From Global to Local and Back: Different Perspectives to Reason about Emerging Behavior

Michele Loreti
joint work with Michela Quadrini and Aniqa Rehman

University of Camerino

Open Problems in Concurrency Theory
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Collective Systems

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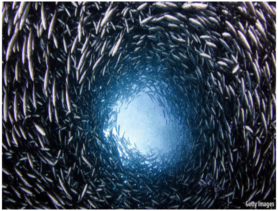
in the natural world ....



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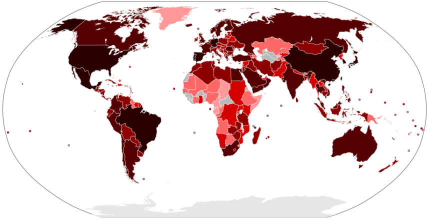

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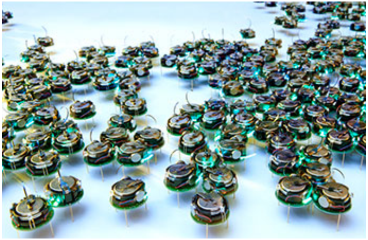
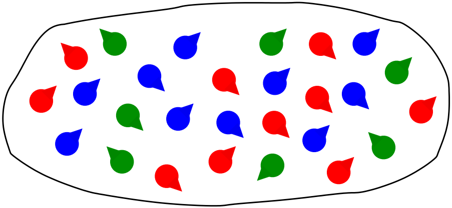


Collective Adaptive Systems

From a computer science perspective these systems can be viewed as being made up of a large number of interacting entities.



Each entity may have its own properties, objectives and actions.

At the system level these combine to create the collective behaviour.

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An action/activity is executed with a probability that depends on the state of entities in the system.

An Example: Red Blue Scenario...

Let us considered a population of agents that can be either red or blue.

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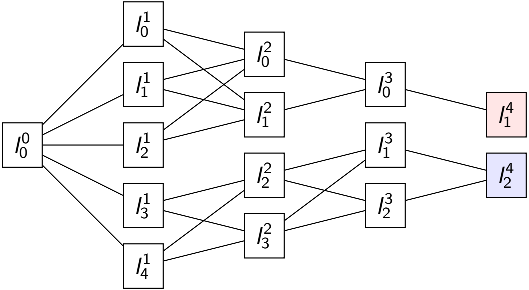
Starting from a common initial area, each agent must reach the goal area of its own colour:



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To reach the target cells each agent:

- follows a random movement;
- the probability to move from one location to one of its neighbours depend on the fraction of agents in the target location.

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The higher is the number of landmarks/barriers at a cell, the higher/lower is the probability that one agent can jump on it.

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Multi-Agent Stochastic Processes. . .

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In this talk, for the sake of simplicity, we focus on Discrete Time Multi-Agent Stochastic Process.

Multi-Agent Discrete Time Markov Chain. . .

We let Δ be a set of agent definitions consisting of:

- a set $\mathcal{S}$ of *agent states*;
- a probability matrix $\mathbf{P} : \mathcal{S}^* \to \mathcal{S} \times \mathcal{S} \to [0, 1]$;
- a set of atomic propositions $\mathcal{AP}$
- a labelling function $\mathcal{L} : \mathcal{S} \to 2^{\mathcal{AP}}$.

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We assume that for any $\vec{q} \in \mathcal{S}^*$ and for any $s \in \mathcal{S}$:

$$\sum_{s' \in \mathcal{S}} \mathbf{P}(\vec{q})[s, s'] = 1$$

Multi-Agent Discrete Time Markov Chain...

A *Multi-Agent Discrete Time Markov Chain* (MA-DTMC) with size N for a agent definition Δ, $\mathcal{M}_{\Delta}^{N}$, is a DTMC $(\mathcal{S}^N, \mathbf{P}^N)$:

$$\mathbf{P}^N[\vec{q}_1, \vec{q}_2] = \Pi_{i=1}^{n} \mathbf{P}(\vec{q}_1)\left[\vec{q}_1[i], \vec{q}_2[i]\right]$$

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Given a $\vec{q} \in \mathcal{S}^N$...

- $\vec{q}[i] \in \mathcal{S}$ denotes the state of agent in position i;
- for any $s \in \mathcal{S}$:

$$\#(\vec{q}, s) = |\{i|\vec{q}[i] = s\}| \qquad \%(\vec{q}, s) = \frac{\#(\vec{q}, s)}{N}$$

Global paths...

A *global* path π over $\mathcal{M}^N$ is a non empty (infinite) sequence of states $\vec{q}_0\vec{q}_1\vec{q}_2\cdots$ of states in $\mathcal{S}^N$ such that, for any i, $\mathbf{P}^N[\vec{q}_i, \vec{q}_{i+1}] > 0$.

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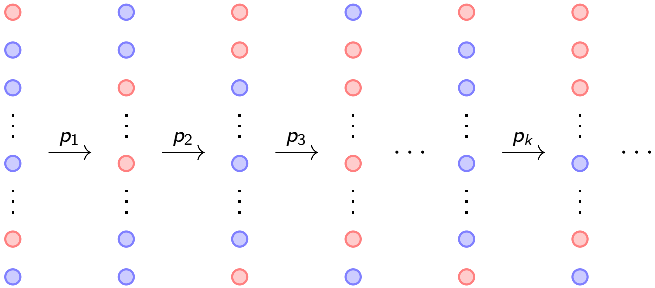

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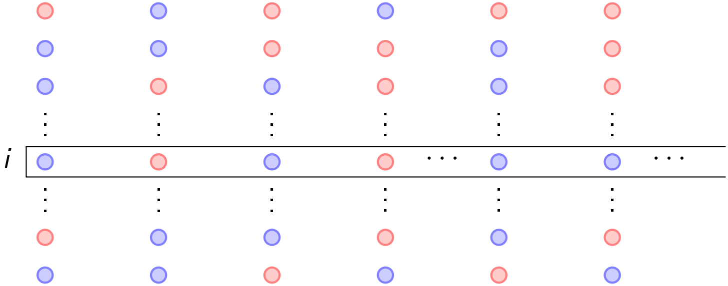

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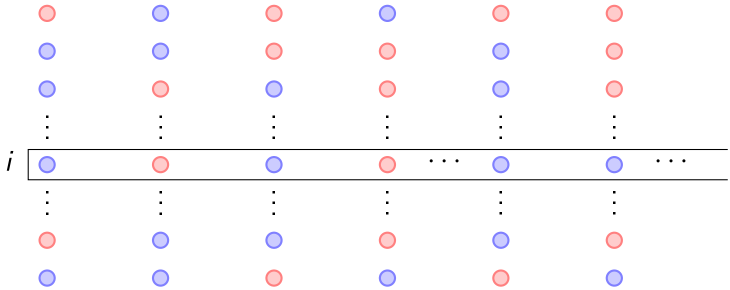

We say that π_ℓ is a *local path* of agent i over $\mathcal{M}^N$ if there exists $\pi \in Paths_{\mathcal{M}^N}$ such that $\pi_\ell = \pi \downarrow i$.

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If an agent becomes a *barrier*, in at most k_2 steps it restarts its journey and in at most k_3 steps it will reach its goal area.

GLoTL: Global and Local Temporal Logic. . .

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GLOBAL FORMULAS

$$\Phi \quad ::= \quad \mathsf{true} \mid \neg\Phi \mid \Phi_1 \wedge \Phi_2 \mid \%[\phi] \bowtie p \mid \mathcal{X}\ \Phi \mid \Phi_1\ \mathcal{U}^{\leq k}\ \Phi_2$$

LOCAL FORMULAS

$$\phi \quad ::= \quad \mathsf{true} \mid \alpha \mid \neg\phi \mid \phi_1 \wedge \phi_2 \mid \mathcal{X}\ \phi \mid \phi_1\ \mathcal{U}^{\leq k}\ \phi_2$$

- A *global formula* Φ, which permits specifying properties of *global computations*;

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- A *local formula* ϕ, is used to specify properties of the single agents.

Global and Local Temporal Logic...

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Operators of both global and local formulas are standard. Both the fragments are a variant of *bounded LTL*.

The only novel operator $\%[\phi] \bowtie p$ is used to specify that, at a given point in the computation, the *fraction of agents* satisfying *local formula* ϕ is $\bowtie p$

Derivable Operators...

Global Formulas

$$\begin{array}{rcl}
\mathsf{false} & = & \neg\mathsf{true} \\
\Phi_1 \vee \Phi_2 & = & \neg(\neg\Phi_1 \wedge \neg\Phi_2) \\
\Phi_1 \rightarrow \Phi_2 & = & \neg\Phi_1 \wedge \Phi_2 \\
\Diamond^{\leq k}\Phi & = & \mathsf{true}\ \mathcal{U}^{\leq k}\ \Phi \\
\Box^{\leq k}\Phi & = & \neg\Diamond^{\leq k}\neg\Phi \\
\exists\phi & = & \%[\phi] > 0 \\
\forall\phi & = & \%[\phi] \geq 1 \\
\%[\phi] \in [v_1, v_2] & = & \%[\phi] \geq v_1 \wedge \%[\phi] \leq v_2
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$$\pi \models^{\mathcal{M}^N,\mathcal{L}} \mathcal{X}\ \Phi \iff \pi[1..] \models^{\mathcal{M}^N,\mathcal{L}} \Phi$$

$$\pi \models^{\mathcal{M}^N,\mathcal{L}} \Phi_1\ \mathcal{U}^{\leq k}\ \Phi_2 \iff$$

$$\exists 0 \leq h \leq k.\ \pi[h..] \models^{\mathcal{M}^N,\mathcal{L}} \Phi_2 \wedge \forall 0 \leq i < h.\ \pi[i..] \models \Phi_1$$

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Satisfaction Probability...

Let $\mathcal{M}^N = (\mathcal{S}^N, \mathbf{P}^N)$ be a MA-DTMC and $\mathcal{L} : \mathcal{S} \to 2^{\mathcal{AP}}$ be a labelling function.

For any $\vec{q} \in \mathcal{S}^N$ and formula Φ we let μ be the function amounting the probability that $\vec{q}$ satisfies Φ:

$$\mu(\mathcal{M}^N, \mathcal{L}, \vec{q}, \Phi) = Pr_{\mathcal{M}^N}\{\pi \in Paths_{\mathcal{M}^N}(\vec{q}) | \pi \models^{\mathcal{M}^N, \mathcal{L}} \Phi\}$$

An Example: Red-Blue scenario

Global and Local Properties

- almost all blue and read agents have reached their goal areas::

$$\Phi_{gb} = (\%[\mathit{blue}@\mathit{goal}] \in [0.5-\varepsilon, 0.5+\varepsilon])$$

$$\Phi_{gr} = (\%[\mathit{read}@\mathit{goal}] \in [0.5-\varepsilon, 0.5+\varepsilon])$$

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- the system is able to reach a configuration where almost all agents have reached their goal area:

$$\Phi_1 = \Diamond^{\leq k_1} (\Phi_{gb} \wedge \Phi_{gr})$$

An Example: Red-Blue scenario

- if an agent becomes a *barrier*, in at most k_2 it restarts its journey and in at most k_3 steps it will reach its goal area.

$$\phi_{bb} = bb \rightarrow \Diamond^{\leq k_2}(\neg bb \wedge \Diamond^{\leq k_3} blue@goal)$$

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- in the next k_4 steps, if the fraction of agents that are barriers is greater than 30% then 75% of them satisfy ϕ_{bb} or ϕ_{rb}:

$$\Phi_2 = \Box^{\leq k_4} (\%[bb \vee br] \geq .30 \rightarrow \%[\phi_{br} \wedge \phi_{rb}] \geq .75)$$

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Global Bisimulation...

Let $\mathcal{M}_{\Delta}^{N} = (\mathcal{S}^N, \mathbf{P}^N)$ be a MA-DTMC model, an equivalence relation $\mathcal{R} \subseteq \mathcal{S}^N \times \mathcal{S}^N$ is a *Global Probabilistic Bisimulation Relation* if and only if for any $(\vec{q_1}, \vec{q_2}) \in \mathcal{R}$:

1. $\forall i \ \mathcal{L}(\vec{q_1}[i]) = \mathcal{L}(\vec{q_2}[i])$;
2. for any equivalence class $\mathcal{C} \in \mathcal{S}^N/\mathcal{R}$

$$\mathbf{P}^N[\vec{q_1}, \mathcal{C}] = \mathbf{P}^N[\vec{q_2}, \mathcal{C}]$$

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We let $\sim_G$ denote the largest *Global Probabilistic Bisimulation Relation*

Local Bisimulation...

Let $\Delta = (\mathcal{S}, \mathbf{P}, \mathcal{AP}, \mathcal{L})$, an equivalence relation $\mathcal{R} \subseteq \mathcal{S} \times \mathcal{S}$ is a *Local Probabilistic Bisimulation Relation* if and only if for any $(s_1, s_2) \in \mathcal{R}$:

1. $\mathcal{L}(s_1) = \mathcal{L}(s_2)$;
2. for any $\vec{q} \in \mathcal{S}^N$ and for any equivalence class $\mathcal{C} \in \mathcal{S}/\mathcal{R}$

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We let $\sim_L$ denote the largest *Local Probabilistic Bisimulation Relation*.

Local Bisimulation...

Let $\Delta = (\mathcal{S}, \mathbf{P}, \mathcal{AP}, \mathcal{L})$, an equivalence relation $\mathcal{R} \subseteq \mathcal{S} \times \mathcal{S}$ is a *Local Probabilistic Bisimulation Relation* if and only if for any $(s_1, s_2) \in \mathcal{R}$:

1. $\mathcal{L}(s_1) = \mathcal{L}(s_2)$;
2. for any $\vec{q} \in \mathcal{S}^N$ and for any equivalence class $\mathcal{C} \in \mathcal{S}/\mathcal{R}$

$$\mathbf{P}^{(\vec{q})}[s_1, \mathcal{C}] = \mathbf{P}^{(\vec{q})}[s_2, \mathcal{C}]$$

We let $\sim_L$ denote the largest *Local Probabilistic Bisimulation Relation*.

Computing Local Bisimulation...

We say that $\Delta = (\mathcal{S}, \mathbf{P}, \mathcal{AP}, \mathcal{L})$ is in polynomial form if for any $s_1, s_2 \in \mathcal{S}$:

$$\mathbf{P}(\tilde{x}_S)[s_1, s_2] = \frac{p(\tilde{x}_S)}{q(\tilde{x}_S)}$$

where $p(\tilde{x}_S)$ and $q(\tilde{x}_S)$ are multivariate polynomials in the variable $\tilde{x}_S$.

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For each $s \in \mathcal{S}$, x_s is associated with the number/fraction of agents in the state s.

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Following the an approach similar to CTTV17[1] we can compute the equivalence classes of $\sim_L$ in $\mathcal{S}$.

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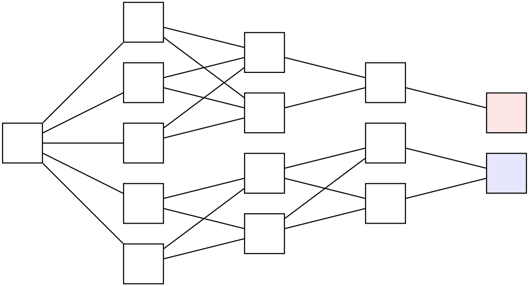
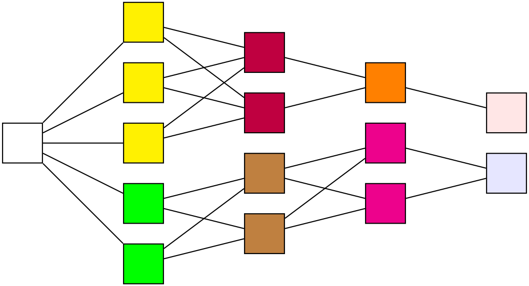
State space reduction…



State space reduction. . .



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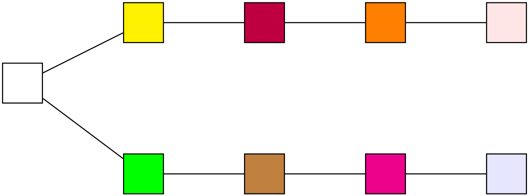

Some results. . .

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$$\forall i.\vec{q}_1[i] \sim_L \vec{q}_2[i] \Longrightarrow \vec{q}_1 \sim_G \vec{q}_2$$

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Concluding remark

In this talk we have presented a methodology that can be used to specify properties of CAS at both global and local level.

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A behavioural equivalence has been proposed to aggregate states at both global and local level.

The proposed equivalences are sound (but not complete) w.r.t. the proposed logic.

thank you!