OPEN PROBLEMS for TIMED SYSTEMS

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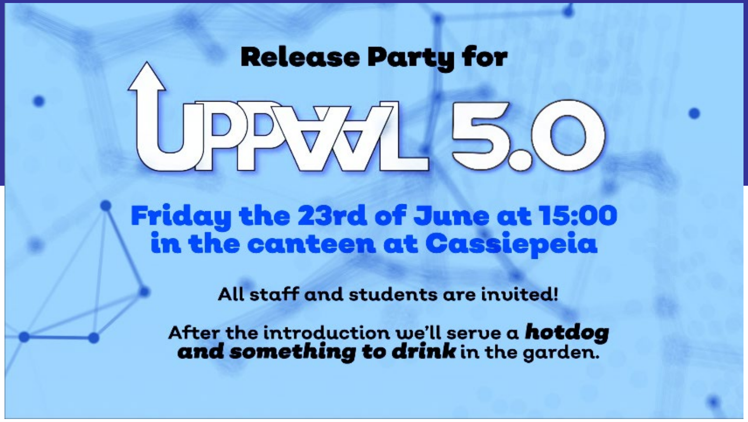
DIREC Digital Research Centre Denmark
erc
S4OS
VILLUM FONDEN



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1995 **CLASSIC**

CORA 2001

TIGA 2005

ECDAR 2010

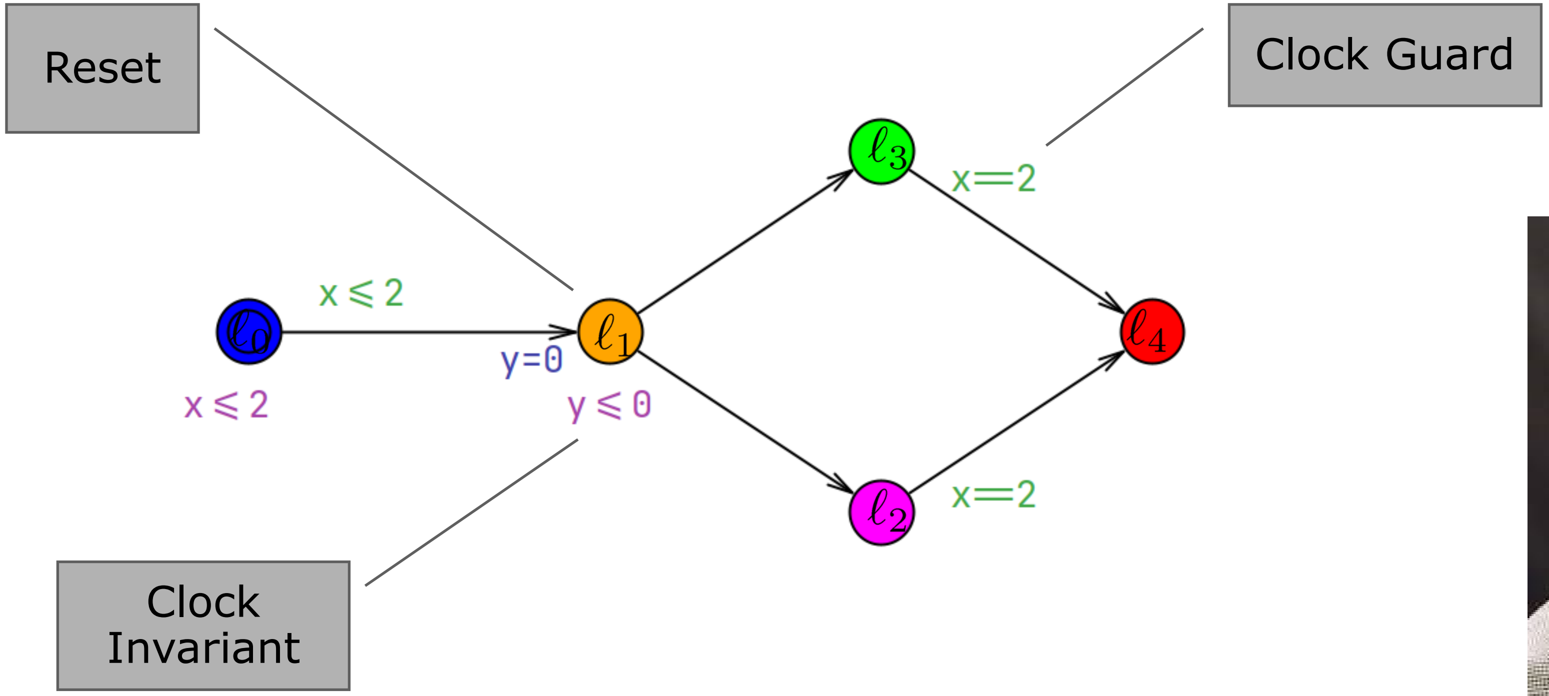
TRON 2004

SMC 2011

2014 **STRATEGO**

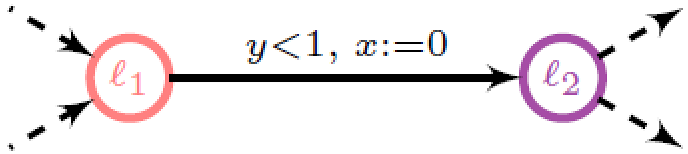
2023 **UPPAAL 5.0**

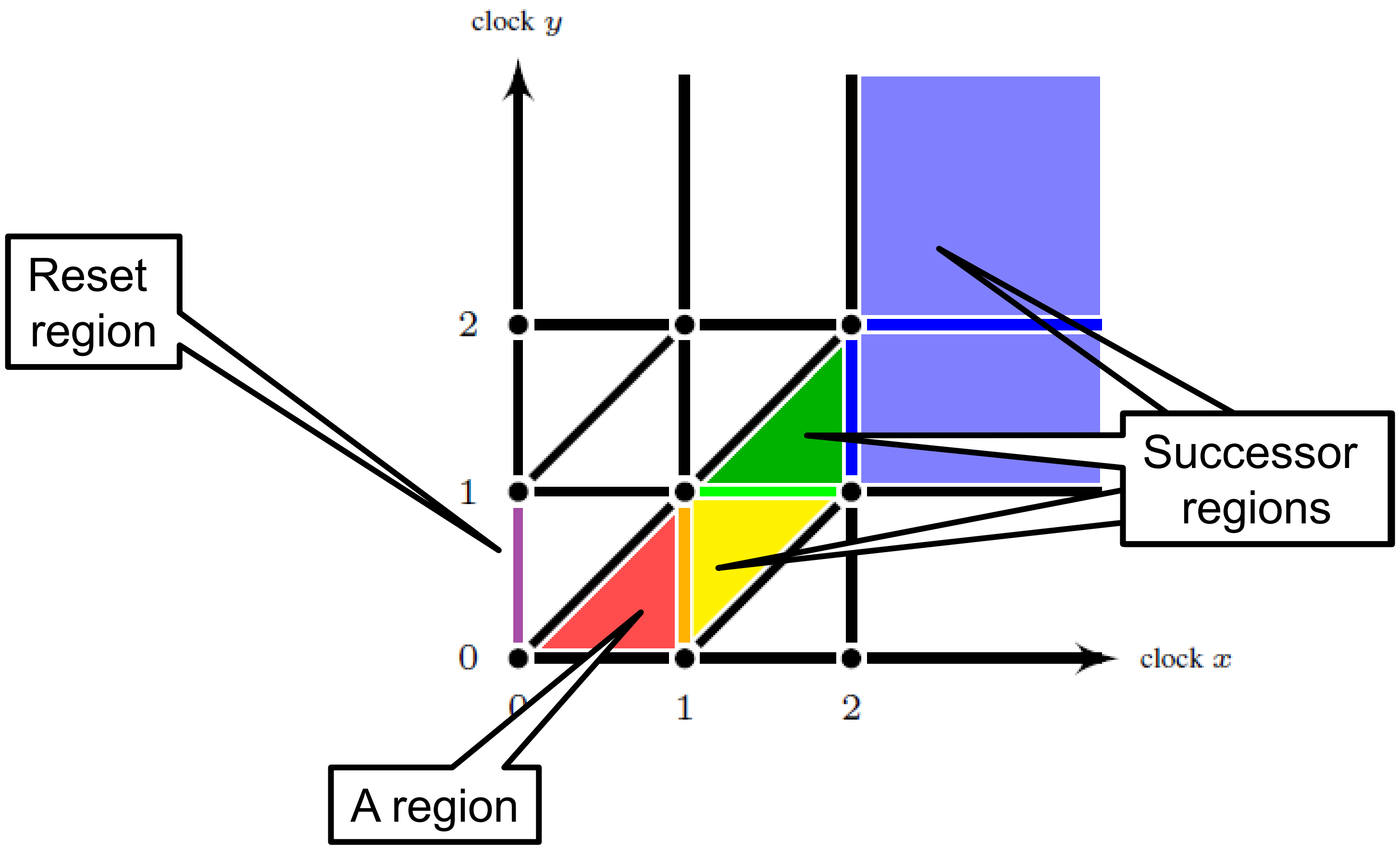
Timed Automata [Alur & Dill'89]



$$
\begin{aligned}
(\ell_0, x=0, y=0) &\xrightarrow{1.9} (\ell_0, x=1.9, y=1.9)\\
&\longrightarrow (\ell_1, x=1.9, y=0)\\
&\longrightarrow (\ell_2, x=1.9, y=0)\\
&\xrightarrow{0.1} (\ell_2, x=2.0, y=0.1)\\
&\longrightarrow (\ell_4, x=2.0, y=0.1)
\end{aligned}
$$

Regions – from infinite to finite





THM [AD90]
Reachability is decidable (and PSPACE-complete) for timed automata

THM [CY90]
Time-optimal reachability is decidable (and PSPACE-complete) for timed automata

Zones – From finite to efficiency

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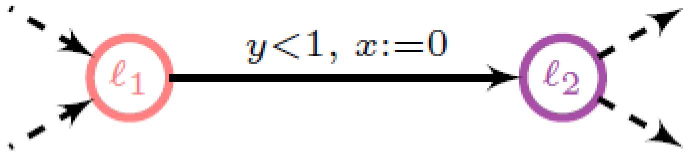

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Reachability is decidable (and PSPACE-complete) for timed automata

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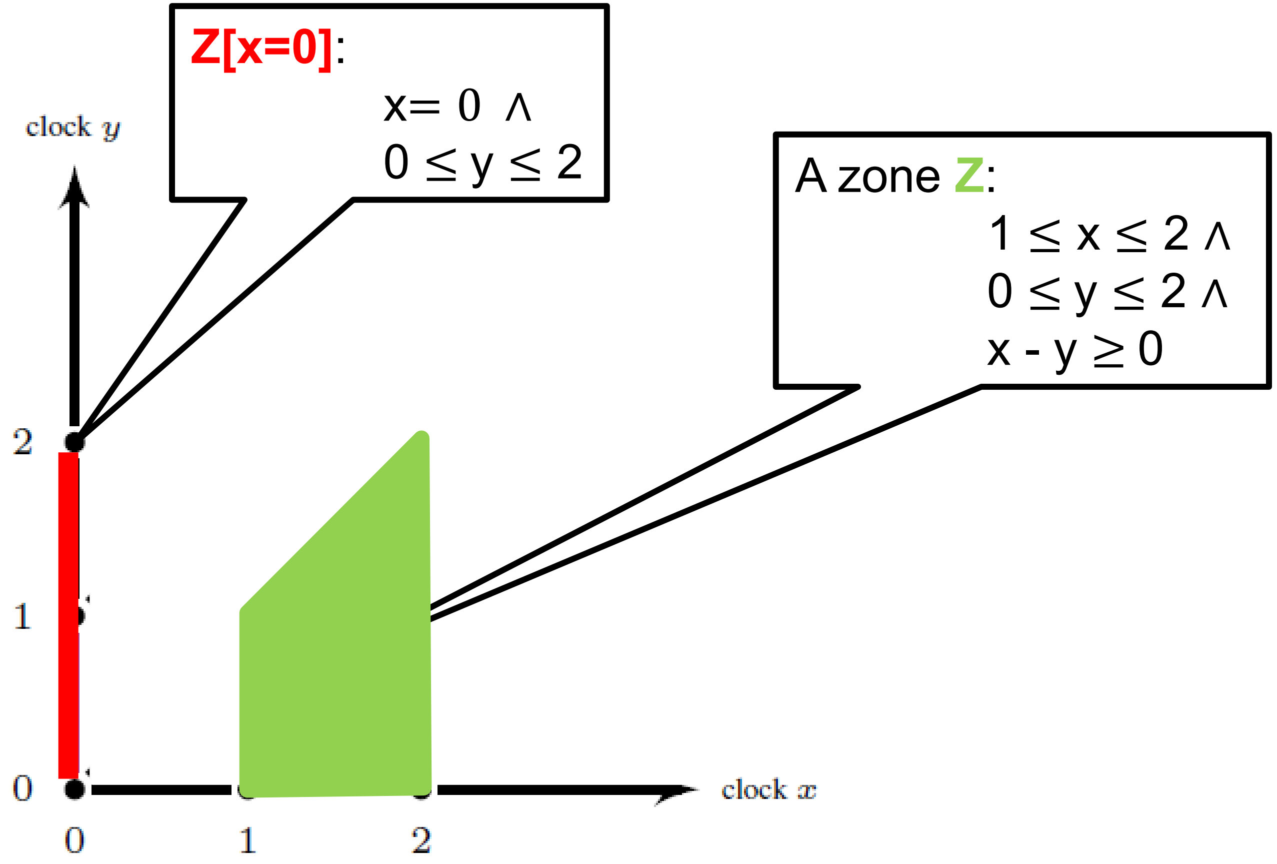
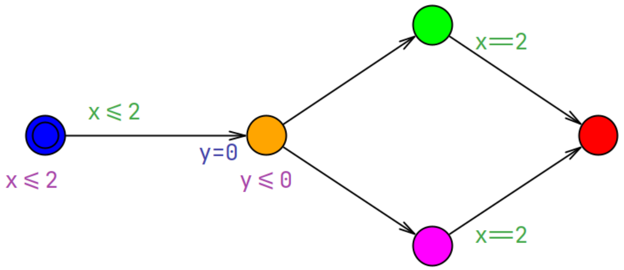
Time-optimal reachability is decidable (and PSPACE-complete) for timed automata



Theorem
The number of regions is $n! \cdot 2^n \cdot \prod_{x \in C}(2c_x + 2)$.

EXTENSIONS



Timed Automata

Cost / Opponent / Stochastic / Parameters

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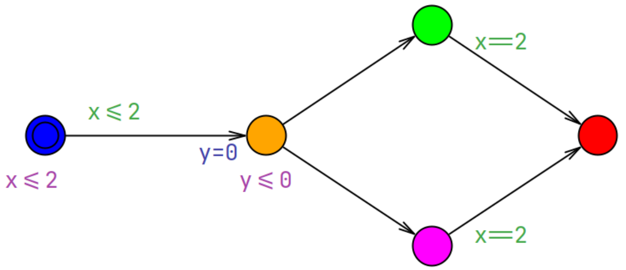
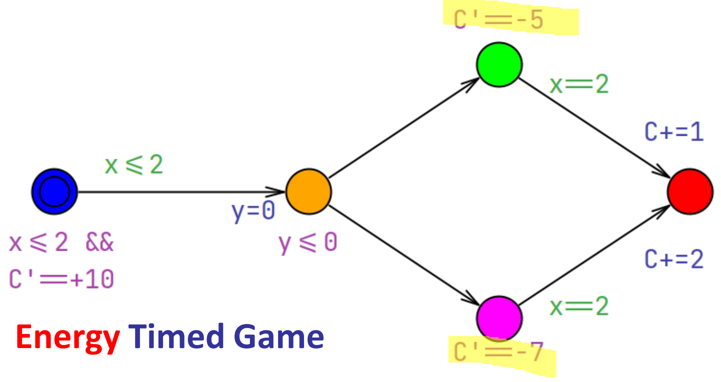
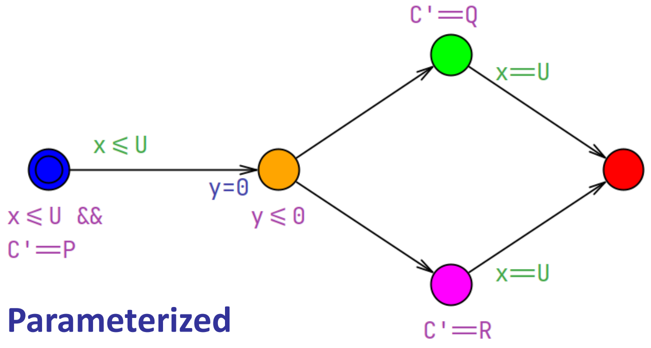


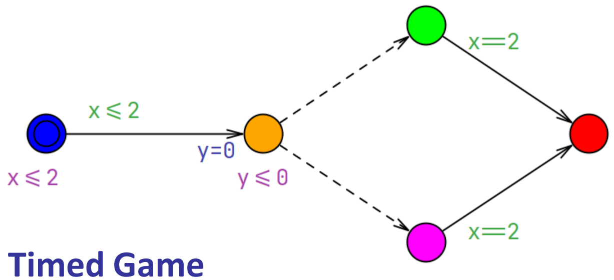
Timed Automata



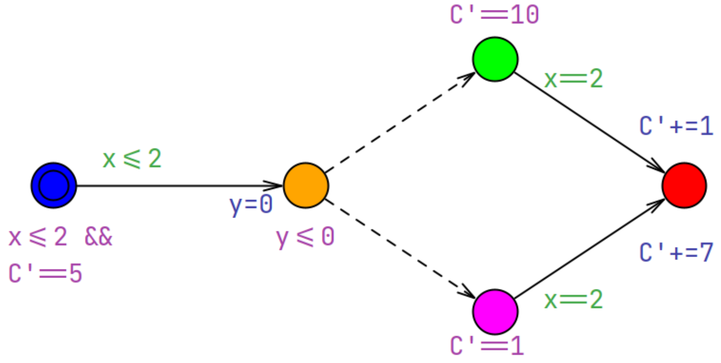
Energy Timed Game

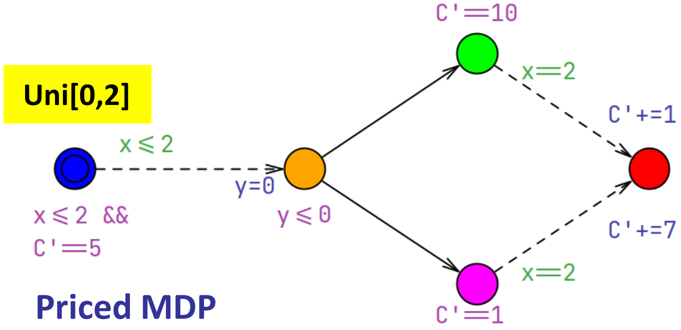


Parameterized Priced TA

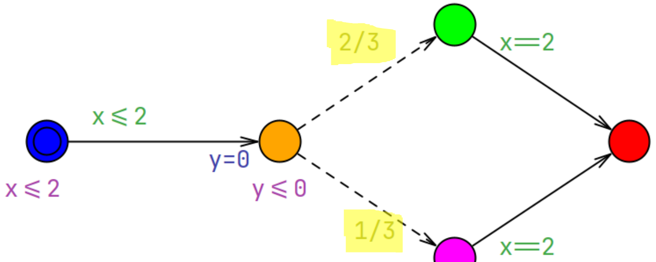


Timed Game

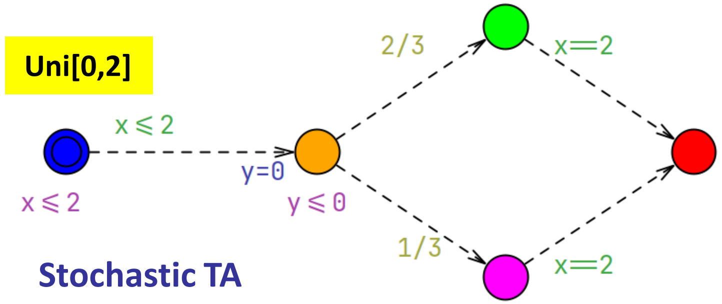




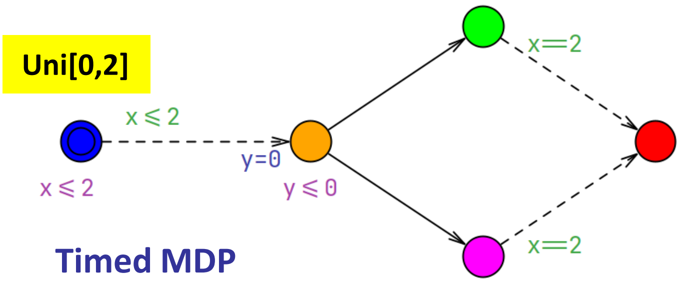
Priced MDP



Probabilistic TA



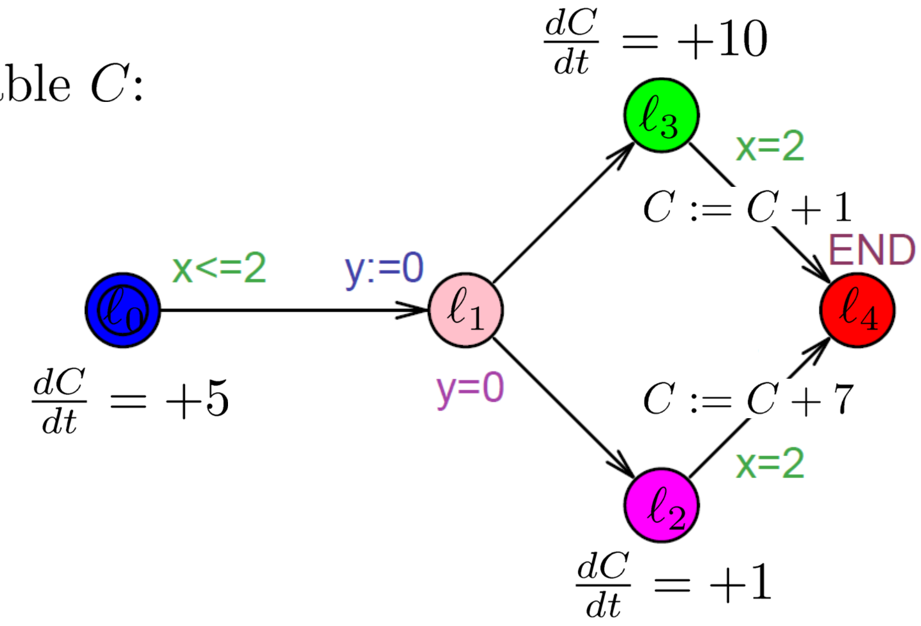
Stochastic TA



Timed MDP

Priced Timed Automata

Observer variable C:



$$(\ell_0, [0,0]) \xrightarrow{1.9}_{9.5} (\ell_0, [1.9, 1.9]) \rightarrow_0 (\ell_1, [1.9, 0]) \rightarrow_0 (\ell_2, [1.9, 0]) \xrightarrow{0.1}_{0.1} (\ell_2, [2, 0.1]) \rightarrow_7 (\ell_4, [2, 0.1])$$

$$\sum C_i = 16.6$$

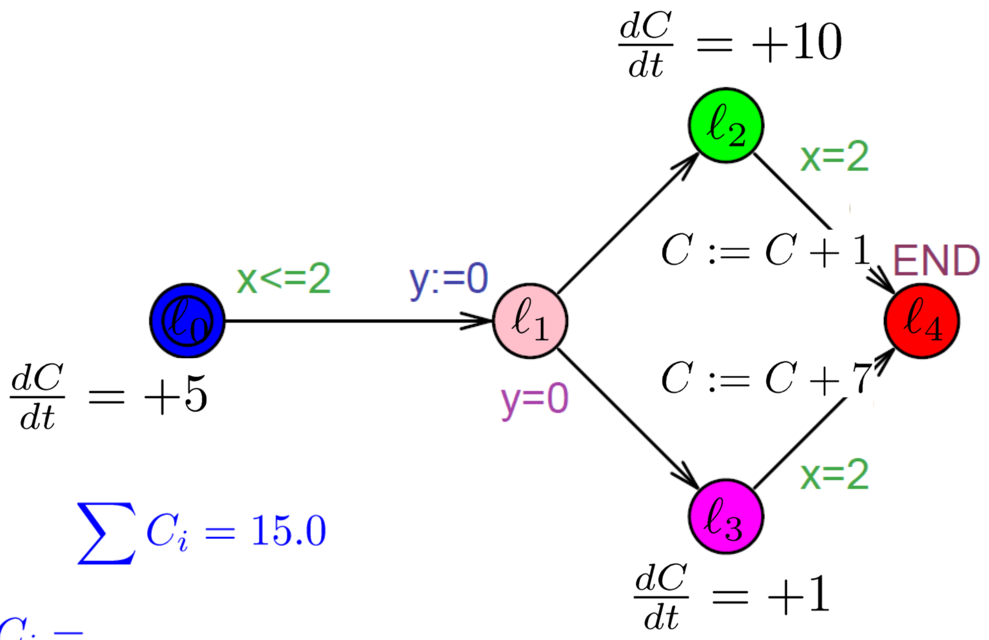
$$(\ell_0, [0,0]) \xrightarrow{1.2}_{6.0} (\ell_0, [1.2, 1.2]) \rightarrow_0 (\ell_1, [1.2, 0]) \rightarrow_0 (\ell_3, [1.2, 0]) \xrightarrow{0.8}_{8.0} (\ell_3, [2, 0.8]) \rightarrow_1 (\ell_4, [2, 0.8])$$

$$\sum C_i = 15.0$$

G Behrmann, A Fehnker, T Hune, K G Larsen, P Pettersson, J Romijn, F W. Vaandrager: Minimum-Cost Reachability for Priced Timed Automata. HSCC01

R Alur, S La Torre, G J. Pappas:Optimal Paths in Weighted Timed Automata. HSCC01

Priced Timed Automata



$\sum C_i = 16.6$ $\quad \sum C_i = 15.0$

$\sum C_i = ..$

Q: What is cheapest cost for reaching ℓ_4 ?

$$\inf_{0 \leq t \leq 2} \min\{5t + 10(2-t) + 1, 5t + (2-t) + 4\} = 9$$

➜ **strategy:** leave immediately ℓ_0, go to ℓ_3, and wait there 2 t.u.

G Behrmann, A Fehnker, T Hune, K G Larsen, P Pettersson, J Romijn, F W. Vaandrager: Minimum-Cost Reachability for Priced Timed Automata. HSCC01

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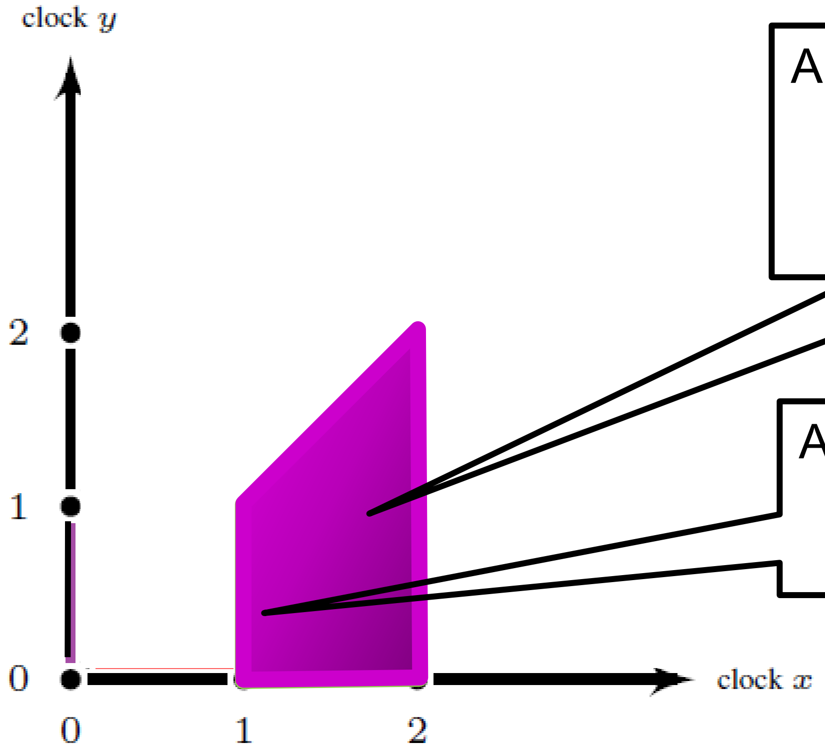
Priced Zone



A zone **Z**:

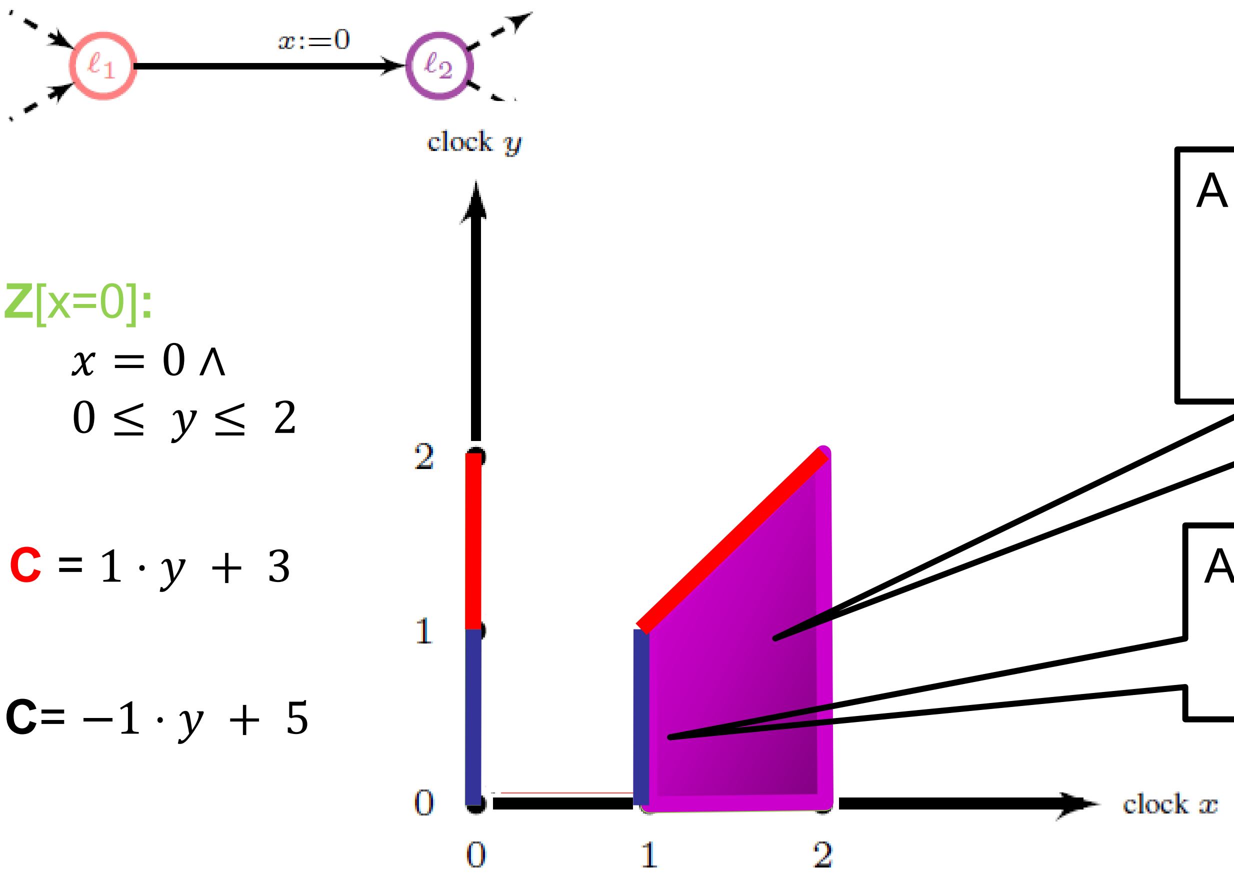
$$1 \leq x \leq 2 \ \wedge$$
$$0 \leq y \leq 2 \ \wedge$$
$$x - y \geq 0$$

A cost function **C**

$$C(x, y) =$$
$$2 \cdot x - 1 \cdot y + 3$$

K G Larsen, G Behrmann, E Brinksma, A Fehnker, T Hune, P Pettersson, J Romijn:
As Cheap as Possible: Efficient Cost-Optimal Reachability for Priced Timed Automata. CAV 2001:

Priced Zone -- Exploration



Z[x=0]:
$x = 0 \wedge$
$0 \leq y \leq 2$

C = $1 \cdot y + 3$

C= $-1 \cdot y + 5$

A zone **Z**:
$1 \leq x \leq 2 \wedge$
$0 \leq y \leq 2 \wedge$
$x - y \geq 0$

A cost function **C**
$C(x, y) =$
$2 \cdot x - 1 \cdot y + 3$

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Symbolic Branch & Bound Algorithm

THM [Behrmann, Fehnker, L ..01] [Alur,Torre,Pappas 01]
Optimal reachability is decidable for PTA

THM [Bouyer, Brojaue, Briuere, Raskin 07]
Optimal reachability is PSPACE-complete for PTA

Cost := ∞
Passed := $\emptyset$
Waiting := $\{(l_0, Z_0)\}$
while Waiting $\neq \emptyset$ **do**
  select (l, Z) from Waiting
  if $l = l_g$ and minCost(Z) < Cost **then**
    Cost := minCost(Z)
  if minCost(Z) + Rem$_{(l,Z)}$ $\geq$ C…
  if for all (l, Z') in Passed: $Z' \not\leq Z$ **then**
    add (l, Z) to Passed
    add all (l', Z') with $(l, Z) \rightarrow (l', Z')$
return Cost

$Z' \leq Z$

Z′* is bigger & cheaper than *Z

$\leq$ is a well-quasi ordering which guarantees termination!

K G Larsen, G Behrmann, E Brinksma, A Fehnker, T Hune, P Pettersson, J Romijn:
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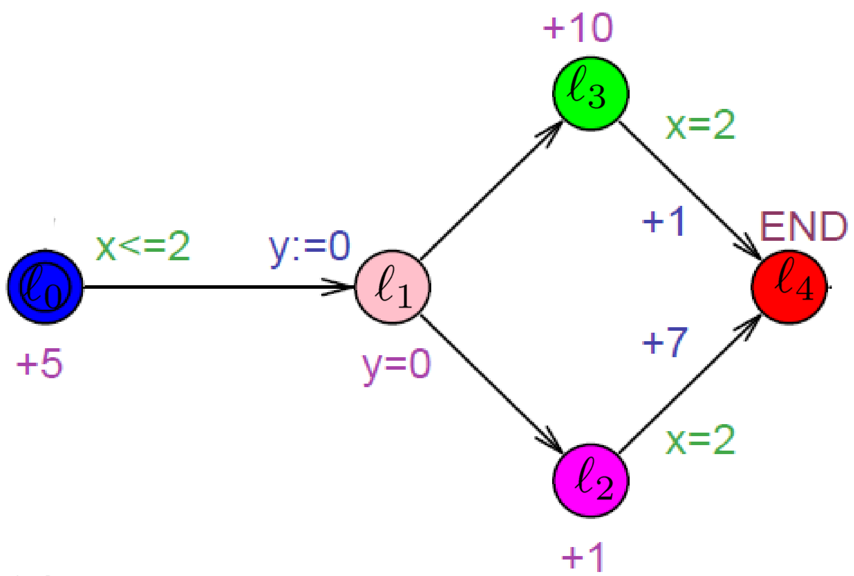
Optimal INFINITE Schedule



THM:[Bouyer,Brinksma, L 05]
The mean payoff optimization problem is PSPACE-c for PTA.
Corner Point Abstract Sound & Complete [Fahrenberg, L 09]
Same for discount-optimalilty.

OPEN: how to obtain efficient symbolic algorithm using **PRICED ZONES ?**

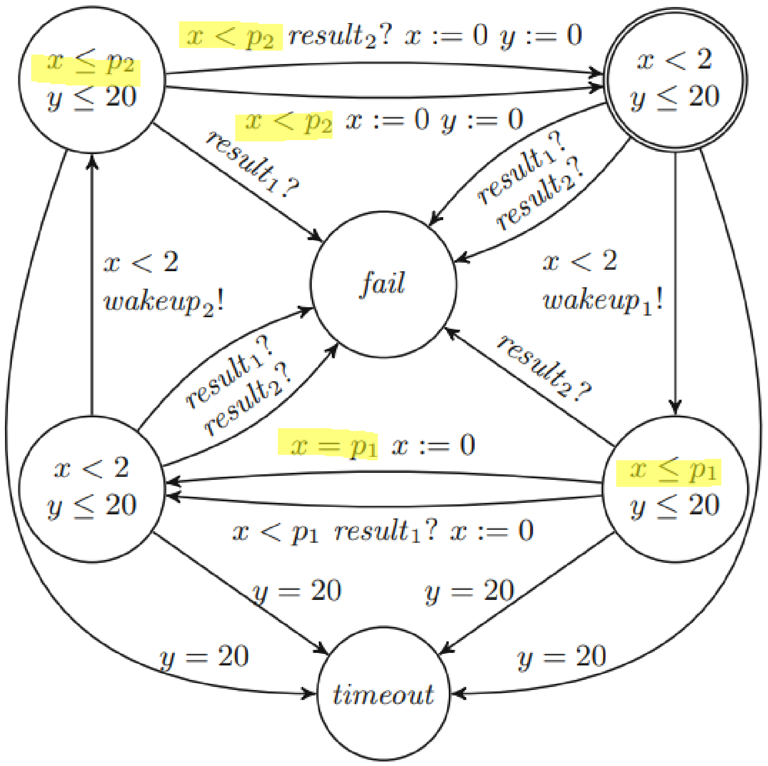
$$(\ell_0,[0,0]) \xrightarrow{1.2}_{6.0} (\ell_0,[1.2,1.2]) \rightarrow_0 (\ell_1,[1.2,0]) \rightarrow_0$$

$$(\ell_3,[1.2,0]) \xrightarrow{0.8}_{8.0} (\ell_3,[2,0.8]) \rightarrow_1 (\ell_4,[2,0.8])$$

$$\rightarrow_{2.0} (\ell_0,[0,0])$$

$$\sum_i C_i / \sum_i t_i = 17/2 = 8.5$$

Parameterized Timed Automata



(c) Controller with parameters p_1 and p_2

Table 1. Decidability of the language (non)emptiness problems

	discrete time integer parameters	continuous time integer parameters	continuous time real parameters
n clocks, m parameters 1 parametric clock only	decidable [3]	decidable	undecidable [17]
3 clocks, 1 parameter	undecidable	undecidable	undecidable [17]
3 clocks, 6 parameters	undecidable [3]	undecidable [3]	undecidable [3]

2 CLOCKS ?

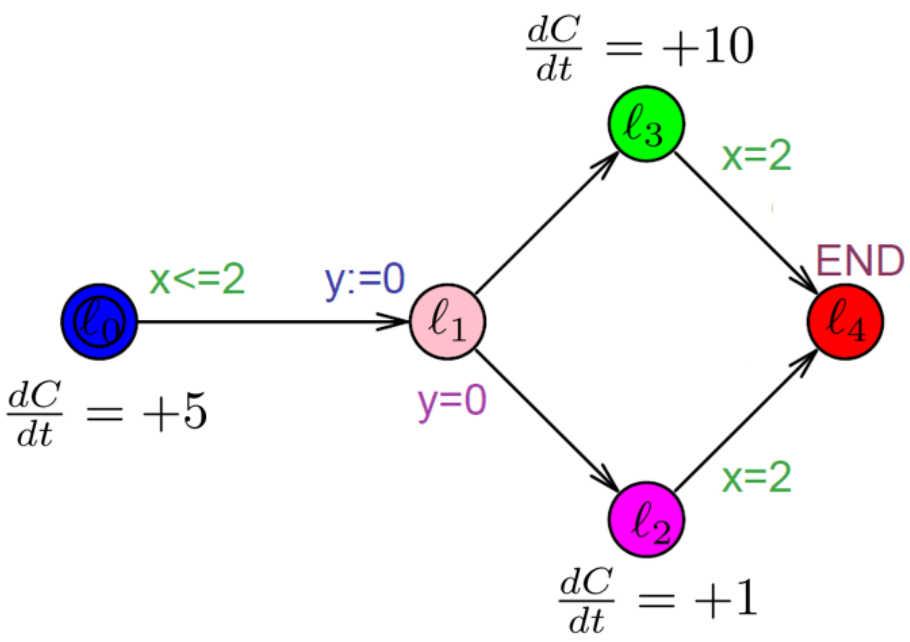
Priced TA?

[3] Alur, R., Henzinger, T., Vardi, M.: Parametric real-time reasoning. STOC93

[17] Miller: Decidability and complexity results for timed automata and semi-linear hybrid automata. HSCC. 2000

Benes, Bezdek, Larsen, Srba: Language Emptiness of Continuous-Time Parametric Timed Automata. ICALP15

Parameterized Priced Timed Automata



affine maps $f : \mathbb{R}^k \to \mathbb{R}$

$$f(\mathbf{x}) = \mathbf{a} \cdot \mathbf{x} + b$$

$$\mathbf{a} = (a_1, \ldots, a_k) \in \mathbb{Q}^k_{\geq 0}$$

$$b \in \mathbb{Q}_{\geq 0}$$

May be $\mathbb{N}_{\geq 0}$

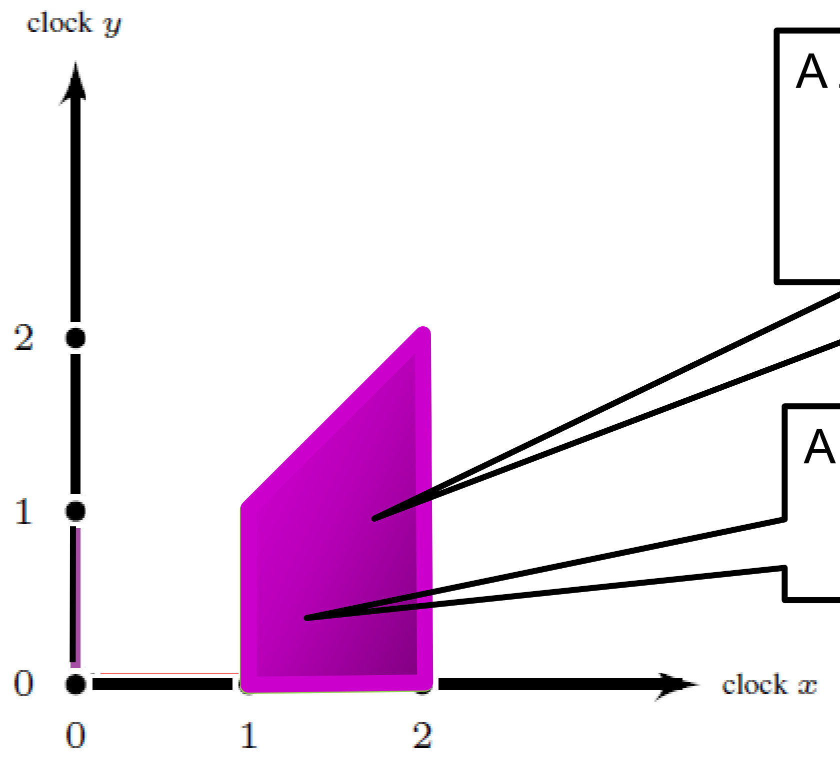
Q: What is cheapest cost for reaching ?

$$\inf_{0 \leq t \leq 2} \min\{\ t \cdot f_1 + (2-t) \cdot f_2 + f_4 \ ,\ \ t \cdot f_1 + (2-t) \cdot f_3 + f_5\ \}$$

Parameterized Priced Zone

affine maps $f : \mathbb{R}^k \rightarrow \mathbb{R}$

$$f(\mathbf{x}) = \mathbf{a} \cdot \mathbf{x} + b$$



A zone **Z**:

$$1 \leq x \leq 2 \ \wedge$$
$$0 \leq y \leq 2 \ \wedge$$
$$x - y \geq 0$$

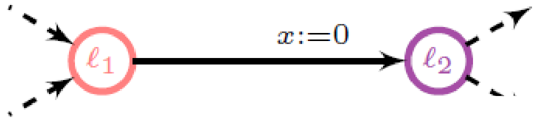
A cost function **C**

$$C(x, y) =$$
$$f_1 \cdot x + f_2 \cdot y + f_3$$

Parameterized Priced Zone -- Exploration

affine maps $f\colon \mathbb{R}^k \to \mathbb{R}$

$$f(\mathbf{x}) = \mathbf{a} \cdot \mathbf{x} + b$$



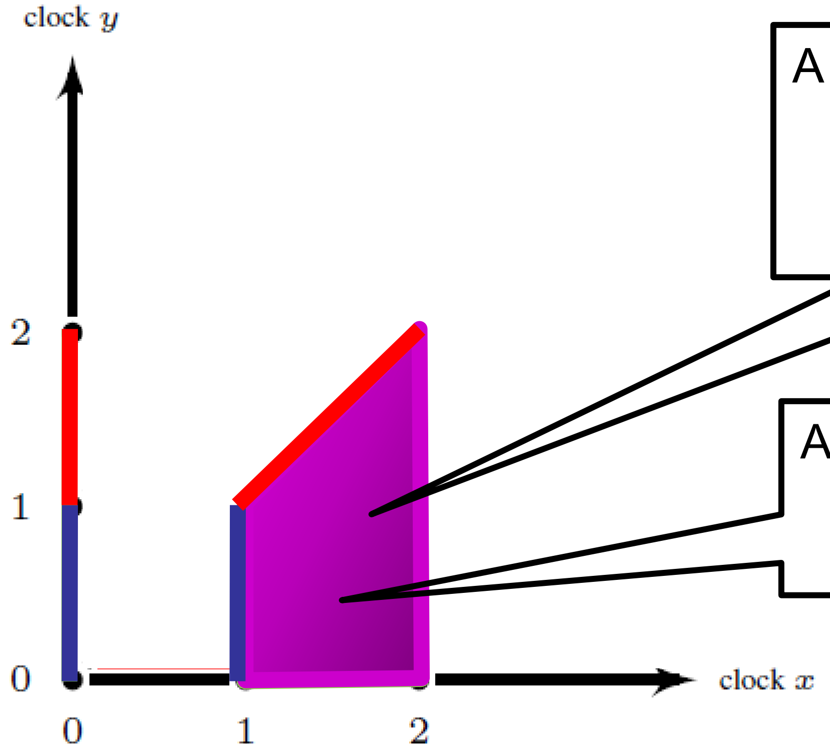
Z[x=0]:

$x = 0 \wedge$

$0 \leq y \leq 2$

C = $(f_1 + f_2) \cdot y + f_3$

C= $f_2 \cdot y + (f_3 + f_1)$



A zone **Z**:

$1 \leq x \leq 2 \wedge$

$0 \leq y \leq 2 \wedge$

$x - y \geq 0$

A cost function **C**

$C(x, y) =$

$f_1 \cdot x + f_2 \cdot y + f_3$

CASE $f_1 > 0$

Symbolic Branch & Bound Algorithm

Cost := ∞

Passed := $\emptyset$

Waiting := $\{(l_0, Z_0)\}$

while Waiting $\neq \emptyset$ **do**

  select (l, Z) from Waiting

  if $l = l_g$ and minCost(Z) < Cost **then**

    Cost := minCost(Z)

  if minCost(Z) + Rem$_{(l,Z)} \geq$ Co…

  if for all (l, Z') in Passed: $Z' \not\leq Z$ **then**

    add (l, Z) to Passed

    add all (l', Z') with $(l, Z) \rightarrow (l', Z')$

return Cost

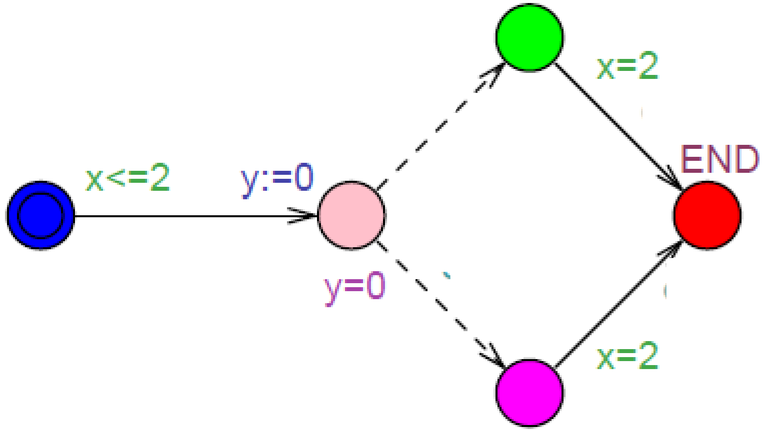
CONJECTURE

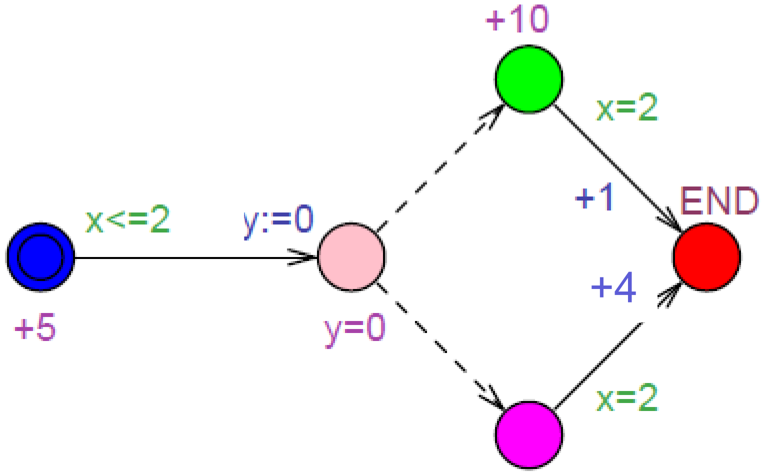
$Z' \leq Z$

Z′* is bigger & cheaper than *Z
for all parameter values

$\leq$ is a well-quasi ordering which guarantees termination!

Priced Timed Games





Time Optimal

$$\inf_{0\le t\le 2} \max\{t + (2 - t), t + (2 - t)\} =$$

Cost Optimal

$$\inf_{0\le t\le 2} \max\{5t + 10(2 - t) + 1, 5t + (2 - t) + 4\} =$$

P Bouyer, F Cassez, E Fleury, K GLarsen:
Synthesis of Optimal Strategies Using HyTech. GDV@CAV 2004

Decidable with **1** clock [BLMR06, HJM13]
- Acyclic [LTMM02]
- Bounded length [ABM04]
- Strong non-zeno cost-behaviour [BCFL04]

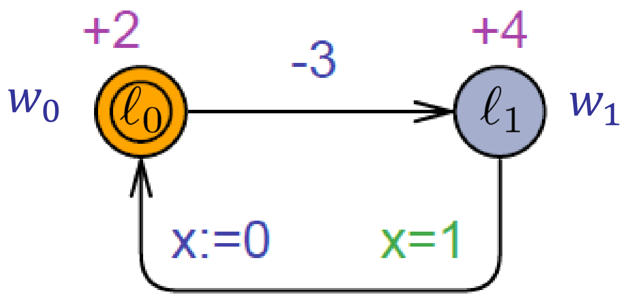
Undecidable with **3** clocks or more [BBR05, BBM06]

Open problem with **2** clocks

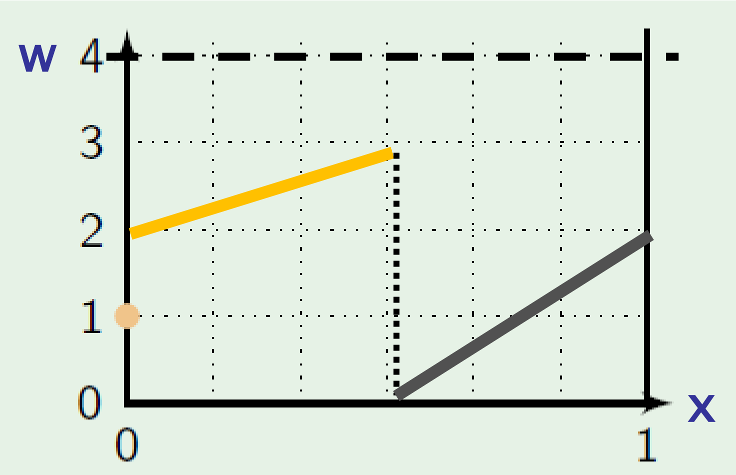
Can we find **decidable practical sub-class**

= URGENT control ?

Energy Timed Automata



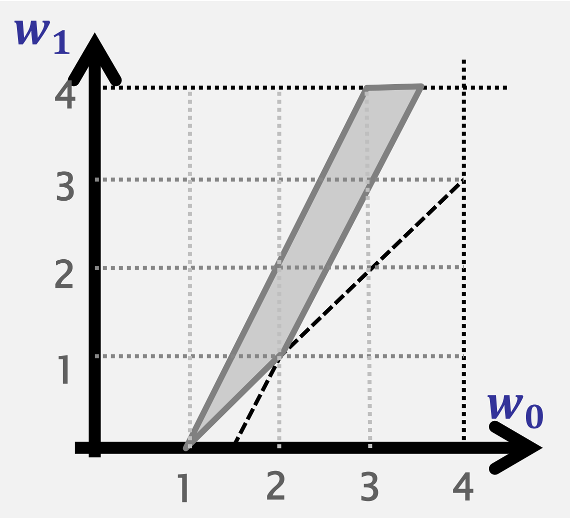
[0,4]



$R(w_0, w_1, h) \triangleq$ **Energy Relation**

$$\exists t.\, 0 \le t \le 1 \;\wedge$$
$$w_0 + 2t \le h \;\wedge$$
$$w_0 + 2t - 3 \ge 0 \;\wedge$$
$$w_0 + 2t - 3 + 4(1-t) \le h$$
$$w_1 = w_0 + 2t - 3 + 4(1-t)$$

$$R(w_0, w_1, 4) = (w_1 \le 4) \wedge (2w_0 \le 3 + w_1) \wedge (w_1 \le w_0 - 2) \wedge (w_0 - 1 \le w_1)$$



P Bouyer, U Fahrenberg, K Larsen, N Markey,.. . Infinite runs in weighted timed automata with energy constraints. 2008.
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P. Bouyer, K. G. Larsen, and N. Markey. Lower-bound constrained runs in weighted timed automata. QEST 2012
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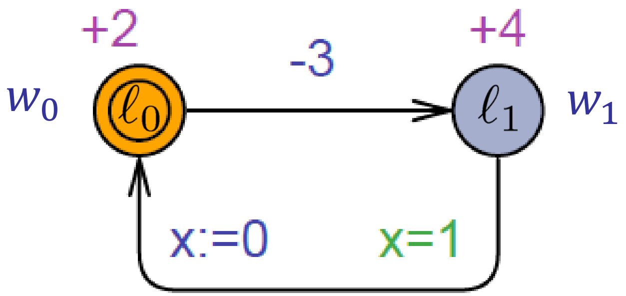
Energy Timed Automata

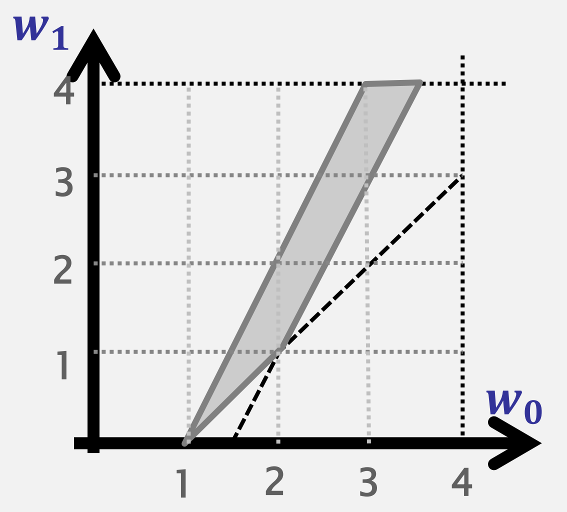


[0,4]

R^ω = max $[a,b]$.
$\forall w_0 \in [a,b]\ \exists w_1 \in [a,b]\ \boldsymbol{R}(w_0, w_1, 4)$

$\boldsymbol{R}(w_0, w_1, \boldsymbol{h}) \triangleq$
$\exists t.\ 0 \le t \le 1\ \wedge$
$w_0 + 2t \le \boldsymbol{h} \wedge$
$w_0 + 2t - 3 \ge 0 \wedge$
$w_0 + 2t - 3 + 4(1-t) \le \boldsymbol{h}$
$w_1 = w_0 + 2t - 3 + 4(1-t)$

$\boldsymbol{R}(w_0, w_1, \boldsymbol{4}) = (w_1 \le 4) \wedge (2w_0 \le 3 + w_1) \wedge$
$(w_1 \le w_0 - 2) \wedge (w_0 - 1 \le w_1)$



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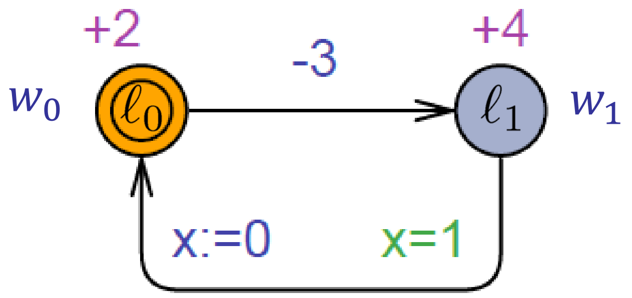
Energy Timed Automata

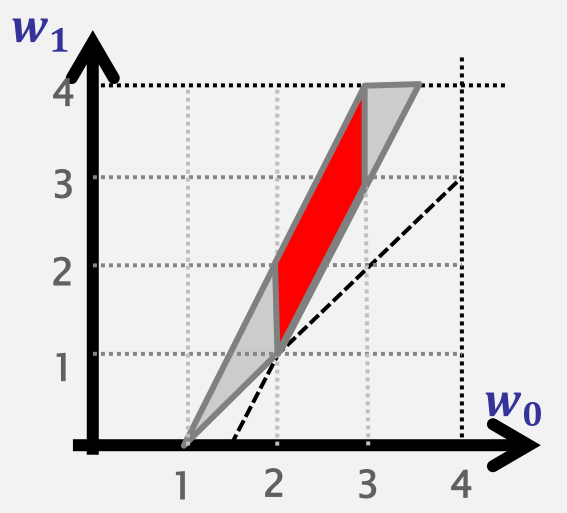


[0,4]

$$R^\omega = \max\ [a,b].\ \forall w_0 \in [a,b]\ \exists w_1 \in [a,b]\ \boldsymbol{R}(w_0, w_1, 4) = [2,3]$$

$$\boldsymbol{R}(w_0, w_1, \boldsymbol{h}) \triangleq \exists t.\, 0 \le t \le 1 \ \wedge\ w_0 + 2t \le \boldsymbol{h} \ \wedge\ w_0 + 2t - 3 \ge 0 \ \wedge\ w_0 + 2t - 3 + 4(1-t) \le \boldsymbol{h} \ \wedge\ w_1 = w_0 + 2t - 3 + 4(1-t)$$

$$\boldsymbol{R}(w_0, w_1, \boldsymbol{4}) = (w_1 \le 4) \wedge (2w_0 \le 3 + w_1) \wedge (w_1 \le w_0 - 2) \wedge (w_0 - 1 \le w_1)$$



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G Bacci, P Bouyer, U Fahrenberg, K.G. Larsen, N Markey, PA Reynier: Optimal and Robust Controller Synthesis - Using Energy Timed Automata with Uncertainty. FM 2018:

Energy Timed Automata

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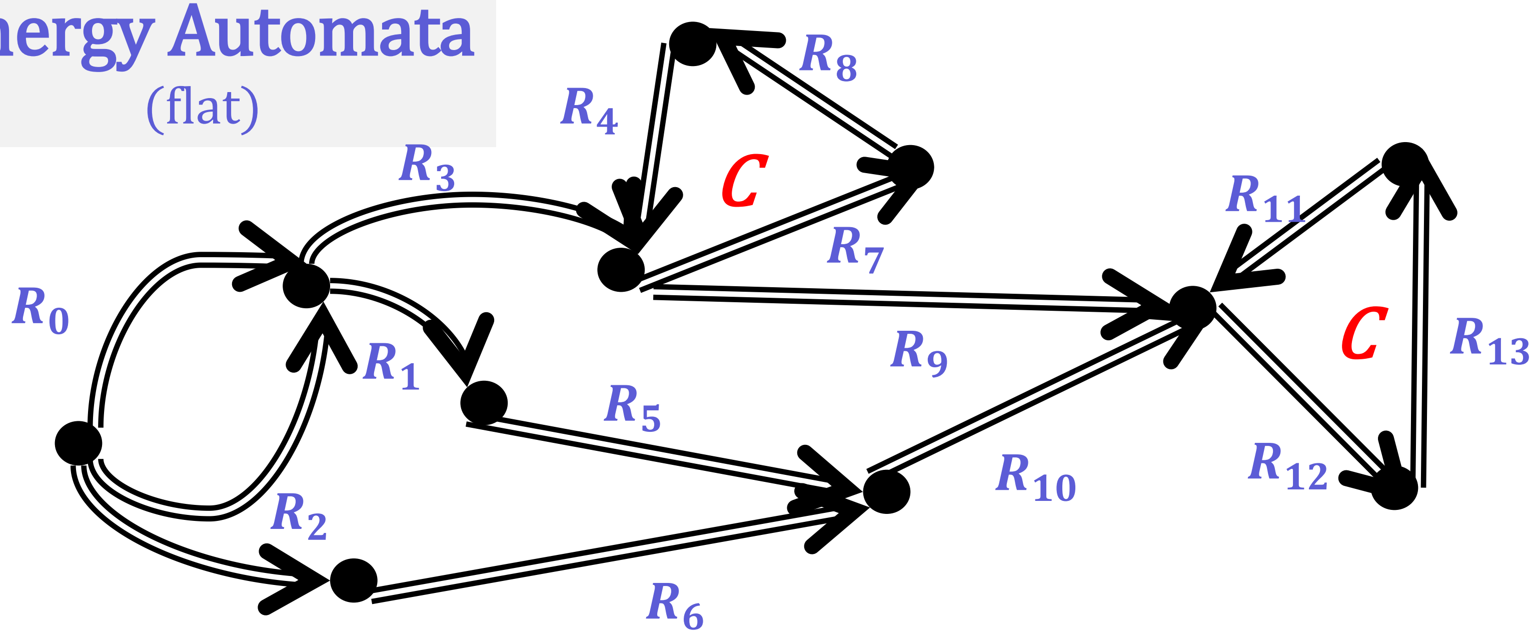
Energy Automata
(flat)

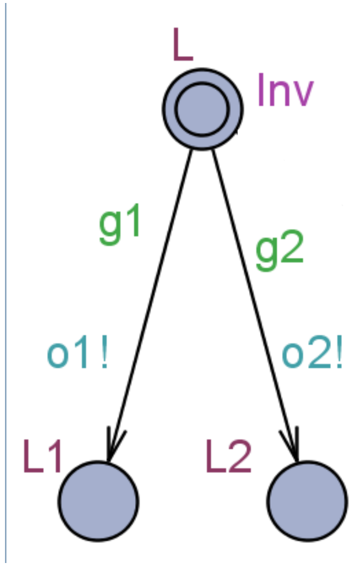


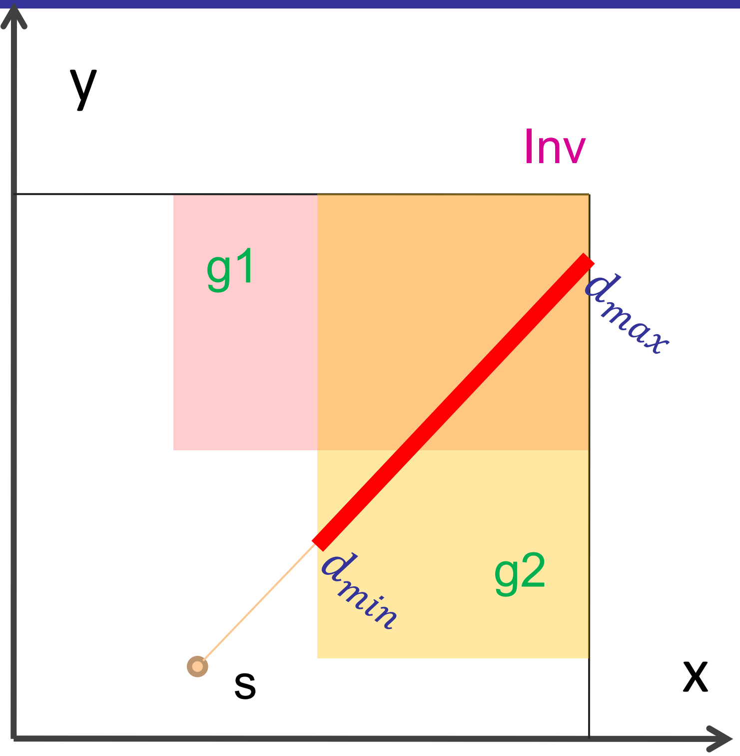
Open Problems:

- Non-flat automata?
- Multiple costs?
- Optimal safe infinite run?
- Synthesis of minimal h ?

1. Forward energy interval I using R_i
2. Whenever a simple cycle C is met check $I \cap R_C^{\omega} \neq \emptyset$
3. Otherwise continue with $I, R_C(I), R_C^2(I), \ldots, R_C^n(I), \ldots$
   (can be proved $= \emptyset$ for some n)

Stochastic Timed Automata





Delay Density Function

$$\mu_s : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$$

Output Probability Function

$$\gamma_s : \Sigma_O \rightarrow [0,1]$$

- E.g. μ_s uniform on $[d_{min}, d_{max}]$
- E.g. γ_s uniform over enabled outputs

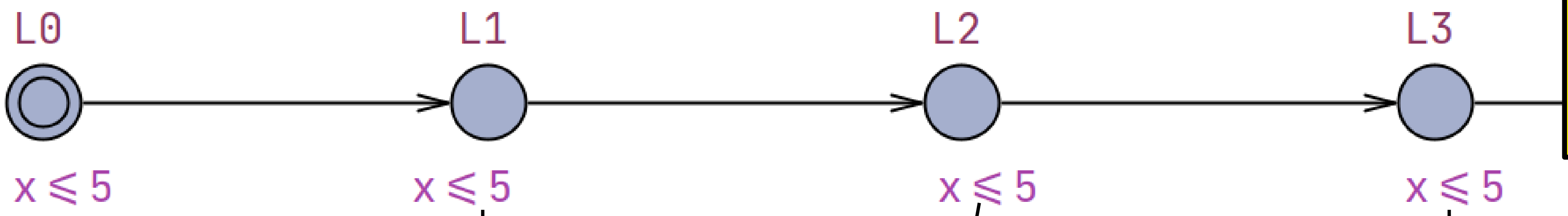
A David, K G. Larsen, A Legay, M Mikucionis, Z Wang: Time for Statistical Model Checking of Real-Time Systems. CAV 2011

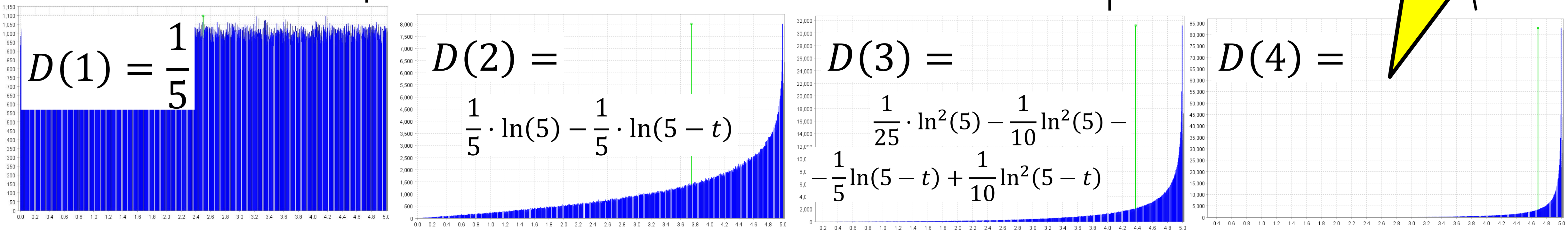
Stochastic Timed Automata – Open Problem

```
def logdensity(x,k,l):
    result = (-1)**k/(l*factorial(k))*(np.log((l-x))**k-np.log(l)**k)

 for j in range(1,k):
result+=(-1)**(k-j+1)*np.log(l)**(k-j)/factorial(k-j)*logdensity(x,j,l)

    return result
```





$D(1) = \frac{1}{5}$

$D(2) = \frac{1}{5} \cdot \ln(5) - \frac{1}{5} \cdot \ln(5-t)$

$D(3) = \frac{1}{25} \cdot \ln^2(5) - \frac{1}{10}\ln^2(5) - \frac{1}{5}\ln(5-t) + \frac{1}{10}\ln^2(5-t)$

$D(4) =$

$$D(1)(t) = \frac{1}{5}\ \textit{if}\ \ t \leq 5; \quad 0\ \textit{ow}.$$

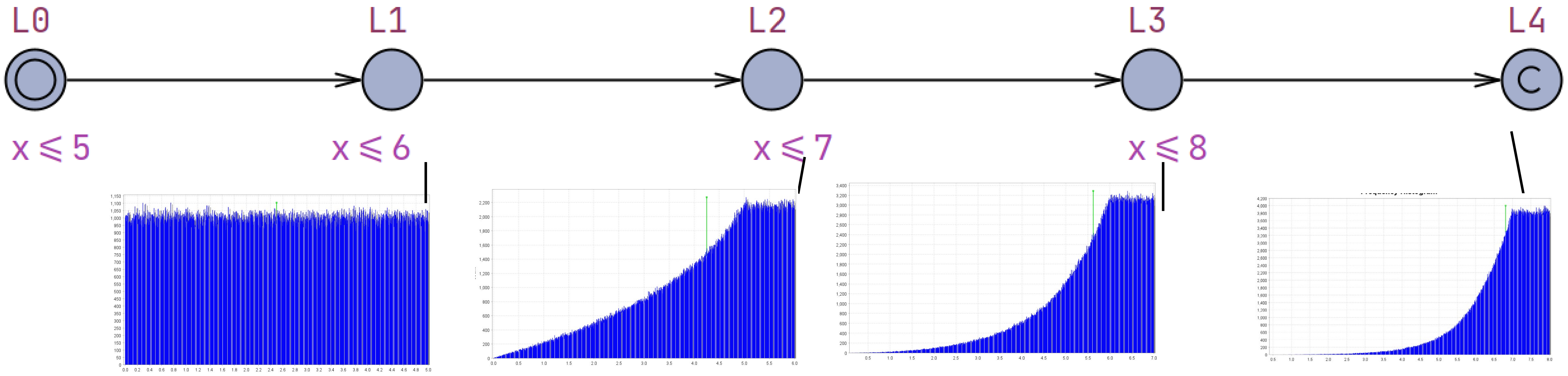
$$D(n+1)(t) = \int_{\tau=0}^{\tau=t} D(n)(\tau) \cdot \frac{1}{5-\tau}\, d\tau$$

$$\Pr\big(\ \langle\rangle(L4 \wedge x \leq 3)\big) \geq \frac{1}{2} \qquad \approx 0.014$$

A David, K G. Larsen, A Legay, M Mikucionis, Z Wang: Time for Statistical Model Checking of Real-Time Systems. CAV 2011

P Bouyer, T Brihaye, M Jurdzinski , Q Menet: Almost-Sure Model-Checking of Reactive Timed Automata. QEST 2012

Stochastic Timed Automata – Open Problem



$D(1)(t) = ?$ $D(2)(t) = ?$ $D(3)(t) = ?$ $D(4)(t) = ?$

$$\Pr(\langle\rangle(L4 \wedge x \leq 3)) \geq \frac{1}{2} \quad \approx 0.0036$$

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Stochastic Timed Automata – Open Problem

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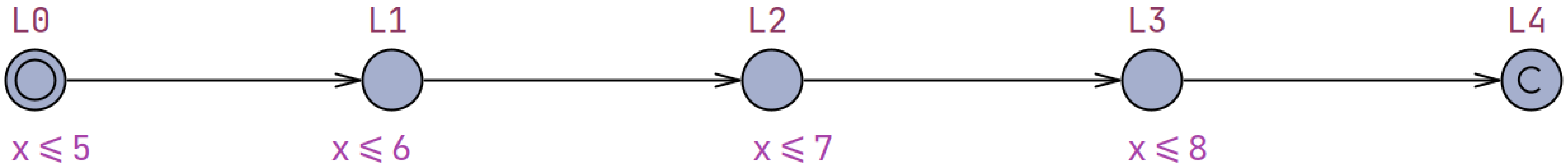

OPEN PROBLEM
Decidability of

$$\Pr(\langle\rangle(G)) \geq p$$

for 1-clock STA

Consider

Guards, Resets, Branching, Cycles, ..

$$\Pr(\ \langle\rangle(\mathrm{L4} \wedge x \leq 3)) \geq \frac{1}{2} \quad \approx 0.0036$$

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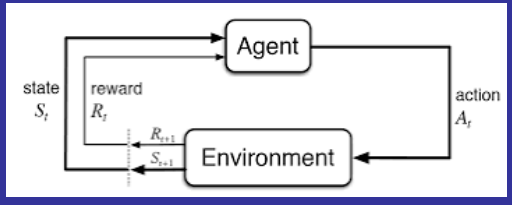
Priced MDP / UPPAAL STRATEGO

UPPAAL STRATEGO



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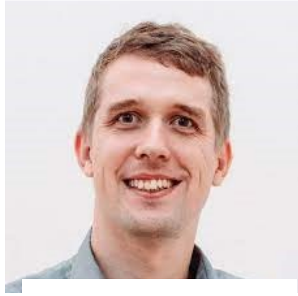
Peter G Jensen


Jakob H Taankvis


Mathias G Sørensen


Andreas B Eriksen

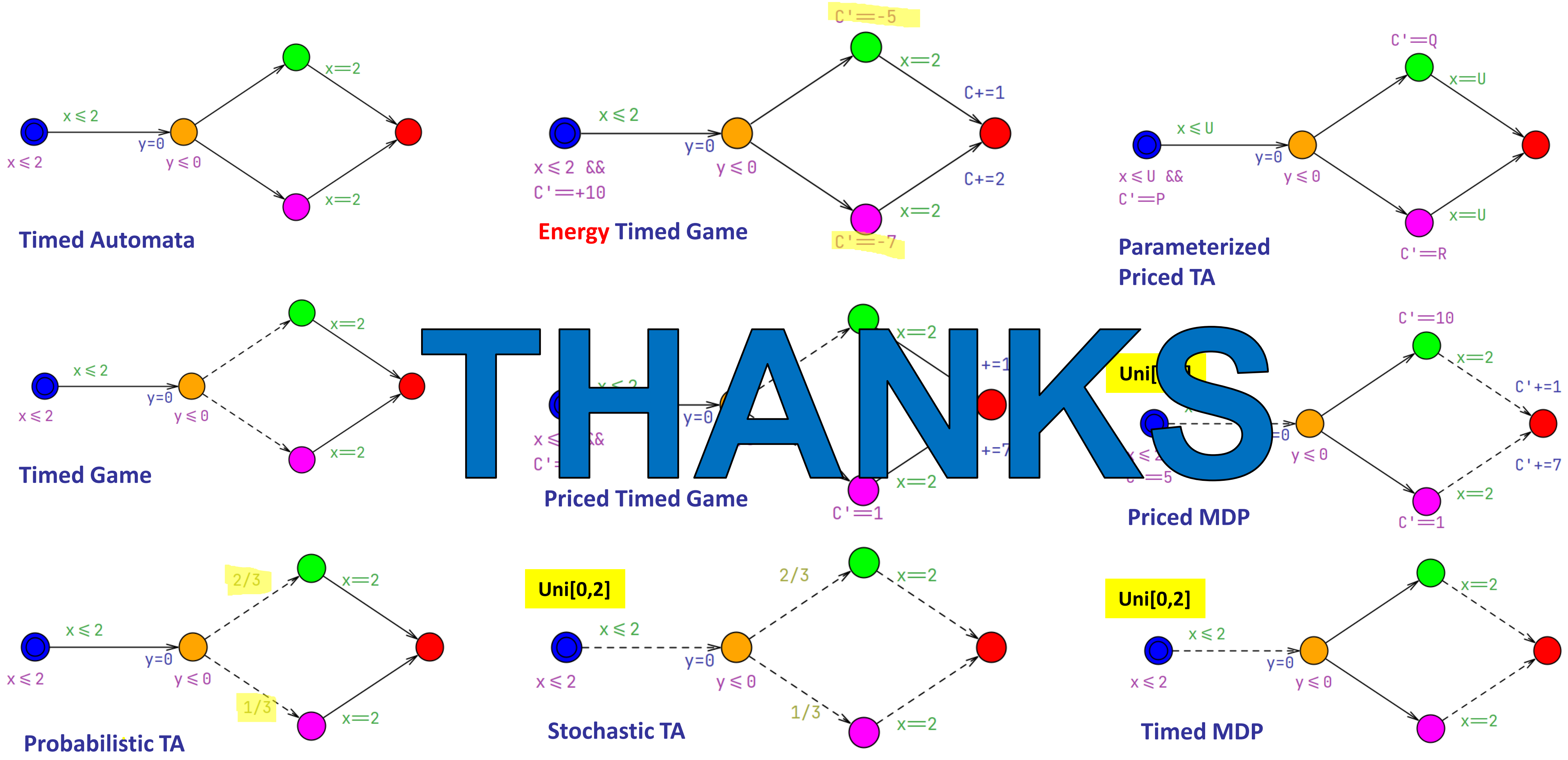
Wordle game

1	T	A	P	E	N
2	P	A	D	D	Y
3					

UPPAAL Stratego suggestions

H	A	R	P	Y	1.803
R	A	S	P	Y	1.806
G	A	S	P	Y	1.995
W	A	S	P	Y	1.997
V	A	M	P	Y	2.234

Cost / Opponent / Stochastic / Parameters



THANKS