expressions

Evaluate distance over time  
(idea  $\exists = \min, \forall = \max$ )

$F^l \text{exp} \mid G^l \text{exp} \mid \text{exp } U^l \text{exp} \mid$

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mathematical expressions

No surprises

$(\sum_k w_k = 1)$

## RobTL expressions

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$\sigma(\text{exp}, \bowtie \zeta)$

conditional expression

if  $\text{exp} \bowtie \zeta$ , then 0

else 1

## RobTL expressions

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## RobTL formulae

$$\varphi ::= \top \mid \Delta(\text{exp}, p) \bowtie \eta \mid \neg\varphi \mid \varphi \wedge \varphi \mid \varphi \mathcal{U}' \varphi$$

Evaluated over an evolution sequence  $\mathcal{S}$  and a time instant  $\tau_0$

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### Atomic formulae

- **exp**: RobTL expression defining the distance we want to analyse
- **p**: perturbation function applied to  $\mathcal{S}$  at time  $\tau_0$

$$p ::= f @ \tau \mid p ; p \mid p^n$$



## RobTL formulae

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### Atomic formulae

- **exp**: RobTL expression defining the distance we want to analyse
- **p**: perturbation function applied to  $\mathcal{S}$  at time  $\tau_0$
- the threshold  $\eta \in [0, 1]$  ( $\bowtie \in \{<, \leq, \geq, >\}$ )

$$p ::= f @ \tau \mid p ; p \mid p^n$$

## *The tool*

# SOFTWARE TOOL FOR THE ANALYSIS OF ROBUSTNESS IN THE UNKNOWN ENVIRONMENT

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# SOFTWARE TOOL FOR THE ANALYSIS OF ROBUSTNESS IN THE UNKNOWN ENVIRONMENT



Source available at <https://github.com/quasylab/jspear>

Demo available at <http://quasylab.unicam.it/stark/>

## The tool: STARK

- Specification language
  - ▶ agents
  - ▶ environment
  - ▶ RobTL expressions
  - ▶ perturbations
  - ▶ RobTL formulae
- Module for the simulation of evolution sequences and their perturbed versions
- Module for the evaluation of RobTL expressions
- Model checker for RobTL formulae

## The tool: STARK

- Specification language
  - ▶ agents
  - ▶ environment
  - ▶ RobTL expressions
  - ▶ perturbations
  - ▶ RobTL formulae
- Module for the simulation of evolution sequences and their perturbed versions
- Module for the evaluation of RobTL expressions (including the evaluation of confidence intervals)
- Model checker for RobTL formulae (three-valued semantics)

*Ongoing/future work*

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- TempOral aNaLYsis ( **TONY** ) module of **STARK**
- Application to biological systems and chemical networks

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- Predictive monitoring?
- Formal framework for AI?
- Analysis of system performance?

