Reversibility in Timed Process Calculi

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Done with Marco Bernardo, Laura Bocchi, Ivan Lenese and Shoji Yuen

OPTC23 meeting @ Bertinoro

Timed Systems - design choices

We address reversibility of real-time systems

Several design choices - **Durationless actions Vs durational actions**

- Durationless actions
  - actions are instantaneous events and time passes in between them
  - functional behaviour and time are orthogonal
- durational actions
  - every action takes a certain amount of time to be performed and time passes only due to action execution
  - functional behavior and time are integrated

Timed Systems - design choices

Several design choices - **Relative time Vs Absolute time**

- Relative time
  - Each timestamp refers to the time instant of the previous observation
- Absolute time
  - all timestamps refer to the starting time of the system execution

Timed Systems - design choices

Several design choices - **Global clock Vs Local clocks**

- Global clock
  - a single clock governs time passing
- Local clocks
  - several clocks associated with the various system parts elapse independent of each other

Timed Systems - design choices

Several interpretations of action execution in terms of whether and when the execution can be delayed

- Eagerness
  - actions must be performed as soon as they become enabled
  - e.g. actions cannot be delayed
- Laziness
  - actions can be delayed arbitrarily long before they are executed
- Maximal progress
  - enabled actions can be delayed unless they are independent of the external environment (e.g., taus)

Our design choices

We study reversibility in a timed calculus with

- Durationless actions VS durational actions
- Relative time VS absolute time
- Global clock VS local clocks

with

eager and lazy interpretation and time additivity

We start from a simple calculus inspired by Moller & Tofts [MT90]

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Reversibility in Concurrent System

Calculi

Reversible Communicating System (RCCS) Danos&Krivine

- Use of explicit memories to keep track of past events
- Suitable for complex languages (e.g., scales with pi-calculus, Erlang)
- Give the first notion of causally consistent reversibility
- **Won CONCUR23 test of time award**

CCS with communication keys (CCSK) Phillips&Ulidowski

- History information directly recorderded into the term
- Use of keys to keep track of synchronisations
- Suitable for CCS-like languages with LTSs

Example

$$a.P + b.Q \xrightarrow{a} P$$

After the computation, we loose information about

- The performed action a
- The other branch b.Q

RCCS

$$m \triangleright (a.P + b.Q)$$

RCCS

$$m \triangleright (a.P + b.Q)$$

Memory monitoring the process

RCCS

$$m \triangleright (a.P + b.Q) \xrightarrow{a[i]} \langle a, b.Q, i \rangle \cdot m \triangleright P$$

Memory monitoring the process

RCCS

$$\underline{m \triangleright} (a.P + b.Q) \xrightarrow{a[i]} \langle \underline{a, b.Q}, i \rangle \cdot m \triangleright P$$

Memory monitoring the process

Information about the previous state

RCCS

$$m \triangleright (a.P + b.Q) \xrightarrow{a[i]} \langle a, b.Q, i \rangle \cdot m \triangleright P \xrightarrow{a[i]} m \triangleright (a.P + b.Q)$$

Memory monitoring the process

Information about the previous state

CCSK

$$a.P + b.Q$$

The two reversible CCSs have been shown to be equivalent LMM2021

CCSK

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No need of extra memories

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CCSK

$$a.P + b.Q \xrightarrow{a[i]} a[i].P + b.Q$$

No need of extra memories

The two reversible CCSs have been shown to be equivalent LMM2021

CCSK

$$a.P + b.Q \xrightarrow{a[i]} a[i].P + b.Q$$

No need of extra memories

History information
directly in the term

The two reversible CCSs have been shown to be equivalent LMM2021

CCSK

$$a.P + b.Q \xrightarrow{a[i]} a[i].P + b.Q \xrightarrow{\underline{a[i]}} a.P + b.Q$$

No need of extra memories

History information directly in the term

The two reversible CCSs have been shown to be equivalent LMM2021

RTPC: reversible timed process calculus

- A simple extension of CCS with time prefix **(n).P**
- (n).P acts as P after n time units
- Reversing à la CCSK

$$
\begin{array}{lll}
P, Q & ::= & \underline{0} \mid a\,.\,P \mid (n)\,.\,P \mid P + Q \mid P \parallel_L Q \\
R, S & ::= & P \mid a[i]\,.\,R \mid (n)^{[i]}.\,R \mid \langle n^{i} \rangle\,.\,R \mid R + S \mid R \parallel_L S
\end{array}
$$

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Past action prefix

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Past action prefix   Past time prefix

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\begin{aligned}
P, Q &::= \underline{0} \mid a\,.\,P \mid (n)\,.\,P \mid P + Q \mid P \parallel_L Q \\
R, S &::= P \mid a[i]\,.\,R \mid (n)^{[i]}\,.\,R \mid \langle n^i \rangle\,.\,R \mid R + S \mid R \parallel_L S
\end{aligned}
$$

Past action prefix Past time prefix Dynamic delay prefix

RTPC - action semantics

δ(n,i) denote (n)[i] or ⟨ni⟩

$$(\textsc{Act1})\quad \frac{\texttt{std}(R)}{a\,.\,R \xrightarrow{a[i]}_{\mathtt{a}} a[i]\,.\,R}$$

$$(\textsc{Act2})\quad \frac{R \xrightarrow{b[j]}_{\mathtt{a}} R' \quad j \neq i}{a[i]\,.\,R \xrightarrow{b[j]}_{\mathtt{a}} a[i]\,.\,R'}$$

$$(\textsc{Act3})\quad \frac{R \xrightarrow{b[j]}_{\mathtt{a}} R'}{\delta(n,i)\,.\,R \xrightarrow{b[j]}_{\mathtt{a}} \delta(n,i)\,.\,R'}$$

$$(\textsc{Cho})\quad \frac{R \xrightarrow{a[i]}_{\mathtt{a}} R' \quad \texttt{npa}(S)}{R + S \xrightarrow{a[i]}_{\mathtt{a}} R' + S}$$

$$(\textsc{Par})\quad \frac{R \xrightarrow{a[i]}_{\mathtt{a}} R' \quad a \notin L \quad i \notin \texttt{keys}_{\mathtt{a}}(S)}{R \parallel_L S \xrightarrow{a[i]}_{\mathtt{a}} R' \parallel_L S}$$

$$(\textsc{Coo})\quad \frac{R \xrightarrow{a[i]}_{\mathtt{a}} R' \quad S \xrightarrow{a[i]}_{\mathtt{a}} S' \quad a \in L}{R \parallel_L S \xrightarrow{a[i]}_{\mathtt{a}} R' \parallel_L S'}$$

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$$(\textsc{Act1}^\bullet)\ \frac{\mathtt{std}(R)}{a[i]\,.\,R \overset{a[i]}{\dashrightarrow}_{\mathsf{a}} a\,.\,R}$$

$$(\textsc{Act2}^\bullet)\ \frac{R \overset{b[j]}{\dashrightarrow}_{\mathsf{a}} R' \quad j \neq i}{a[i]\,.\,R \overset{b[j]}{\dashrightarrow}_{\mathsf{a}} a[i]\,.\,R'}$$

$$(\textsc{Act3}^\bullet)\ \frac{R \overset{b[j]}{\dashrightarrow}_{\mathsf{a}} R'}{\delta(n,i)\,.\,R \overset{b[j]}{\dashrightarrow}_{\mathsf{a}} \delta(n,i)\,.\,R'}$$

$$(\textsc{Cho}^\bullet)\ \frac{R \overset{a[i]}{\dashrightarrow}_{\mathsf{a}} R' \quad \mathtt{npa}(S)}{R + S \overset{a[i]}{\dashrightarrow}_{\mathsf{a}} R' + S}$$

$$(\textsc{Par}^\bullet)\ \frac{P \overset{a[i]}{\dashrightarrow}_{\mathsf{a}} P' \quad a \notin L \quad i \notin \mathtt{keys}_{\mathsf{a}}(S)}{R \parallel_L S \overset{a[i]}{\dashrightarrow}_{\mathsf{a}} R' \parallel_L S}$$

$$(\textsc{Coo}^\bullet)\ \frac{R \overset{a[i]}{\dashrightarrow}_{\mathsf{a}} R' \quad S \overset{a[i]}{\dashrightarrow}_{\mathsf{a}} S' \quad a \in L}{R \parallel_L S \overset{a[i]}{\dashrightarrow}_{\mathsf{a}} R' \parallel_L S'}$$

RTPC - time semantics 1/2

δ(n,i) denote (n)[i] or ⟨ni⟩

(IDLING1) $\underline{0} \xrightarrow{(n)^{[i]}}_{\mathtt{d}} \langle n^i \rangle \,.\, \underline{0}$

(IDLING2) $$\frac{\mathtt{std}(R)}{a\,.\,R \xrightarrow{(n)^{[i]}}_{\mathtt{d}} \langle n^i \rangle \,.\, a\,.\,R}$$

(IDLING3) $$\frac{\mathtt{std}(R) \quad a \neq \tau}{a\,.\,R \xrightarrow{(n)^{[i]}}_{\mathtt{d}} \langle n^i \rangle \,.\, a\,.\,R}$$

(DELAY1) $$\frac{\mathtt{std}(R)}{(n)\,.\,R \xrightarrow{(n)^{[i]}}_{\mathtt{d}} (n)^{[i]}\,.\,R}$$

(DELAY2) $$\frac{R \xrightarrow{(n)^{[j]}}_{\mathtt{d}} R'}{a[i]\,.\,R \xrightarrow{(n)^{[j]}}_{\mathtt{d}} a[i]\,.\,R'}$$

(DELAY3) $$\frac{R \xrightarrow{(m)^{[j]}}_{\mathtt{d}} R' \quad j \neq i}{\delta(n,i)\,.\,R \xrightarrow{(m)^{[j]}}_{\mathtt{d}} \delta(n,i)\,.\,R'}$$

RTPC - time semantics 1/2

δ(n,i) denote (n)[i] or ⟨ni⟩

Laziness

(IDLING1) $\underline{0} \xrightarrow{(n)^{[i]}}_{\mathtt{d}} \langle n^i \rangle \,.\, \underline{0}$

(IDLING2) $$\frac{\mathtt{std}(R)}{a \,.\, R \xrightarrow{(n)^{[i]}}_{\mathtt{d}} \langle n^i \rangle \,.\, a \,.\, R}$$

(IDLING3) $$\frac{\mathtt{std}(R) \quad a \neq \tau}{a \,.\, R \xrightarrow{(n)^{[i]}}_{\mathtt{d}} \langle n^i \rangle \,.\, a \,.\, R}$$

(DELAY1) $$\frac{\mathtt{std}(R)}{(n) \,.\, R \xrightarrow{(n)^{[i]}}_{\mathtt{d}} (n)^{[i]} \,.\, R}$$

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RTPC - time semantics 1/2

δ(n,i) denote (n)[i] or ⟨ni⟩

Laziness

Maximal Progress

(IDLING1) $\underline{0} \xrightarrow{(n)^{[i]}}_{\mathtt{d}} \langle n^i \rangle . \underline{0}$

(IDLING2) $\dfrac{\mathtt{std}(R)}{a\,.\,R \xrightarrow{(n)^{[i]}}_{\mathtt{d}} \langle n^i \rangle . a\,.\,R}$

(IDLING3) $\dfrac{\mathtt{std}(R) \quad a \neq \tau}{a\,.\,R \xrightarrow{(n)^{[i]}}_{\mathtt{d}} \langle n^i \rangle . a\,.\,R}$

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Maximal Progress

(IDLING1) $\underline{0} \xrightarrow{(n)^{[i]}}_{\mathtt{d}} \langle n^{i} \rangle . \underline{0}$

(IDLING2) $$\frac{\mathtt{std}(R)}{a . R \xrightarrow{(n)^{[i]}}_{\mathtt{d}} \langle n^{i} \rangle . a . R}$$

(IDLING3) $$\frac{\mathtt{std}(R) \quad a \neq \tau}{a . R \xrightarrow{(n)^{[i]}}_{\mathtt{d}} \langle n^{i} \rangle . a . R}$$

(DELAY1) $$\frac{\mathtt{std}(R)}{(n) . R \xrightarrow{(n)^{[i]}}_{\mathtt{d}} (n)^{[i]} . R}$$

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(DELAY3) $$\frac{R \xrightarrow{(m)^{[j]}}_{\mathtt{d}} R' \quad j \neq i}{\delta(n,i) . R \xrightarrow{(m)^{[j]}}_{\mathtt{d}} \delta(n,i) . R'}$$

(IDLING1$^{\bullet}$) $\langle n^{i} \rangle . \underline{0} \overset{(n)^{[i]}}{\dashrightarrow}_{\mathtt{d}} \underline{0}$

(IDLING2$^{\bullet}$) $$\frac{\mathtt{std}(R)}{\langle n^{i} \rangle . a . R \overset{(n)^{[i]}}{\dashrightarrow}_{\mathtt{d}} a . R}$$

(IDLING3$^{\bullet}$) $$\frac{\mathtt{std}(R) \quad a \neq \tau}{\langle n^{i} \rangle . a . R \overset{(n)^{[i]}}{\dashrightarrow}_{\mathtt{d}} a . R}$$

(DELAY1$^{\bullet}$) $$\frac{\mathtt{std}(R)}{(n)^{[i]} . R \overset{(n)^{[i]}}{\dashrightarrow}_{\mathtt{d}} (n) . R}$$

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RTPC - time semantics 2/2

$$(\textsc{TAdd1})\ \frac{R \xrightarrow{(m)^{[j]}}_{\mathtt{d}} R' \quad \mathtt{std}(R) \quad j \neq i}{(n)\,.\,R \xrightarrow{(n+m)^{[i]}}_{\mathtt{d}} (n)^{[i]}.\,R'}$$

$$(\textsc{TAdd2})\ \frac{\mathtt{std}(R) \quad n = n_1 + n_2}{(n)\,.\,R \xrightarrow{(n_1)^{[i]}}_{\mathtt{d}} (n_1)^{[i]}.\,(n_2)\,.\,R}$$

$$(\textsc{TCho1})\ \frac{R \xrightarrow{(n)^{[i]}}_{\mathtt{d}} R' \quad S \xrightarrow{(n)^{[i]}}_{\mathtt{d}} S' \quad \mathbf{npa}(R+S)}{R + S \xrightarrow{(n)^{[i]}}_{\mathtt{d}} R' + S'}$$

$$(\textsc{TCho2})\ \frac{R \xrightarrow{(n)^{[i]}}_{\mathtt{d}} R' \quad \neg\mathbf{npa}(R) \quad \mathbf{npa}(S)}{R + S \xrightarrow{(n)^{[i]}}_{\mathtt{d}} R' + S}$$

$$(\textsc{TCoo})\ \frac{R \xrightarrow{(n)^{[i]}}_{\mathtt{d}} R' \quad S \xrightarrow{(n)^{[i]}}_{\mathtt{d}} S'}{R \parallel_L S \xrightarrow{(n)^{[i]}}_{\mathtt{d}} R' \parallel_L S'}$$

$$(\textsc{TAdd1}^\bullet)\ \frac{R \dashrightarrow^{(m)^{[j]}}_{\mathtt{d}} R' \quad \mathtt{std}(R') \quad j \neq i}{(n)^{[i]}.\,R \dashrightarrow^{(n+m)^{[i]}}_{\mathtt{d}} (n)\,.\,R'}$$

$$(\textsc{TAdd2}^\bullet)\ \frac{\mathtt{std}(R) \quad n = n_1 + n_2}{(n_1)^{[i]}.\,(n_2)\,.\,R \dashrightarrow^{(n_1)^{[i]}}_{\mathtt{d}} (n)\,.\,R}$$

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RTPC - time semantics 2/2

Time Additivity

$$(\text{TA}\textsc{dd}1)\ \frac{R \xrightarrow{(m)^{[j]}}_{\mathtt{d}} R' \quad \mathtt{std}(R) \quad j \neq i}{(n).R \xrightarrow{(n+m)^{[i]}}_{\mathtt{d}} (n)^{[i]}.R'}$$

$$(\text{TA}\textsc{dd}2)\ \frac{\mathtt{std}(R) \quad n = n_1 + n_2}{(n).R \xrightarrow{(n_1)^{[i]}}_{\mathtt{d}} (n_1)^{[i]}.(n_2).R}$$

$$(\text{TC}\textsc{ho}1)\ \frac{R \xrightarrow{(n)^{[i]}}_{\mathtt{d}} R' \quad S \xrightarrow{(n)^{[i]}}_{\mathtt{d}} S' \quad \mathtt{npa}(R+S)}{R+S \xrightarrow{(n)^{[i]}}_{\mathtt{d}} R'+S'}$$

$$(\text{TC}\textsc{ho}2)\ \frac{R \xrightarrow{(n)^{[i]}}_{\mathtt{d}} R' \quad \neg\mathtt{npa}(R) \quad \mathtt{npa}(S)}{R+S \xrightarrow{(n)^{[i]}}_{\mathtt{d}} R'+S}$$

$$(\text{TC}\textsc{oo})\ \frac{R \xrightarrow{(n)^{[i]}}_{\mathtt{d}} R' \quad S \xrightarrow{(n)^{[i]}}_{\mathtt{d}} S'}{R \parallel_L S \xrightarrow{(n)^{[i]}}_{\mathtt{d}} R' \parallel_L S'}$$

$$(\text{TA}\textsc{dd}1^\bullet)\ \frac{R \overset{(m)^{[j]}}{\dashrightarrow}_{\mathtt{d}} R' \quad \mathtt{std}(R') \quad j \neq i}{(n)^{[i]}.R \overset{(n+m)^{[i]}}{\dashrightarrow}_{\mathtt{d}} (n).R'}$$

$$(\text{TA}\textsc{dd}2^\bullet)\ \frac{\mathtt{std}(R) \quad n = n_1 + n_2}{(n_1)^{[i]}.(n_2).R \overset{(n_1)^{[i]}}{\dashrightarrow}_{\mathtt{d}} (n).R}$$

$$(\text{TC}\textsc{ho}1^\bullet)\ \frac{R \overset{(n)^{[i]}}{\dashrightarrow}_{\mathtt{d}} R' \quad S \overset{(n)^{[i]}}{\dashrightarrow}_{\mathtt{d}} S' \quad \mathtt{npa}(R+S)}{R+S \overset{(n)^{[i]}}{\dashrightarrow}_{\mathtt{d}} R'+S'}$$

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Dynamic delay operator ?

$$a \,.\, (n) \parallel_{\emptyset} (n)$$

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$$a[i] \,.\, (n) \parallel_{\emptyset} (n)$$

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$$a[i] \,.\, (n)^{[j]} \parallel_{\emptyset} (n)^{[j]}$$

Dynamic delay operator ?

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$$a[i]\,.\,(n) \parallel_{\emptyset} (n)$$

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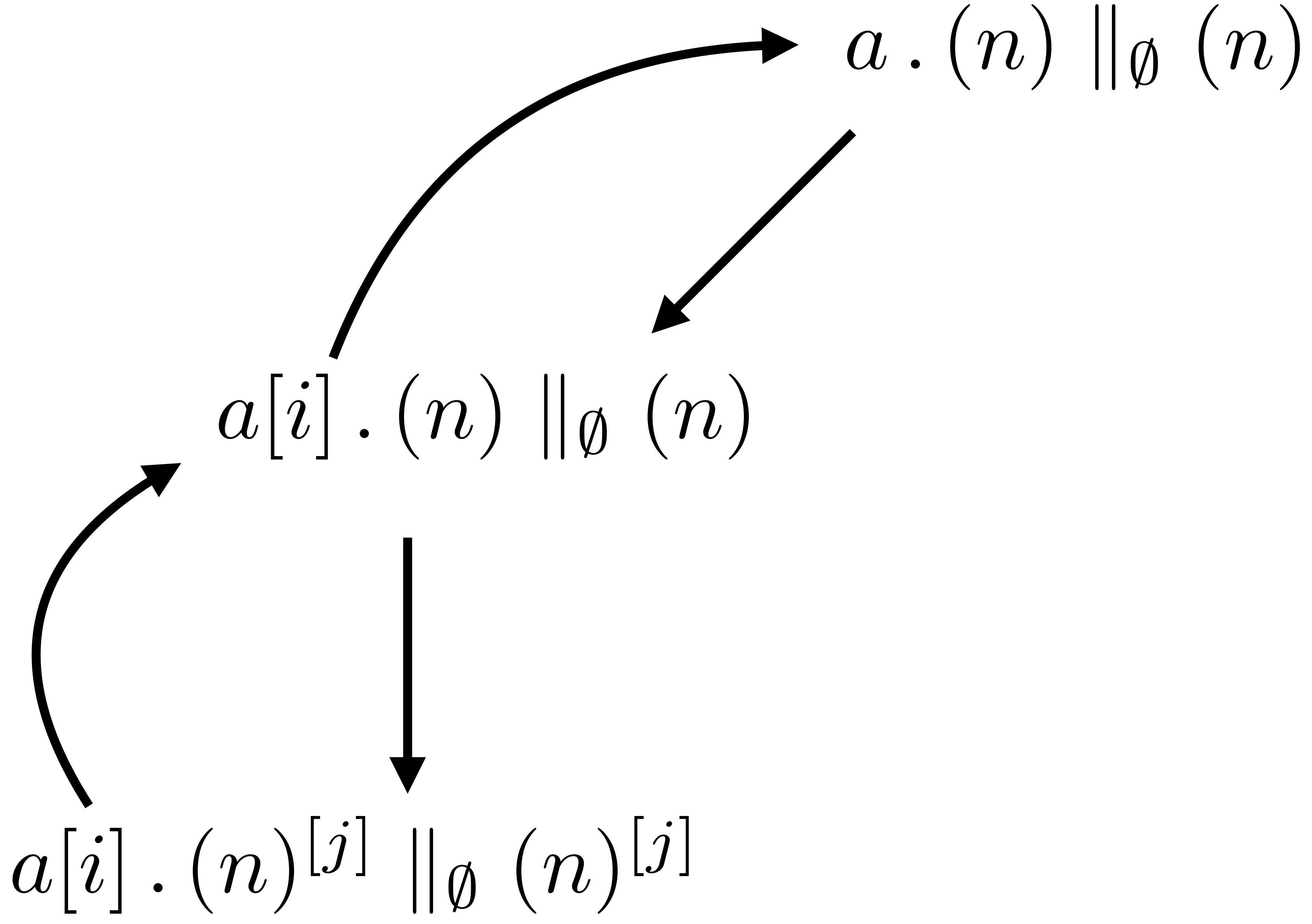
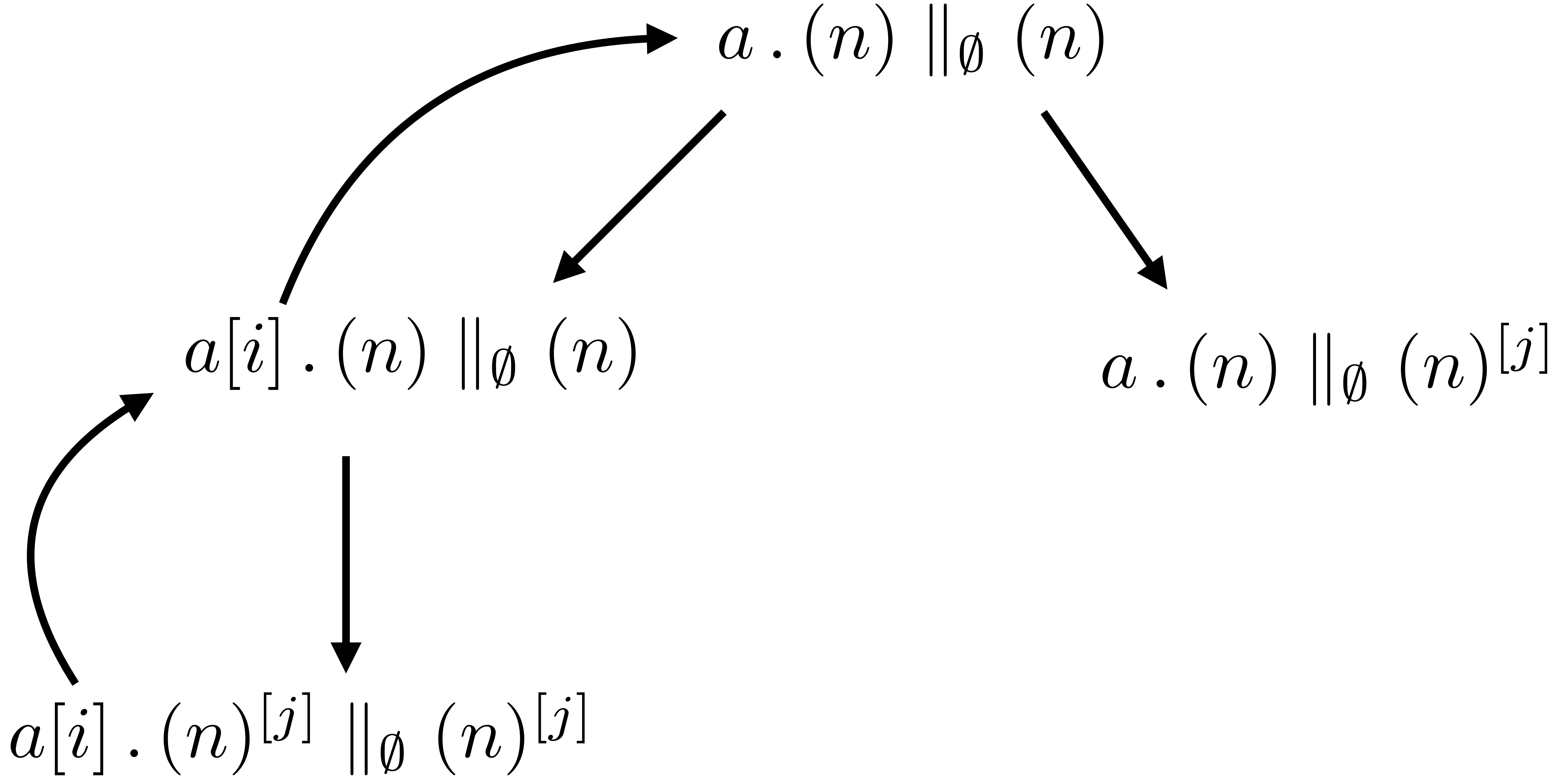
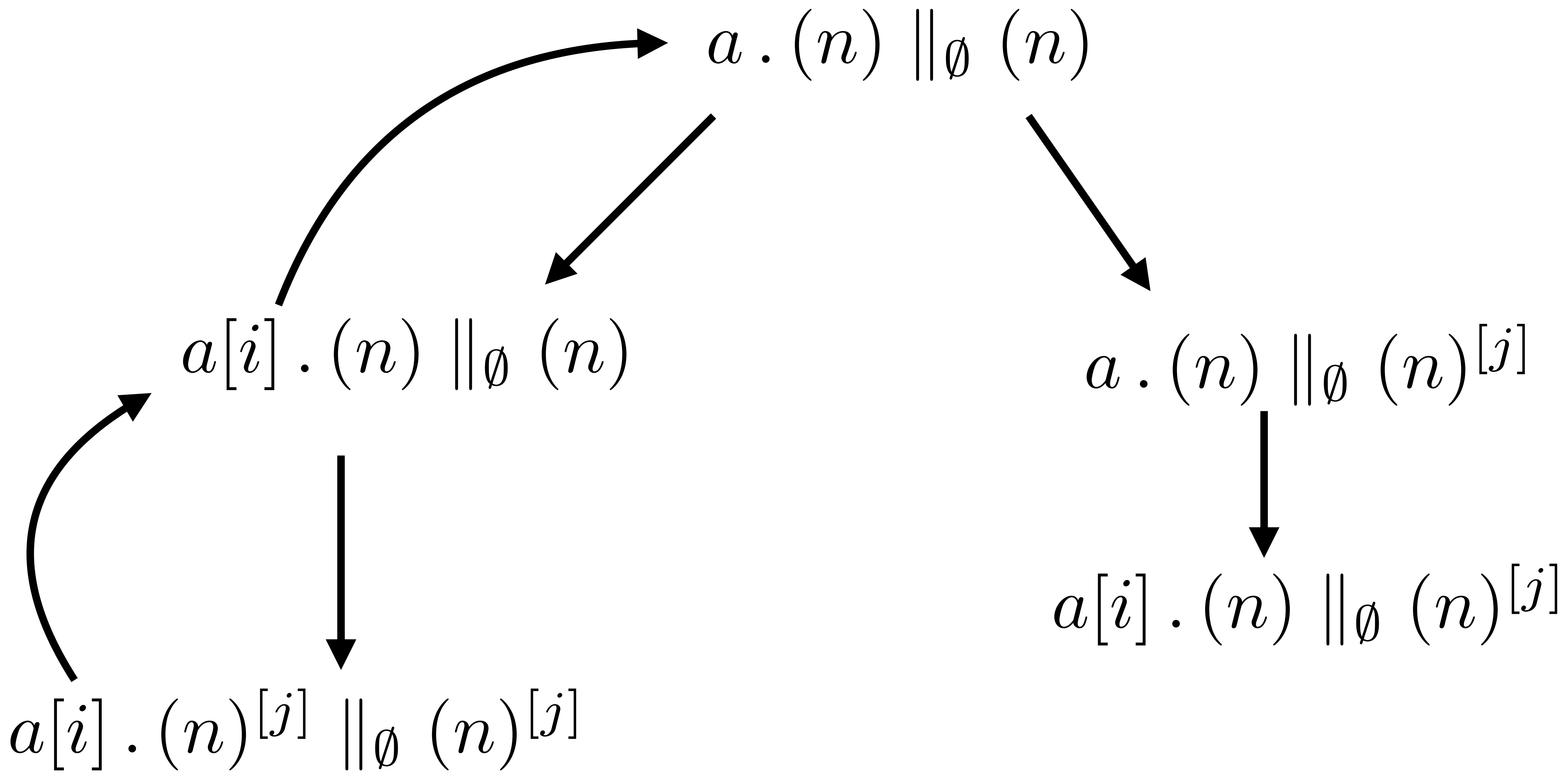
Dynamic delay operator ?



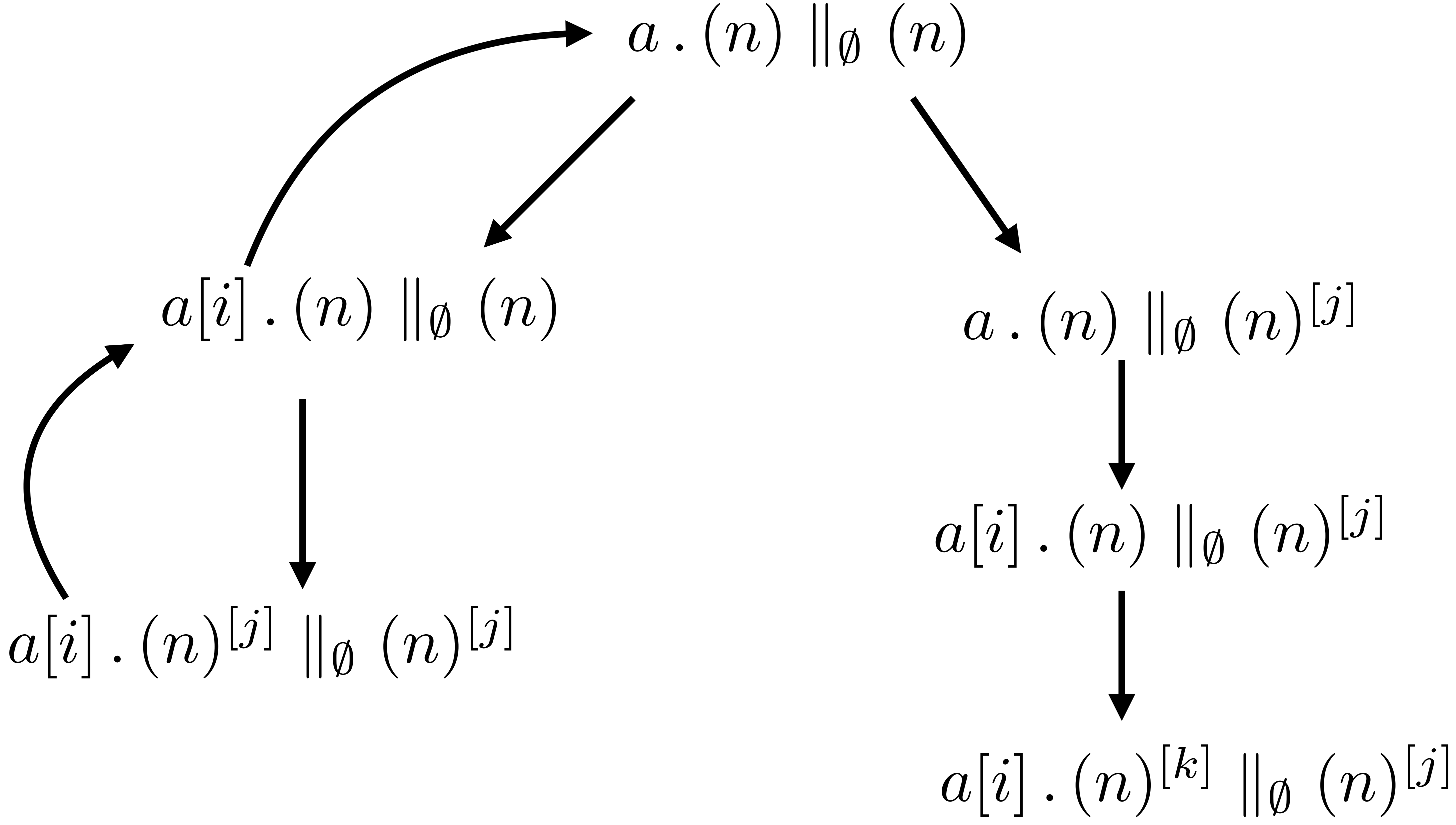
Dynamic delay operator ?



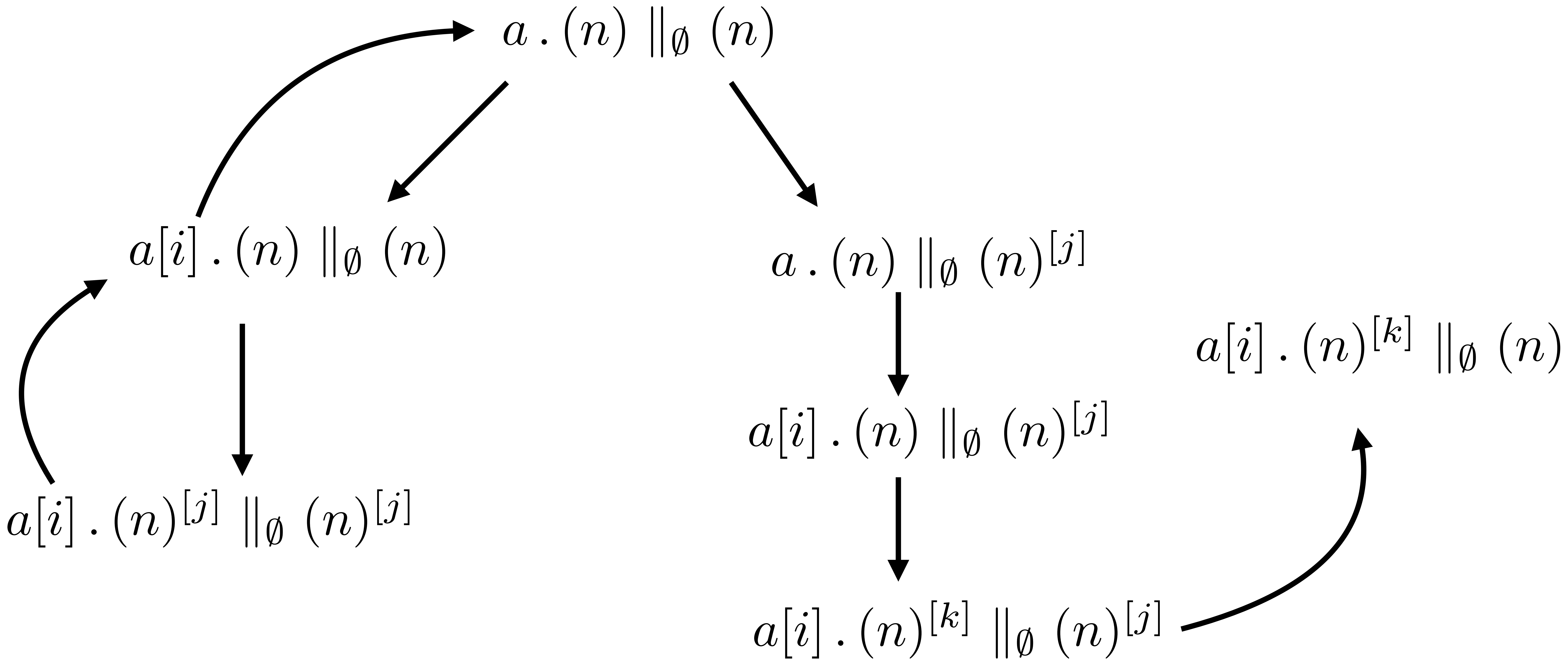
Dynamic delay operator ?



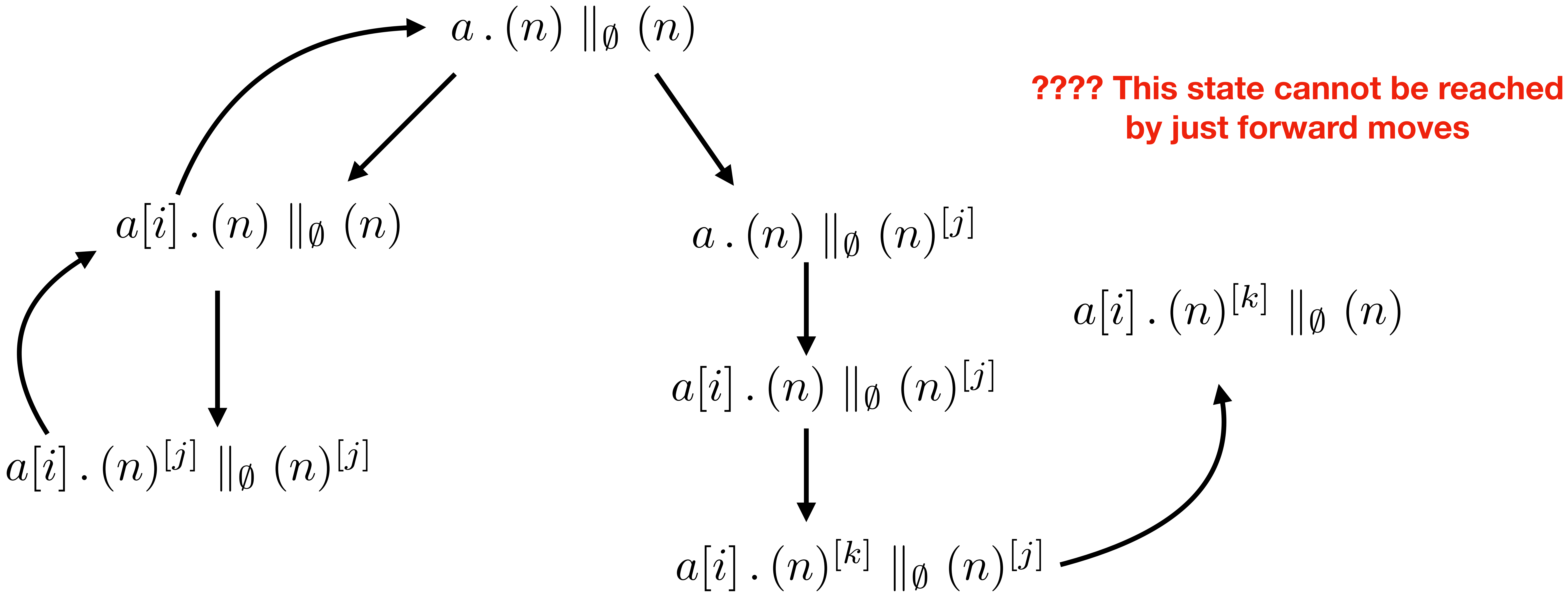
Dynamic delay operator ?



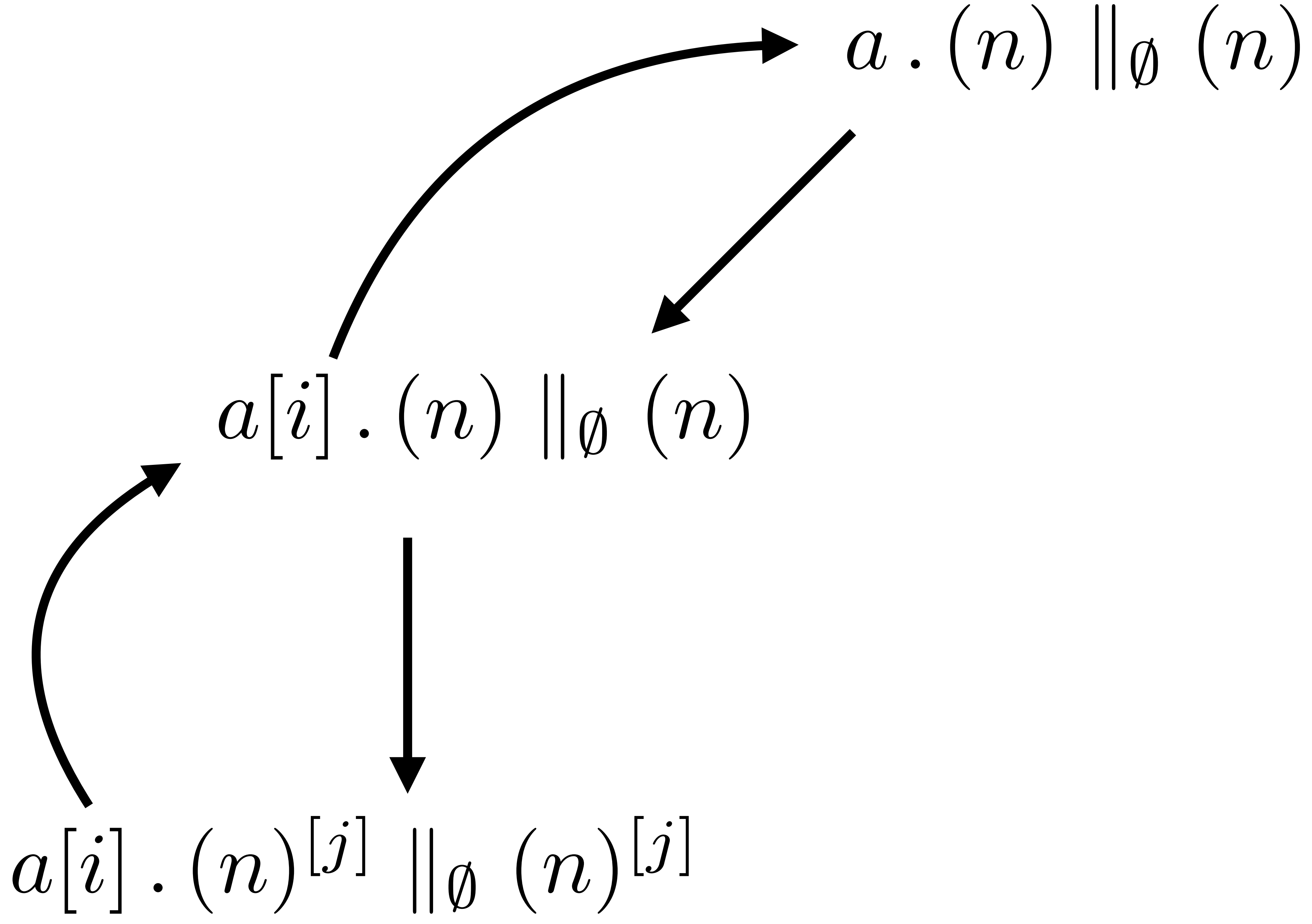
Dynamic delay operator ?



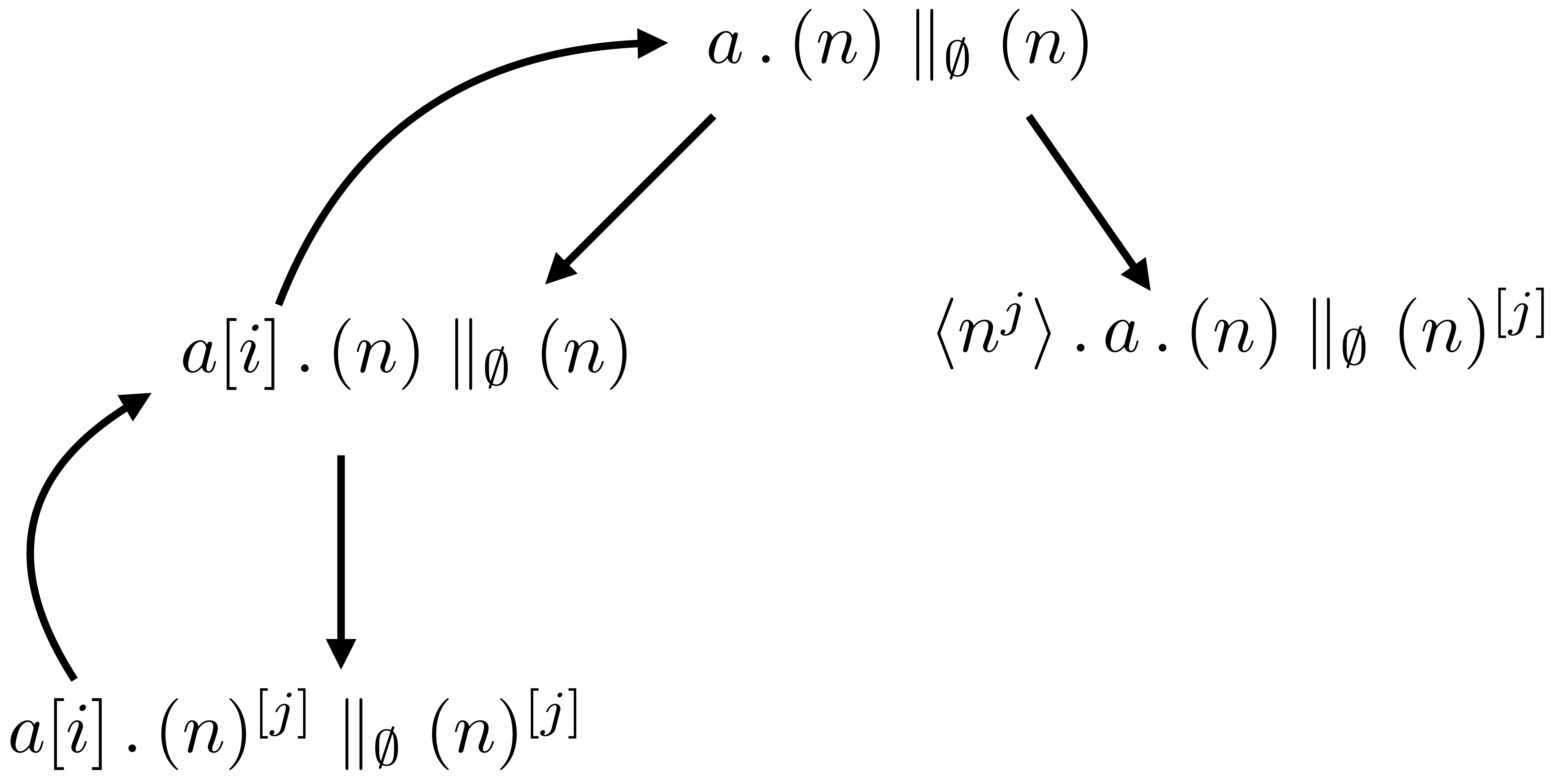
Dynamic delay operator ?



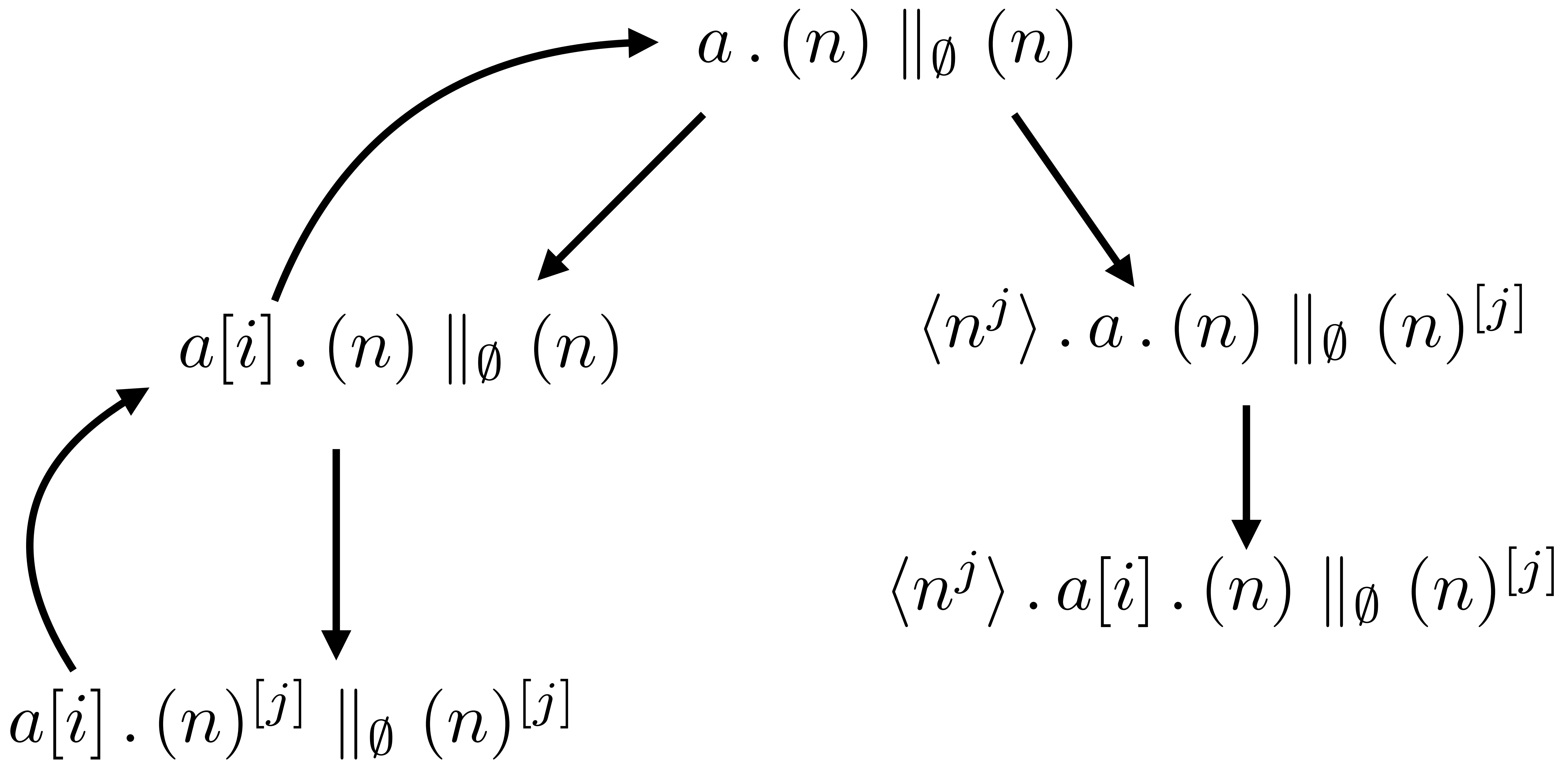
Dynamic delay operator LBMY22



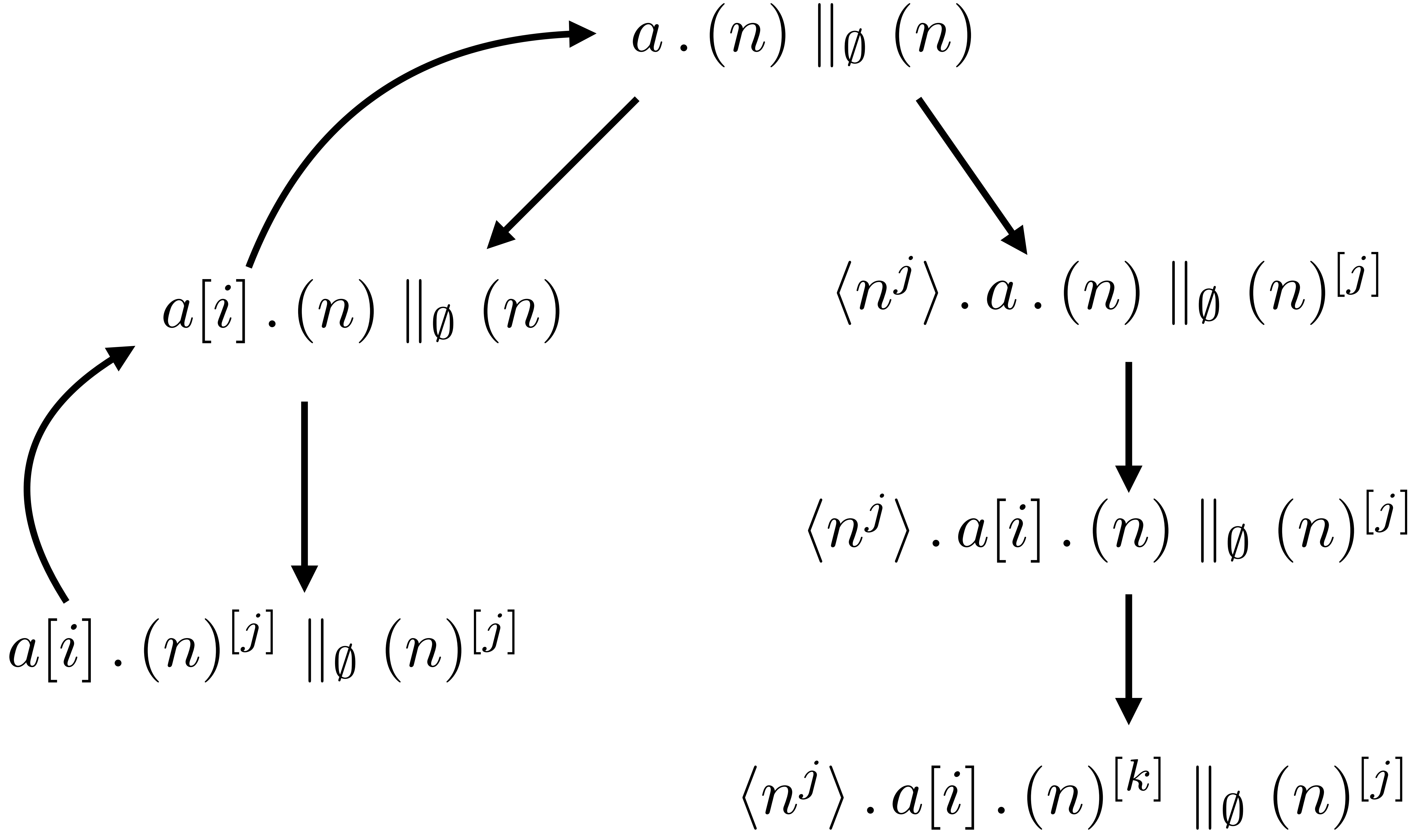
Dynamic delay operator LBMY22



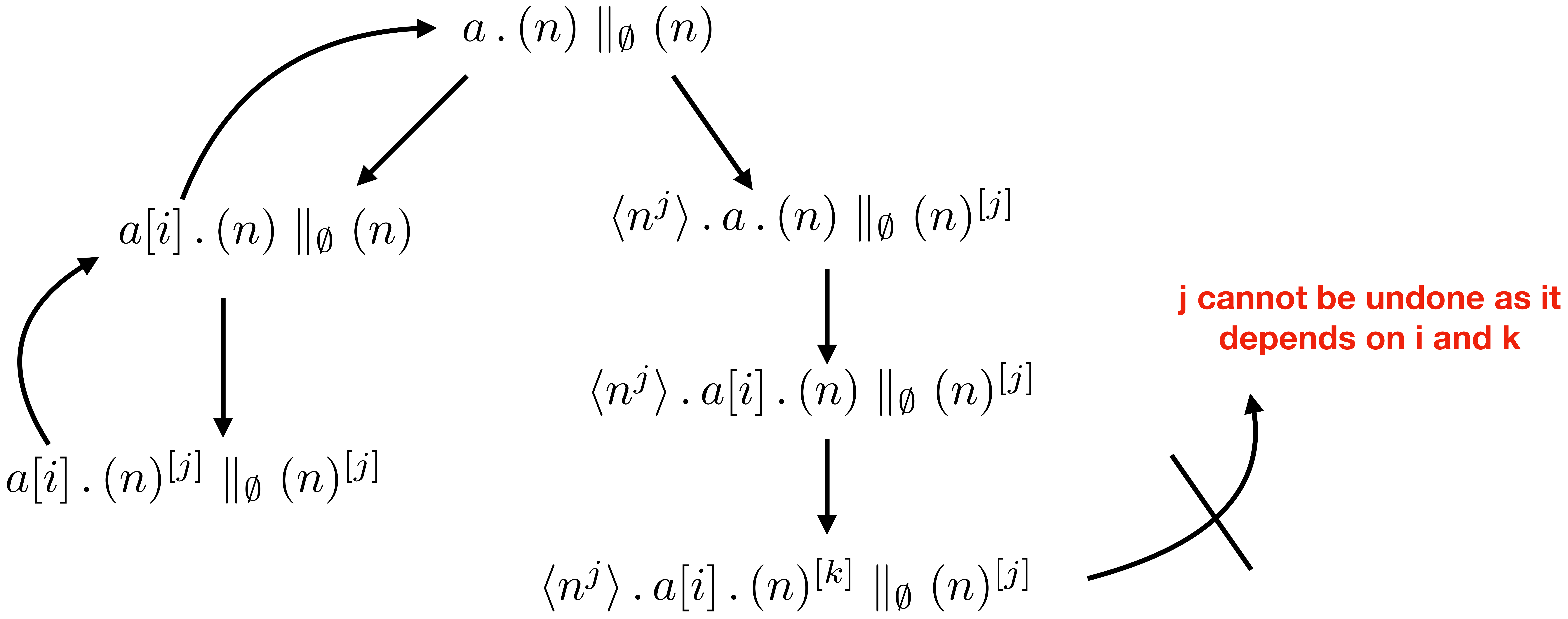
Dynamic delay operator LBMY22



Dynamic delay operator LBMY22



Dynamic delay operator LBMY22



Properties

Proposition 1 (time determinism). *Let* $R, S_1, S_2 \in \mathbb{P}$, $n \in \mathbb{N}_{>0}$, $i_1, i_2 \in \mathcal{K}$, *and* $j \in \mathcal{K}$ *be not occurring associated with past delays in* S_1 *and* S_2. *Then:*

– *If* $R \xrightarrow{(n)^{[i_1]}}_{\mathtt{d}} S_1$ *and* $R \xrightarrow{(n)^{[i_2]}}_{\mathtt{d}} S_2$, *then* $S_1\{^j/_{i_1}\} = S_2\{^j/_{i_2}\}$.

– *If* $R \overset{(n)^{[i_1]}}{\dashrightarrow}_{\mathtt{d}} S_1$ *and* $R \overset{(n)^{[i_2]}}{\dashrightarrow}_{\mathtt{d}} S_2$, *then* $S_1 = S_2$. ■

Proposition 2 (time additivity). *Let* $R, S, S' \in \mathbb{P}$, $n, h \in \mathbb{N}_{>0}$, *and* $i \in \mathcal{K}$. *Then:*

– $R \xrightarrow{(n)^{[i]}}_{\mathtt{d}} S$ *iff* $R \xrightarrow{(m_1)^{[i_1]}}_{\mathtt{d}} \dots \xrightarrow{(m_h)^{[i_h]}}_{\mathtt{d}} S'$ *with* $\sum_{1 \le l \le h} m_l = n$.

– $R \overset{(n)^{[i]}}{\dashrightarrow}_{\mathtt{d}} S$ *iff* $R \overset{(m_1)^{[i_1]}}{\dashrightarrow}_{\mathtt{d}} \dots \overset{(m_h)^{[i_h]}}{\dashrightarrow}_{\mathtt{d}} S$ *with* $\sum_{1 \le l \le h} m_l = n$. ■

Axioms of reversibility

In LPU20 gives a general framework to prove causal consistency

- On a LTS with an independence relation and on which loop lemma holds
- **Square property** (SP), **well-foundness** (WF) and **backward transition independence** (BTI)

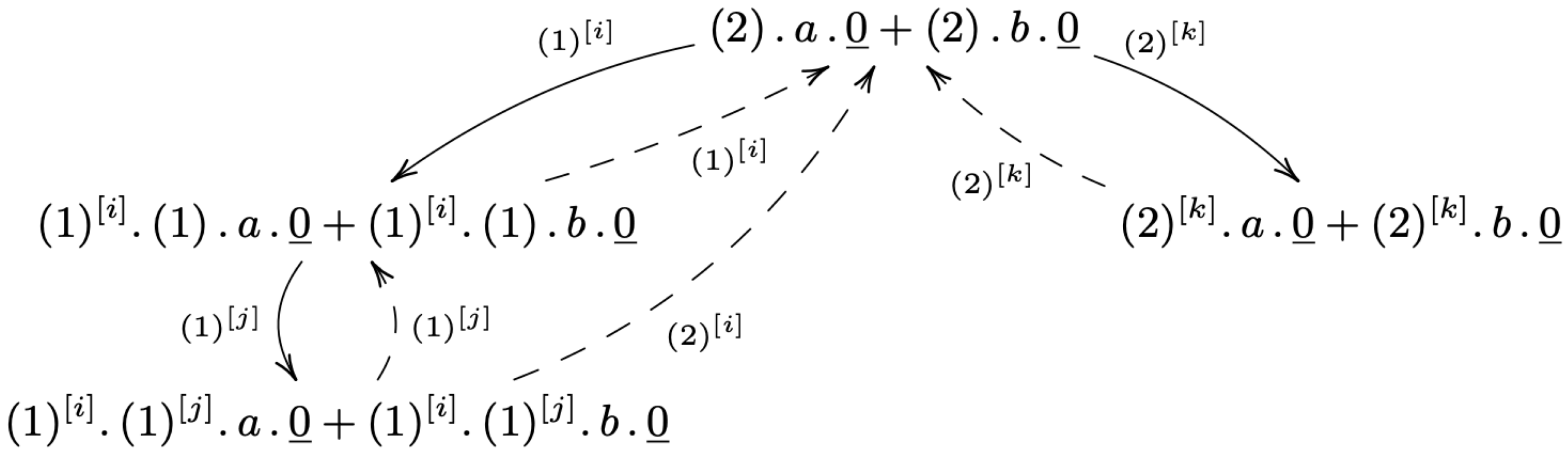
  In RTPC

  - Loop Lemma does not hold
    - Any transition can be undone
  - Square property does hold
    - Concurrent transitions can be swapped
  - WF does hold
    - History information is finite
  - BTI holds as long as at least one of the two coinitial backward transitions is not a delay transition

Loop Property (weak)

Proposition 3 (loop property). *Let* $R, S \in \mathbb{P}$, $a[i], (n)^{[i]} \in \mathcal{L}$, *and* $h \in \mathbb{N}_{>0}$. *Then:*

- $R \xrightarrow{a[i]}_{\mathtt{a}} S$ *iff* $S \overset{a[i]}{\dashrightarrow}_{\mathtt{a}} R$.
- *if* $R \xrightarrow{(n)^{[i]}}_{\mathtt{d}} S$ *then* $S \overset{(m_1)^{[i_1]}}{\dashrightarrow}_{\mathtt{d}} \ldots \overset{(m_h)^{[i_h]}}{\dashrightarrow}_{\mathtt{d}} R$ *with* $\sum_{1 \leq l \leq h} m_l = n$.
- *if* $R \overset{(n)^{[i]}}{\dashrightarrow}_{\mathtt{d}} S$ *then* $S \xrightarrow{(m_1)^{[i_1]}}_{\mathtt{d}} \ldots \xrightarrow{(m_h)^{[i_h]}}_{\mathtt{d}} R$ *with* $\sum_{1 \leq l \leq h} m_l = n$. ∎



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$(1)^{[i]}$
$(2)^{[k]}$
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$(1)^{[i]} . (1) . a . \underline{0} + (1)^{[i]} . (1) . b . \underline{0}$
$(2)^{[k]} . a . \underline{0} + (2)^{[k]} . b . \underline{0}$
$(1)^{[j]}$
$(1)^{[j]}$
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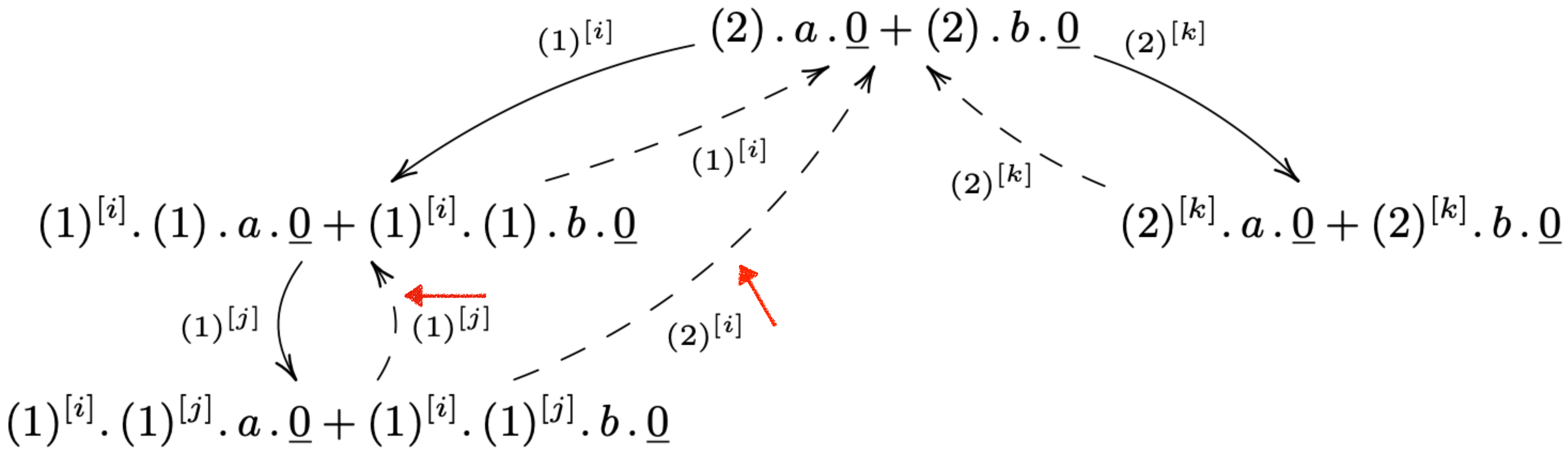
$(1)^{[i]} . (1)^{[j]} . a . \underline{0} + (1)^{[i]} . (1)^{[j]} . b . \underline{0}$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$(2)^{[k]}.a.\underline{0} + (2)^{[k]}.b.\underline{0}$
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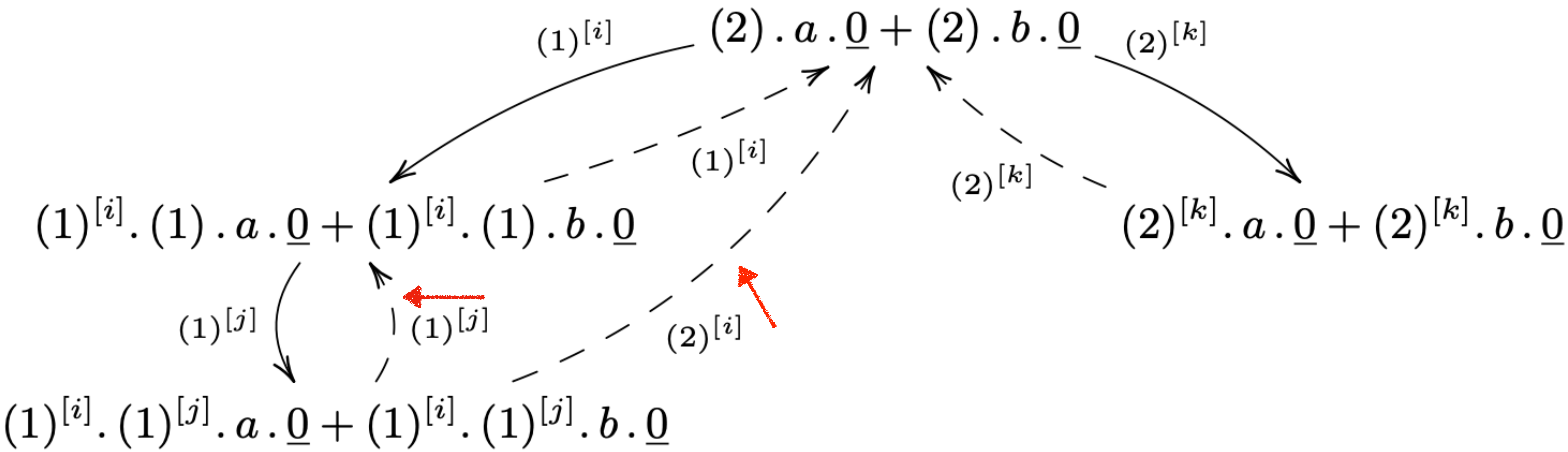
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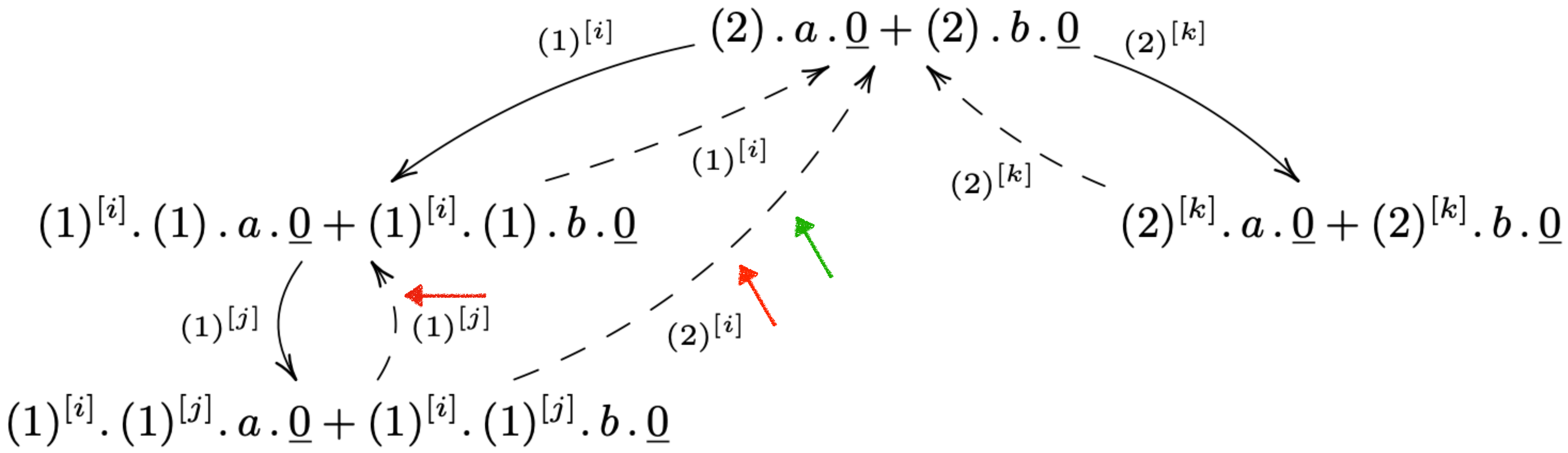
(2)[k]. a . 0 + (2)[k]. b . 0
(1)[j]
(1)[j]
(2)[i]
(1)[i]. (1)[j]. a . 0 + (1)[i]. (1)[j]. b . 0

No BTI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(1)[i] . (1) . a . 0 + (1)[i] . (1) . b . 0
(2)[k] . a . 0 + (2)[k] . b . 0
(1)[j]
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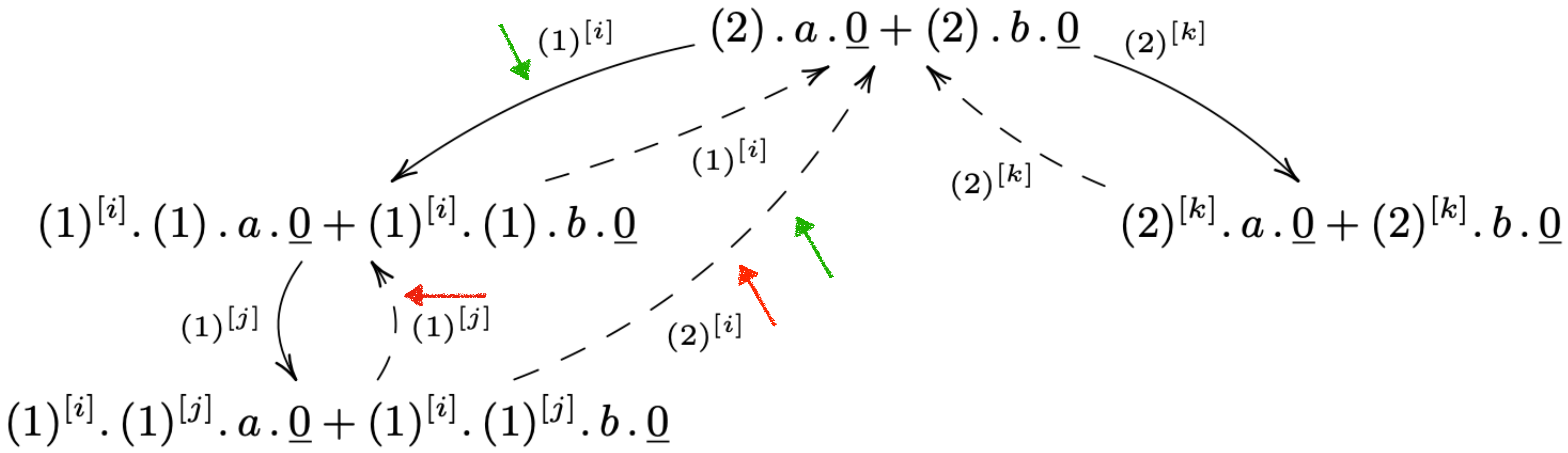
(2)[i]
(1)[i] . (1)[j] . a . 0 + (1)[i] . (1)[j] . b . 0

No BTI



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$(2)^{[k]} . a . \underline{0} + (2)^{[k]} . b . \underline{0}$
$(1)^{[j]}$
$(1)^{[j]}$
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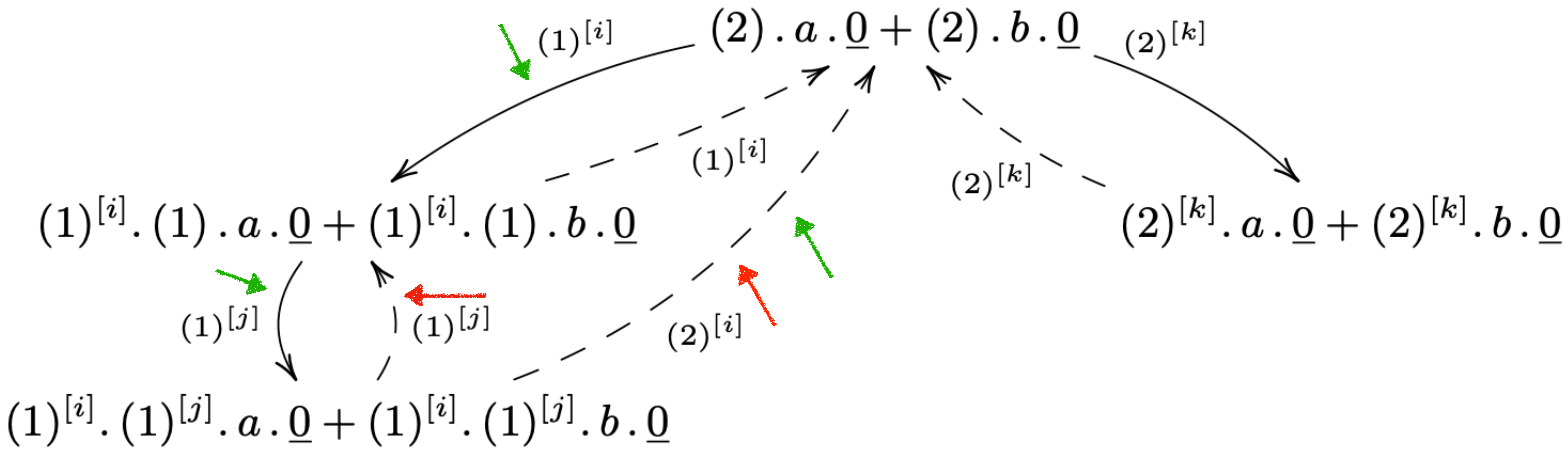
$(1)^{[i]} . (1)^{[j]} . a . \underline{0} + (1)^{[i]} . (1)^{[j]} . b . \underline{0}$

No BTI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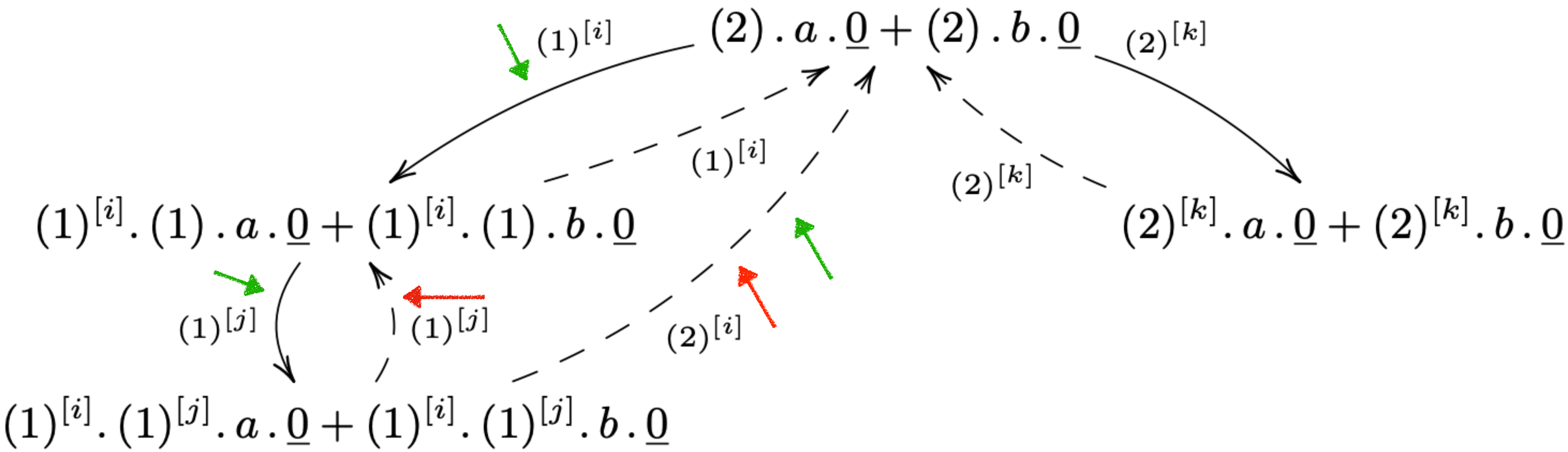
(2)[k]
(1)[i]
(2)[k]
(1)[i]. (1) . a . 0 + (1)[i]. (1) . b . 0
(2)[k]. a . 0 + (2)[k]. b . 0
(1)[j]
(1)[j]
(2)[i]
(1)[i]. (1)[j]. a . 0 + (1)[i]. (1)[j]. b . 0

No BTI

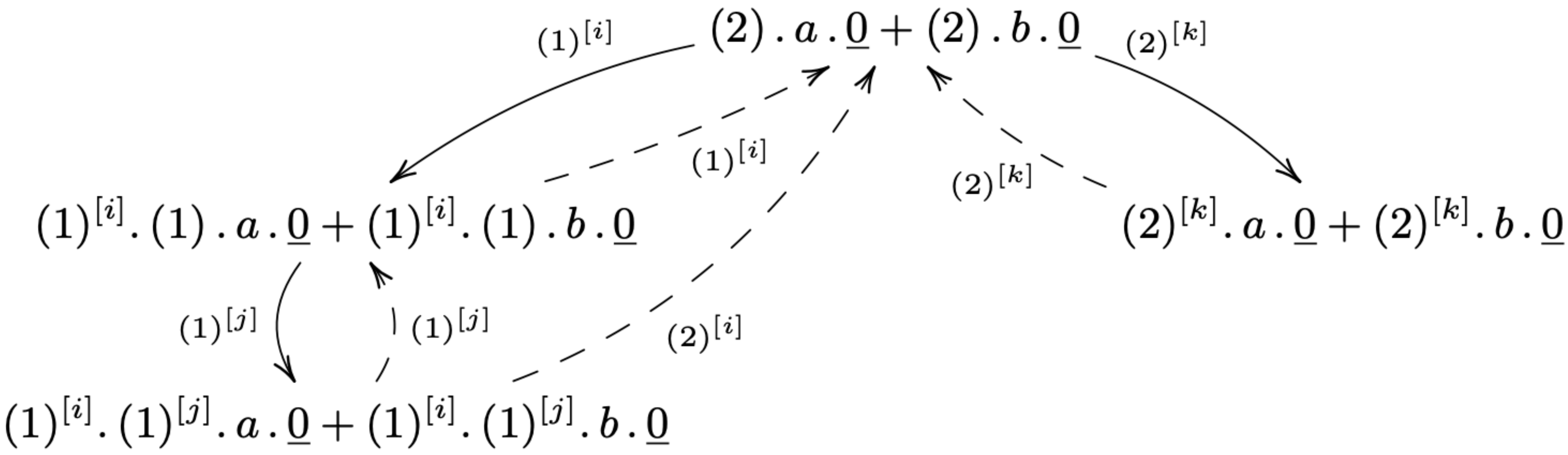


No BTI

No classical loop lemma

Backward triangularity

Lemma 3 (backward triangularity). *Let* $R \in \mathbb{P}$. *Whenever* $R \xrightarrow{(n)^{[i]}}_{\mathtt{d}} S_1$ *and* $R \xrightarrow{(m)^{[j]}}_{\mathtt{d}} S_2$ *with* $m > n$, *then* $S_1 \xrightarrow{(m-n)^{[k]}}_{\mathtt{d}} S_2$. ■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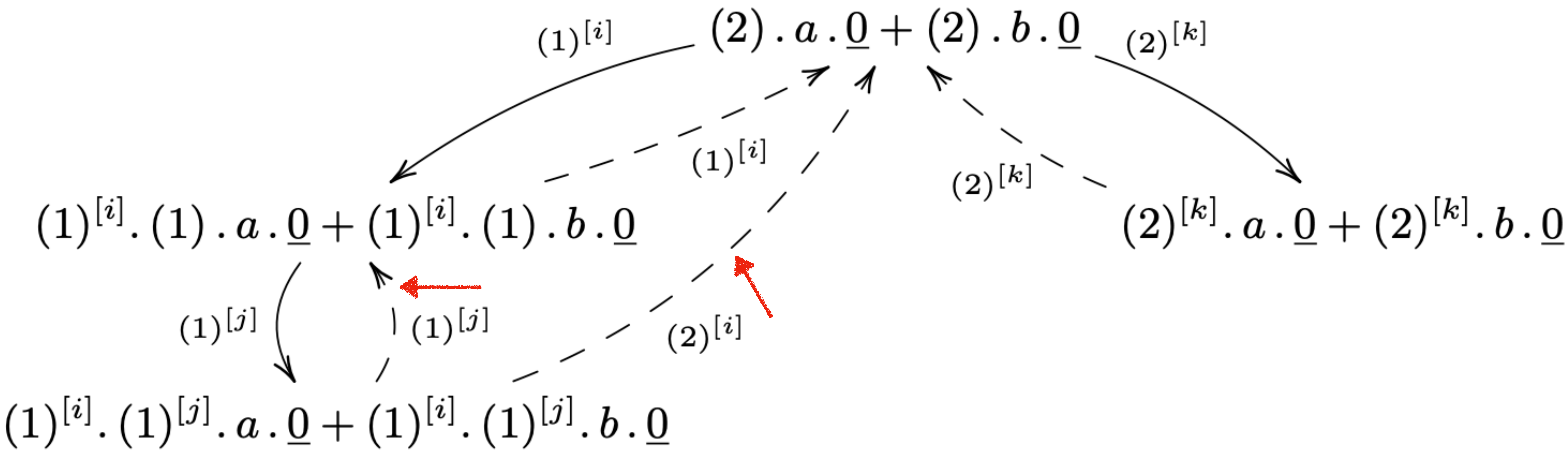
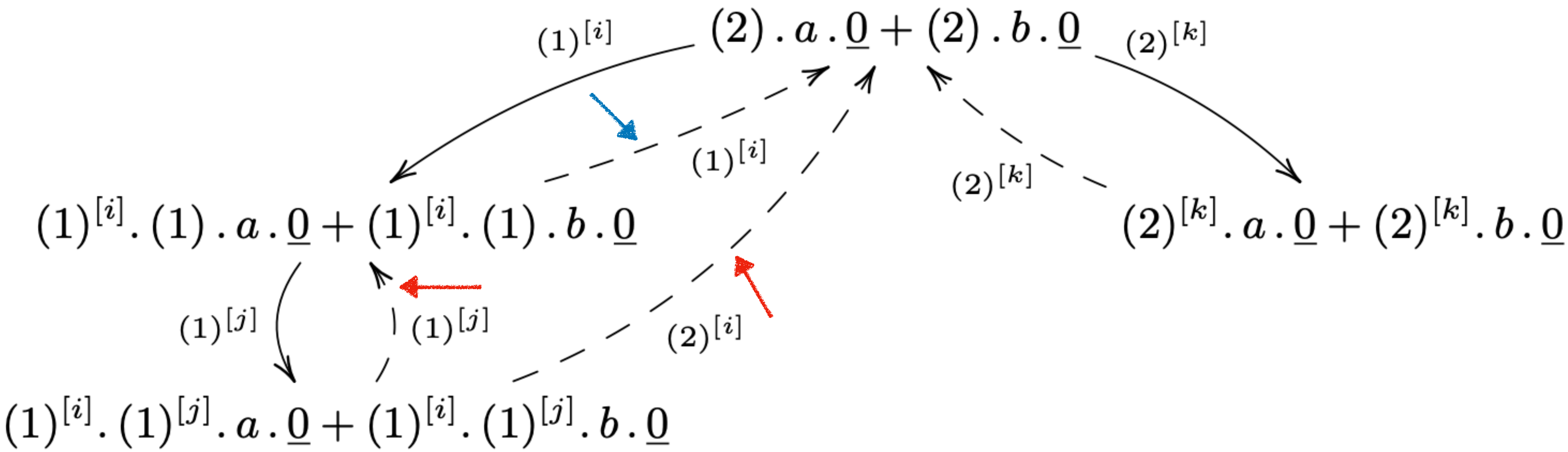
$(1)^{[i]}$
$(2)^{[k]}$
$(1)^{[i]} . (1) . a . \underline{0} + (1)^{[i]} . (1) . b . \underline{0}$
$(2)^{[k]} . a . \underline{0} + (2)^{[k]} . b . \underline{0}$
$(1)^{[j]}$
$(1)^{[j]}$
$(2)^{[i]}$
$(1)^{[i]} . (1)^{[j]} . a . \underline{0} + (1)^{[i]} . (1)^{[j]} . b . \underline{0}$



$(2).a.\underline{0} + (2).b.\underline{0}$
$(1)^{[i]}$
$(2)^{[k]}$
$(1)^{[i]}$
$(2)^{[k]}$
$(1)^{[i]}.(1).a.\underline{0} + (1)^{[i]}.(1).b.\underline{0}$
$(2)^{[k]}.a.\underline{0} + (2)^{[k]}.b.\underline{0}$
$(1)^{[j]}$
$(1)^{[j]}$
$(2)^{[i]}$
$(1)^{[i]}.(1)^{[j]}.a.\underline{0} + (1)^{[i]}.(1)^{[j]}.b.\underline{0}$



$(2).a.\underline{0}+(2).b.\underline{0}$
$(1)^{[i]}$
$(2)^{[k]}$
$(1)^{[i]}$
$(2)^{[k]}$
$(1)^{[i]}.(1).a.\underline{0}+(1)^{[i]}.(1).b.\underline{0}$
$(2)^{[k]}.a.\underline{0}+(2)^{[k]}.b.\underline{0}$
$(1)^{[j]}$
$(1)^{[j]}$
$(2)^{[i]}$
$(1)^{[i]}.(1)^{[j]}.a.\underline{0}+(1)^{[i]}.(1)^{[j]}.b.\underline{0}$

Causal equivalence

Definition 2 (causal equivalence). Causal equivalence $\asymp$ *is the smallest equivalence relation over paths that is closed under composition and satisfies the following:*

1. $\theta_1\theta_2' \asymp \theta_2\theta_1'$ *for any two coinitial concurrent action transitions* $\theta_1 : R \xmapsto{a[i]} R_1$ *and* $\theta_2 : R \xmapsto{b[j]} R_2$ *and any two cofinal action transitions* $\theta_2' : R_1 \xmapsto{b[j]} S$ *and* $\theta_1' : R_2 \xmapsto{a[i]} S$ *respectively composable with the previous ones.*
2. $\theta\overline{\theta} \asymp \epsilon$ *and* $\overline{\theta}\theta \asymp \epsilon$ *for any transition* θ.
3. $\omega_1\overline{\omega_2} \asymp \epsilon$ *and* $\overline{\omega_2}\omega_1 \asymp \epsilon$ *for any two coinitial and cofinal forward paths* ω_1 *and* ω_2 *with delay transitions only such that* $\mathtt{time}(\omega_1) = \mathtt{time}(\omega_2)$. ■

Causal equivalence

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2. $\theta\overline{\theta} \asymp \epsilon$ *and* $\overline{\theta}\theta \asymp \epsilon$ *for any transition* θ*.*
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Properties

Lemma 5 (parabolic lemma). *For any path ω, there exist two forward paths ω_1 and ω_2 such that $\omega \asymp \overline{\omega_1}\omega_2$ and $|\omega_1| + |\omega_2| \leq |\omega|$.* ■

We could not exploit the axioms of Lanese, Phillips & Ulidowski to prove PB

Theorem 1 (causal consistency). *Let ω_1 and ω_2 be two paths. Then $\omega_1 \asymp \omega_2$ iff ω_1 and ω_2 are both coinitial and cofinal.* ■

We could not exploit the axioms of Lanese, Phillips & Ulidowski to prove CC

PB + WF (+ ~~looplemma~~) = CC

Timeout operator

Under maximal progress, we can encode a timeout operator Timeout(P, Q, n)

- It allows the process P to communicate with the environment within n units of time
- After this time has passed, and P has not communicated yet, the process Q takes control

$$\textsc{Timeout}(P, Q, n) = P + (n)\,.\,\tau\,.\,Q$$

Erlang receive

```
process_A() ->
  receive
    X -> handleMsg()
    after 50 ->
      handleTimeout()
  end end.
```

```
process_B(Pid) ->
  timer:sleep(100),
  Pid ! Msg end.

PidA=spawn(?MODULE, process_A, []),
spawn(?MODULE, process_B, [PidA]).
```

Process A awaits a message from the environment (B)

- If the message is received within 50 ms then handleMsg() is called
- Otherwise handleTimeout() is called

RTPC example

$$A = \textsc{Timeout}(a \,.\, P, Q, 50)$$
$$B = (100) \,.\, a \,.\, \underline{0}$$

P is the process encoding handleMsg()

Q is the process encoding handleTimeout()

$$A \parallel_{\{a\}} B \xrightarrow{(50)^{[i]}}_{\mathtt{d}} (a \,.\, P + (50)^{[i]} . \tau \,.\, Q) \parallel_{\{a\}} ((50)^{[i]} . (50) \,.\, a \,.\, \underline{0})$$
$$\xrightarrow{\tau[j]}_{\mathtt{a}} (a \,.\, P + (50)^{[i]} . \tau[j] \,.\, Q) \parallel_{\{a\}} ((50)^{[i]} . (50) \,.\, a \,.\, \underline{0})$$

Conclusions

- We have studied reversibility in a calculus with
  - Eagerness / laziness and where time is modelled via numeric delays
  - Numeric delays are subject to time additivity
- Because of time additivity
  - Loop property is stated differently
  - BTI does not hold

As future work, we plan to investigate suitable notions of bisimilarity for RTPC

Better way to deal with dynamic delay prefix (use zone?)

Study reversibility and dense time

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