

Quantitative Weakest Pre vs. Strongest Post

Linpeng Zhang **Benjamin Lucien Kaminski**



UNIVERSITÄT
DES
SAARLANDES



Open Problems in Concurrency Theory

June 28, 2023, Bertinoro, Italy

Roadmap

- 1 Revisiting Weakest Preconditions
- 2 Quantitative Weakest Pre
- 3 Quantitative Strongest Post
- 4 On Liberal vs. Non-liberal predicate/quantity transformers
 - Even Strongest Liberal Postconditions
(the ones Dijkstra & Scholten forgot (or didn't care) about)
- 5 Thoughts on Hoare logic and Incorrectness Logic

Weakest Preconditions

— revisited —

Weakest Preconditions – A Coarse Weather Forecast

Objective: Forecast tomorrow's weather in Bertinoro

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- **Predicates** (F, G, \dots) : *Mappings* φ : States \rightarrow {true, false} describing sets of states

Weakest Preconditions

[Dijkstra'75]

Question: Will it be warm tomorrow in Bertinoro?

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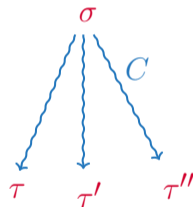
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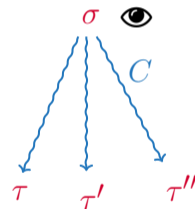
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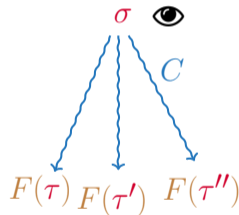
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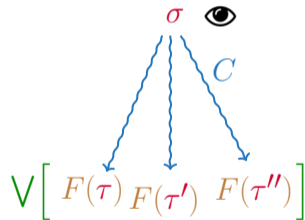
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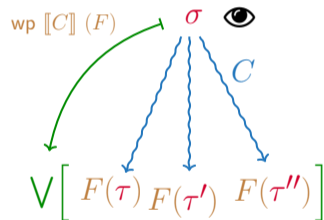
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$\text{wp } \llbracket C \rrbracket : \text{Pred} \rightarrow \text{Pred}$



Weakest Precondition Examples

$$x := 2$$

$$x := 27$$

$$x := x + 1$$

Weakest Precondition Examples

$x := 2$
 $\parallel x > 25$

$x := 27$

$x := x + 1$

Weakest Precondition Examples

// **false**

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// $x > 25$

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Weakest Precondition Examples

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// $x > 25$

// true

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// $x > 25$

// $x > 24$

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Quantitative Weakest Pre

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Quantitative Weakest Pre – A *Fine-grained* Weather Forecast

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- **Quantities** (f, g, \dots) : *Mappings* f : States $\rightarrow \mathbb{R} \cup \{-\infty, +\infty\}$

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[Dijkstra'75, Kozen'83, McIver&Morgan'05, OOPSLA'22]



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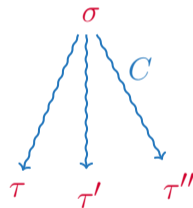
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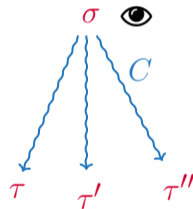
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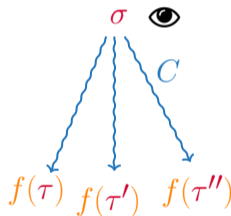
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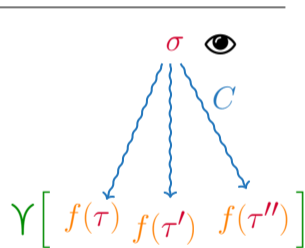
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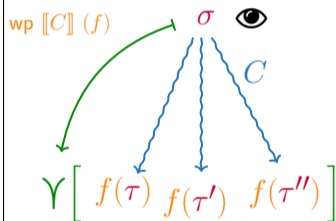
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— our main contribution —

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[OOPSLA'22]

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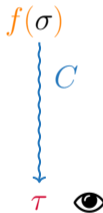


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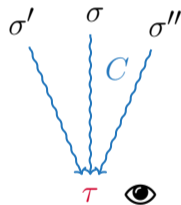


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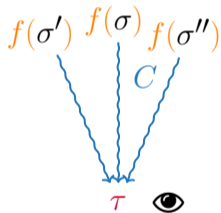


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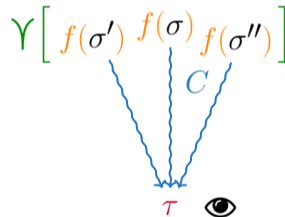


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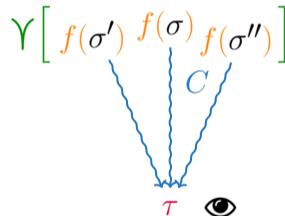
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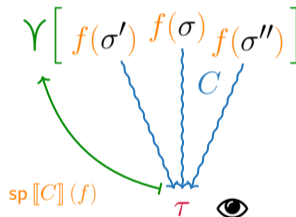
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Quantitative Strongest Post Example

$$x := x + 1$$

Quantitative Strongest Post Example

// x

$x := x + 1$

Quantitative Strongest Post Example

/// x

$x := x + 1$

/// $\mathcal{Q}v: [x = (x + 1) [x/v]] \wedge x [x/v]$

Quantitative Strongest Post Example

/// x

$x := x + 1$

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/// $\mathcal{Q}v: [x = v + 1] \wedge v$

Quantitative Strongest Post Example

/// x

$x := x + 1$

/// $\mathcal{Q}v: [x = (x + 1) [x/v]] \wedge x [x/v]$

/// $\mathcal{Q}v: [x = v + 1] \wedge v$

/// $x - 1$

Quantitative Strongest Post Example

/// **x**

$x := x + 1$;

/// $\mathcal{Q}v: [x = (x + 1) [x/v]] \wedge x [x/v]$

/// $\mathcal{Q}v: [x = v + 1] \wedge v$

/// **$x - 1$**

$x := x - y$

Quantitative Strongest Post Example

// x

$x := x + 1$;

// $\mathcal{Q}v$: $[x = (x + 1) [x/v]] \wedge x [x/v]$

// $\mathcal{Q}v$: $[x = v + 1] \wedge v$

// $x - 1$

$x := x - y$

// $x + y - 1$

Quantitative Strongest Post and the Flow of Quantitative Information

/// **x**

$x := x + 1$;

/// $\mathcal{Q}v: [x = (x + 1) [x/v]] \wedge x [x/v]$

/// $\mathcal{Q}v: [x = v + 1] \wedge v$

/// **$x - 1$**

$x := x - y$

/// **$x + y - 1$**

Quantitative Strongest Post and the Flow of Quantitative Information

Concrete State	Flowing Quantity
----------------	------------------

/// x

$x := x + 1$;

/// $\mathcal{Q}v: [x = (x + 1) [x/v]] \wedge x [x/v]$

/// $\mathcal{Q}v: [x = v + 1] \wedge v$

/// $x - 1$

$x := x - y$

/// $x + y - 1$

Quantitative Strongest Post and the Flow of Quantitative Information

/// x

$x := x + 1$;

/// $\mathcal{Q}v: [x = (x + 1) [x/v]] \wedge x [x/v]$

/// $\mathcal{Q}v: [x = v + 1] \wedge v$

/// $x - 1$

$x := x - y$

/// $x + y - 1$

Concrete State

$x: 16 \quad y: 3$

**Flowing
Quantity**

Quantitative Strongest Post and the Flow of Quantitative Information

/// x

$x := x + 1$;

/// $\mathcal{Q}v: [x = (x + 1) [x/v]] \wedge x [x/v]$

/// $\mathcal{Q}v: [x = v + 1] \wedge v$

/// $x - 1$

$x := x - y$

/// $x + y - 1$

Concrete State

$x: 16 \quad y: 3$

**Flowing
Quantity**

16

Quantitative Strongest Post and the Flow of Quantitative Information

/// x

$x := x + 1$;

/// $\mathcal{Q}v: [x = (x + 1) [x/v]] \wedge x [x/v]$

/// $\mathcal{Q}v: [x = v + 1] \wedge v$

/// $x - 1$

$x := x - y$

/// $x + y - 1$

Concrete State

$x: 16 \quad y: 3$

$x: 17 \quad y: 3$

**Flowing
Quantity**

16

Quantitative Strongest Post and the Flow of Quantitative Information

/// x

$x := x + 1$;

/// $\mathcal{Q}v: [x = (x + 1) [x/v]] \wedge x [x/v]$

/// $\mathcal{Q}v: [x = v + 1] \wedge v$

/// $x - 1$

$x := x - y$

/// $x + y - 1$

Concrete State

$x: 16 \quad y: 3$

$x: 17 \quad y: 3$

**Flowing
Quantity**

16

16

Quantitative Strongest Post and the Flow of Quantitative Information

	Concrete State	Flowing Quantity
$\mathbb{I} \ x$ $x := x + 1 ;$ $\mathbb{I} \ \mathcal{Z}v: [x = (x + 1) [x/v]] \wedge x [x/v]$ $\mathbb{I} \ \mathcal{Z}v: [x = v + 1] \wedge v$	$x: 16 \quad y: 3$	16
$\mathbb{I} \ x - 1$ $x := x - y$	$x: 17 \quad y: 3$	16
$\mathbb{I} \ x + y - 1$	$x: 14 \quad y: 3$	

Quantitative Strongest Post and the Flow of Quantitative Information

	Concrete State	Flowing Quantity
$\mathbb{I} \ x$ $x := x + 1 ;$	$x: 16 \quad y: 3$	16
$\mathbb{I} \ \mathcal{Z}v: [x = (x + 1) [x/v]] \wedge x [x/v]$		
$\mathbb{I} \ \mathcal{Z}v: [x = v + 1] \wedge v$		
$\mathbb{I} \ x - 1$ $x := x - y$	$x: 17 \quad y: 3$	16
$\mathbb{I} \ x + y - 1$	$x: 14 \quad y: 3$	16

Liberal vs. Non-liberal

Quantitative Weakest Pre

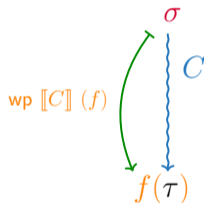
[Dijkstra'75, Kozen'83, McIver&Morgan'05, OOPSLA'22]

- **Given:** Program C and (post)quantity $f: \text{States} \rightarrow \mathbb{R} \cup \{-\infty, +\infty\}$
- **Question:** What is the anticipated value of f after executing C ?

Quantitative Weakest Pre

[Dijkstra'75, Kozen'83, McIver&Morgan'05, OOPSLA'22]

- **Given:** Program C and (post)quantity $f: \text{States} \rightarrow \mathbb{R} \cup \{-\infty, +\infty\}$
- **Question:** What is the anticipated value of f after executing C ?



- **Weakest pre:** Mapping from initial state σ to anticipated value of f evaluated in final state reached after execution of C on σ .

Quantitative Weakest Pre

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- **Question:** What is the anticipated value of f after executing C ?



- **Weakest pre:** Mapping from initial state σ to anticipated value of f evaluated in final state reached after execution of C on σ .
 - Assign $-\infty$ to $\text{wp} \llbracket C \rrbracket (f) (\sigma)$ if C does not terminate on σ .

Quantitative Weakest Liberal Pre

[Dijkstra'75, Kozen'83, McIver&Morgan'05, OOPSLA'22]

- **Given:** Program C and (post)quantity $f: \text{States} \rightarrow \mathbb{R} \cup \{-\infty, +\infty\}$
- **Question:** What is the anticipated value of f after executing C ?



- **Weakest liberal pre:** Mapping from initial state σ to anticipated value of f evaluated in final state reached after execution of C on σ .
 - Assign $-\infty$ to $\text{wlp } \llbracket C \rrbracket (f) (\sigma)$ if C does not terminate on σ .

Quantitative Weakest Liberal Pre

[Dijkstra'75, Kozen'83, McIver&Morgan'05, OOPSLA'22]

- **Given:** Program C and (post)quantity $f: \text{States} \rightarrow \mathbb{R} \cup \{-\infty, +\infty\}$
- **Question:** What is the anticipated value of f after executing C ?



- **Weakest liberal pre:** Mapping from initial state σ to anticipated value of f evaluated in final state reached after execution of C on σ .
 - Assign $+\infty$ to $wlp[C](f)(\sigma)$ if C does not terminate on σ .

Quantitative Strongest Post

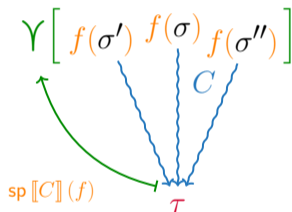
[Dijkstra'75, Kozen'83, Mclver&Morgan'05, OOPSLA'22]

- **Given:** Program C and (pre)quantity $f: \text{States} \rightarrow \mathbb{R} \cup \{-\infty, +\infty\}$
- **Question:** What is the retroicipated value of f before executing C ?

Quantitative Strongest Post

[Dijkstra'75, Kozen'83, Melver&Morgan'05, OOPSLA'22]

- **Given:** Program C and (pre)quantity $f: \text{States} \rightarrow \mathbb{R} \cup \{-\infty, +\infty\}$
- **Question:** What is the retroicipated value of f before executing C ?

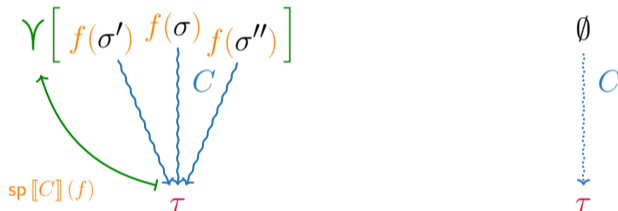


- **Strongest post:** Mapping from final state τ to maximal retroicipated value of f evaluated in initial state(s) that can reach τ by executing C .

Quantitative Strongest Post

[Dijkstra'75, Kozen'83, Mclver&Morgan'05, OOPSLA'22]

- **Given:** Program C and (pre)quantity $f: \text{States} \rightarrow \mathbb{R} \cup \{-\infty, +\infty\}$
- **Question:** What is the retroicipated value of f before executing C ?

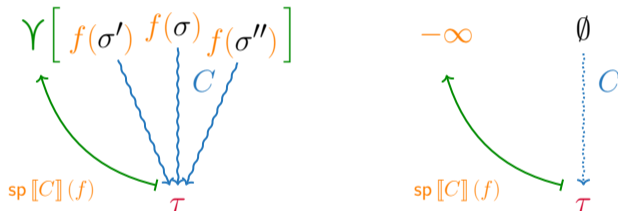


- **Strongest post:** Mapping from final state τ to maximal retroicipated value of f evaluated in initial state(s) that can reach τ by executing C .

Quantitative Strongest Post

[Dijkstra'75, Kozen'83, Melver&Morgan'05, OOPSLA'22]

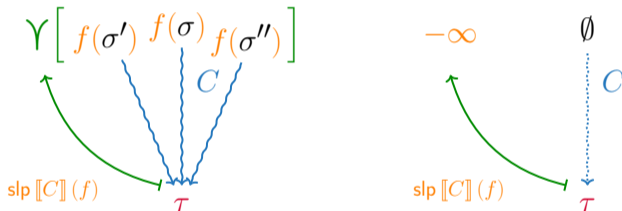
- **Given:** Program C and (pre)quantity $f: \text{States} \rightarrow \mathbb{R} \cup \{-\infty, +\infty\}$
- **Question:** What is the retroicipated value of f before executing C ?



- **Strongest post:** Mapping from final state τ to maximal retroicipated value of f evaluated in initial state(s) that can reach τ by executing C .
 - Assign $-\infty$ to $\text{sp}[C](f)(\tau)$ if τ is unreachable by executing C .

Quantitative Strongest Liberal Post [Dijkstra'75, Kozen'83, Melver&Morgan'05, OOPSLA'22]

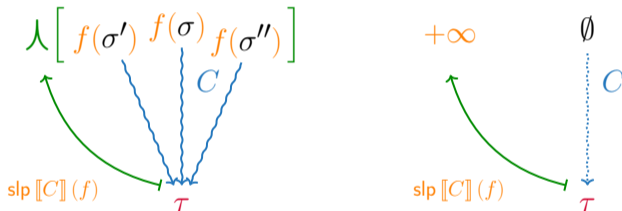
- **Given:** Program C and (pre)quantity $f: \text{States} \rightarrow \mathbb{R} \cup \{-\infty, +\infty\}$
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- **Strongest liberal post:** Mapping from final state τ to min. retroicip. value of f evaluated in initial state(s) that can reach τ by executing C .
 - Assign $-\infty$ to $\text{slp}[C](f)(\tau)$ if τ is unreachable by executing C .

Quantitative Strongest Liberal Post [Dijkstra'75, Kozen'83, Melver&Morgan'05, OOPSLA'22]

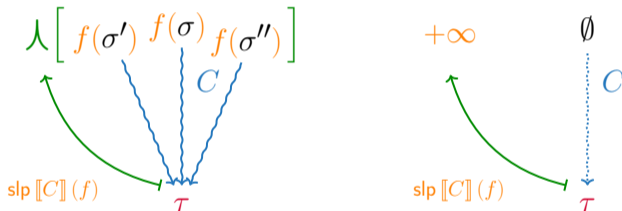
- **Given:** Program C and (pre)quantity $f: \text{States} \rightarrow \mathbb{R} \cup \{-\infty, +\infty\}$
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- **Strongest liberal post:** Mapping from final state τ to min. retroicip. value of f evaluated in initial state(s) that can reach τ by executing C .
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- **Strongest liberal post:** Mapping from final state τ to min. retroicip. value of f evaluated in initial state(s) that can reach τ by executing C .
 - Assign $+\infty$ to $\text{slp}[[C]](f)(\tau)$ if τ is unreachable by executing C .

Quantitative Strongest (Liberal) Post Examples

Reversible Assignment:

sp:

$x := x + 1$

slp:

$x := x + 1$

Quantitative Strongest (Liberal) Post Examples

Reversible Assignment:

sp:

// x

$x := x + 1$

slp:

// x

$x := x + 1$

Quantitative Strongest (Liberal) Post Examples

Reversible Assignment:

sp:

$\parallel x$

$x := x + 1$

$\parallel \mathcal{Q}v: [x = v + 1] \wedge v$

slp:

$\parallel x$

$x := x + 1$

Quantitative Strongest (Liberal) Post Examples

Reversible Assignment:

sp:

$\parallel x$

$x := x + 1$

$\parallel \exists v: [x = v + 1] \wedge v$

$\parallel x - 1$

slp:

$\parallel x$

$x := x + 1$

Quantitative Strongest (Liberal) Post Examples

Reversible Assignment:

sp:

$\parallel x$

$x := x + 1$

$\parallel \mathcal{Z}v: [x = v + 1] \wedge v$

$\parallel x - 1$

slp:

$\parallel x$

$x := x + 1$

$\parallel \mathcal{L}v: [x \neq v + 1] \vee v$

Quantitative Strongest (Liberal) Post Examples

Reversible Assignment:

sp:

// x

$x := x + 1$

// $\mathcal{Z}v: [x = v + 1] \wedge v$

// $x - 1$

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// x

$x := x + 1$

// $\mathcal{L}v: [x \neq v + 1] \vee v$

// $x - 1$

Quantitative Strongest (Liberal) Post Examples

Reversible Assignment:

sp:

// x

$x := x + 1$

// $\mathcal{Q}v: [x = v + 1] \wedge v$

// $x - 1$

slp:

// x

$x := x + 1$

// $\mathcal{L}v: [x \neq v + 1] \vee v$

// $x - 1$

Irreversible Assignment:

sp:

$x := 2$

slp:

$x := 2$

Quantitative Strongest (Liberal) Post Examples

Reversible Assignment:

sp:

// x

$x := x + 1$

// $\mathcal{Q}v: [x = v + 1] \wedge v$

// $x - 1$

slp:

// x

$x := x + 1$

// $\mathcal{L}v: [x \neq v + 1] \vee v$

// $x - 1$

Irreversible Assignment:

sp:

// x

$x := 2$

slp:

// x

$x := 2$

Quantitative Strongest (Liberal) Post Examples

Reversible Assignment:

sp:

$\parallel x$

$x := x + 1$

$\parallel \mathcal{Q}v: [x = v + 1] \wedge v$

$\parallel x - 1$

slp:

$\parallel x$

$x := x + 1$

$\parallel \mathcal{L}v: [x \neq v + 1] \vee v$

$\parallel x - 1$

Irreversible Assignment:

sp:

$\parallel x$

$x := 2$

$\parallel \mathcal{Q}v: [x = 2] \wedge v$

slp:

$\parallel x$

$x := 2$

Quantitative Strongest (Liberal) Post Examples

Reversible Assignment:

sp:

// x

$x := x + 1$

// $\exists v: [x = v + 1] \wedge v$

// $x - 1$

slp:

// x

$x := x + 1$

// $\exists v: [x \neq v + 1] \vee v$

// $x - 1$

Irreversible Assignment:

sp:

// x

$x := 2$

// $\exists v: [x = 2] \wedge v$

// $[x = 2]$

slp:

// x

$x := 2$

Quantitative Strongest (Liberal) Post Examples

Reversible Assignment:

sp:

$\parallel x$

$x := x + 1$

$\parallel \mathcal{Q}v: [x = v + 1] \wedge v$

$\parallel x - 1$

slp:

$\parallel x$

$x := x + 1$

$\parallel \mathcal{L}v: [x \neq v + 1] \vee v$

$\parallel x - 1$

Irreversible Assignment:

sp:

$\parallel x$

$x := 2$

$\parallel \mathcal{Q}v: [x = 2] \wedge v$

$\parallel [x = 2]$

slp:

$\parallel x$

$x := 2$

$\parallel \mathcal{L}v: [x \neq 2] \vee v$

Quantitative Strongest (Liberal) Post Examples

Reversible Assignment:

sp:

// x

$x := x + 1$

// $\mathcal{Q}v: [x = v + 1] \wedge v$

// $x - 1$

slp:

// x

$x := x + 1$

// $\mathcal{L}v: [x \neq v + 1] \vee v$

// $x - 1$

Irreversible Assignment:

sp:

// x

$x := 2$

// $\mathcal{Q}v: [x = 2] \wedge v$

// $[x = 2]$

slp:

// x

$x := 2$

// $\mathcal{L}v: [x \neq 2] \vee v$

// $[x \neq 2]$

Quantitative Strongest Post Loop Example

$hi := hi + 5$

```
while (  $lo < hi$  ) {  
     $lo := lo + 1$  }
```

Quantitative Strongest Post Loop Example

/// hi

hi := hi + 5

```
while(lo < hi) {  
    lo := lo + 1 }
```

Quantitative Strongest Post Loop Example

// hi

hi := hi + 5

// hi - 5

```
while(lo < hi) {  
    lo := lo + 1 }
```


Quantitative Strongest Post Loop Example

/// hi

hi := hi + 5

/// hi - 5

while (lo < hi) {
 lo := lo + 1 }

/// [lo ≥ hi] ∧ (hi - 5)

Weakest Pre vs. Strongest Post

Does what?

liberal vs. non-liberal

is about

Weakest Pre vs. Strongest Post

	Does what?	liberal vs. non-liberal	is about
weakest pre	predicts / anticipates	(non)termination	being able to reach a specified <i>final</i> state / coreachability

Weakest Pre vs. Strongest Post

	Does what?	liberal vs. non-liberal	is about
weakest pre	predicts / anticipates	(non)termination	being able to reach a specified <i>final</i> state / coreachability
strongest post	retrodicts / retrocipates	(un)reachability	being reachable from a specified <i>initial</i> state / reachability

Weakest Pre vs. Strongest Post

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Search on German Language...

"ist (passiv) erreichbar" zu "Erreichbarkeit" wie "kann (aktiv) erreichen" zu "???"

Asked 1 year, 1 month ago Modified 1 year, 1 month ago Viewed 111 times

0

noun

Ich suche ein Wort xyz (ein Nomen), das sich zu *erreichend* (im Sinne von A erreicht (aktiv) B und im Sinne von A kann einen Weg hin zu B zurücklegen) verhält wie *Erreichbarkeit* zu *erreichbar*. So etwas wie *Erreichendheit* oder *-keit*, nur dass sich die beiden sehr falsch anhören.

Share Edit Delete Flag

asked Apr 10, 2022 at 14:36

edited Apr 10, 2022 at 22:49

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Es gibt genau zwei Substantive, die vom Verb »erreichen« abgeleitet sind:

Erreichbarkeit

Dieses Substantiv beschreibt eine **Möglichkeit**, dass etwas am Ende eines Vorgangs steht.

1. Das was erreicht werden soll (z.B. ein Berggipfel) existierte bereits vorher, lag damals aber in der Ferne. Jetzt befindet man sich genau am selben Ort.
2. Das was erreicht werden soll (z.B. die Fusion zweier Unternehmen) existierte vorher nicht, sondern wurde erst durch den Vorgang erschaffen.

Erreichung

Dieses Substantiv beschreibt den sehr kurzen **Vorgang**, in dem die zuvor beschriebene Möglichkeit zur Realität wird.

Beide Substantive drehen sich also um das Ziel eines Vorgangs. Einmal als noch nicht realisierte Möglichkeit, das andere mal als Vorgang der Realisierung.

Andere Substantive, die vom Verb *erreichen* abgeleitet sind, gibt es nicht.

answered Apr 11, 2022 at 9:04

Hubert Schönast
117k ● 15 ● 203 ● 389

...eichen" zu "???"

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Galois Connections between Weakest Pre and Strongest Post

Dijkstra's classical predicate transformers:

$$G \implies \text{wlp } \llbracket C \rrbracket (F) \quad \text{iff} \quad \text{sp } \llbracket C \rrbracket (G) \implies F$$

Galois Connections between Weakest Pre and Strongest Post

Dijkstra's classical predicate transformers:

$$G \implies \text{wlp } \llbracket C \rrbracket (F) \quad \text{iff} \quad \text{sp } \llbracket C \rrbracket (G) \implies F$$

Our quantity transformers:

$$g \preceq \text{wlp } \llbracket C \rrbracket (f) \quad \text{iff} \quad \text{sp } \llbracket C \rrbracket (g) \preceq f$$

Galois Connections between Weakest Pre and Strongest Post

Dijkstra's classical predicate transformers:

$$G \implies \text{wlp } \llbracket C \rrbracket (F) \quad \text{iff} \quad \text{sp } \llbracket C \rrbracket (G) \implies F$$

Our quantity transformers:

$$g \preceq \text{wlp } \llbracket C \rrbracket (f) \quad \text{iff} \quad \text{sp } \llbracket C \rrbracket (g) \preceq f$$

$$\text{wp } \llbracket C \rrbracket (g) \preceq f \quad \text{iff} \quad g \preceq \text{slp } \llbracket C \rrbracket (f)$$

Thoughts on Hoare Logic and Incorrectness Logic

Total vs. Partial Correctness Hoare Logic

Total vs. Partial Correctness Hoare Logic

Total Correctness Hoare Triples:

$\langle \varphi \rangle C \langle \psi \rangle$ valid for **total** correctness iff $\varphi \implies \text{wp } [C] (\psi)$

Total vs. Partial Correctness Hoare Logic

Total Correctness Hoare Triples:

$\langle \varphi \rangle C \langle \psi \rangle$ valid for **total** correctness iff $\varphi \implies \text{wp } \llbracket C \rrbracket (\psi)$

Partial Correctness:

$\langle \varphi \rangle C \langle \psi \rangle$ valid for **partial** correctness iff $\varphi \implies \text{wlp } \llbracket C \rrbracket (\psi)$

Incorrectness Logic [O'Hearn'19]

Incorrectness Triples:

$[\varphi] C [\psi]$ valid for incorrectness iff $\psi \implies \text{sp} \llbracket C \rrbracket (\varphi)$

Incorrectness Logic [O'Hearn'19]

Incorrectness Triples:

$[\varphi] C [\psi]$ valid for **incorrectness** iff $\psi \implies \text{sp} \llbracket C \rrbracket (\varphi)$

Partial/Liberal/??? Incorrectness Triples:

$[\varphi] C [\psi]$ valid for **partial** incorrectness iff $\psi \implies \text{slp} \llbracket C \rrbracket (\varphi)$

Claim: There are Precisely 6 Fundamentally Different Hoare Logics

	implication	defines
G	\implies $\text{wp} \llbracket C \rrbracket (F)$	total correctness
G	\implies $\text{wlp} \llbracket C \rrbracket (F)$	partial correctness
$\text{wp} \llbracket C \rrbracket (F)$	\implies G	partial incorrectness
$\text{wlp} \llbracket C \rrbracket (F)$	\implies G	???
F	\implies $\text{sp} \llbracket C \rrbracket (G)$	(total) incorrectness
F	\implies $\text{slp} \llbracket C \rrbracket (G)$	partial incorrectness
$\text{sp} \llbracket C \rrbracket (G)$	\implies F	partial correctness
$\text{slp} \llbracket C \rrbracket (G)$	\implies F	???

Backup Slides

Rules for Strongest (Liberal) Post

C	$\mathbf{sp} \llbracket C \rrbracket (f)$	$\mathbf{slp} \llbracket C \rrbracket (f)$
diverge	$-\infty$	$+\infty$
$x := e$	$\mathcal{Z}\alpha: [x = e [x/\alpha]] \wedge f [x/\alpha]$	$\mathcal{L}\alpha: [x \neq e [x/\alpha]] \vee f [x/\alpha]$
$C_1 \mathbin{;} C_2$	$\mathbf{sp} \llbracket C_2 \rrbracket (\mathbf{sp} \llbracket C_1 \rrbracket (f))$	$\mathbf{slp} \llbracket C_2 \rrbracket (\mathbf{slp} \llbracket C_1 \rrbracket (f))$
$\{C_1\} \square \{C_2\}$	$\mathbf{sp} \llbracket C_1 \rrbracket (f) \vee \mathbf{sp} \llbracket C_2 \rrbracket (f)$	$\mathbf{slp} \llbracket C_1 \rrbracket (f) \wedge \mathbf{slp} \llbracket C_2 \rrbracket (f)$
if $(\varphi) \{C_1\}$ else $\{C_2\}$	$\mathbf{sp} \llbracket C_1 \rrbracket ([\varphi] \wedge f) \vee \mathbf{sp} \llbracket C_2 \rrbracket ([\neg\varphi] \wedge f)$	$\mathbf{slp} \llbracket C_1 \rrbracket ([\neg\varphi] \vee f) \wedge \mathbf{slp} \llbracket C_2 \rrbracket ([\varphi] \vee f)$
while $(\varphi) \{C'\}$	$[\neg\varphi] \wedge (\mathbf{lfp} Y. f \vee \mathbf{sp} \llbracket C' \rrbracket ([\varphi] \wedge Y))$	$[\varphi] \vee (\mathbf{gfp} Y. f \wedge \mathbf{slp} \llbracket C' \rrbracket ([\neg\varphi] \vee Y))$