Refreshing an Old Problem: Relating Nets and Event Structures in the Reversible Setting

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Starting point

Petri nets (PN) and event structures (ES) are two well known and studied models for concurrency

PN on the operational side, ES on the denotational one

Question: how PN and ES cope with reversibility and how to relate them?

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Question: how PN and ES cope with reversibility and how to relate them?

lot of work has been made by others, I'm working on this mainly with Claudio Mezzina and Hernán Melgratti focussing on the Petri Nets side

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configurations furnish a denotational view of the system

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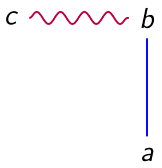

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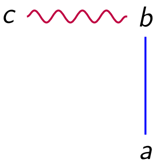

$$
\begin{tikzcd}
\emptyset \arrow[r] \arrow[d] \arrow[dr] & \{a\} \arrow[r] \arrow[d] & \{a, b\} \\
\{c\} \arrow[r] & \{a, c\} &
\end{tikzcd}
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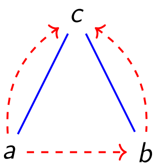

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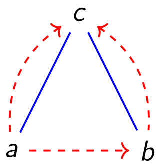

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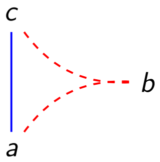
an inhibitor ES ($\vdash\!\!\circ(\{a\}, b, \{c\})$, $\vdash\!\!\circ(\emptyset, c, \{a\})$)



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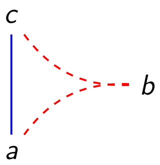

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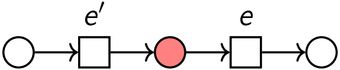
A closer look to dependencies in nets

$e < e'$ can be represented as



the partial order is mimicked by the flow relation, there is a shared place between the two events

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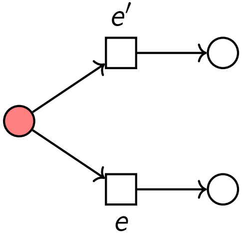
$e \# e'$ can be represented as



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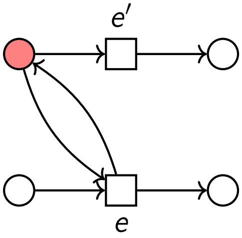

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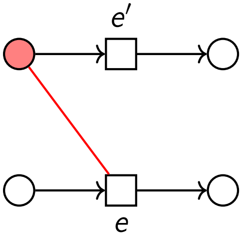
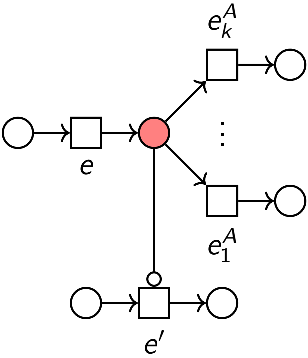


dependencies are obtained still via a shared place and read arc can be used as well

A closer look to dependencies in nets

$\vdash\!\!\multimap(\{e\}, e', A)$ can be represented as



dependencies are obtained via a shared place, but some dependencies arise from inhibitor arcs

A closer look to dependencies in nets

and you can continue with the same machinery for other kind of dependencies....

Occurrence Nets

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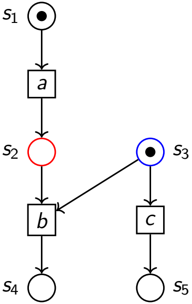

s_2 says that b depends on a and s_3 says that b and c are in conflict

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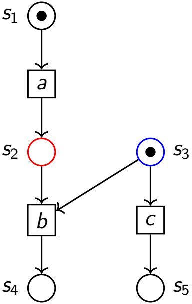

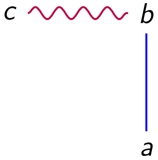

A correspondence between the net and the prime event structures is established and well know

Occurrence Nets

It is possible to add some inhibitor or read arcs to ON to establish similar results for AES or IES

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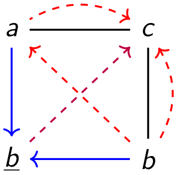

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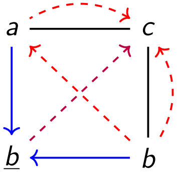

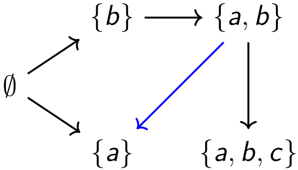

Reversing ON?

We do have an ON modeling the forward flow, can we add some reversing transitions

It would be nice to have ON (maybe enriched to take into account other kind of causality) and add the reversing transitions

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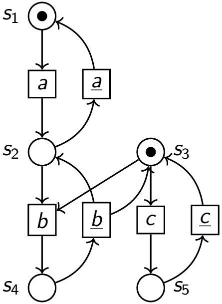

On occurrence nets add the reversing transitions

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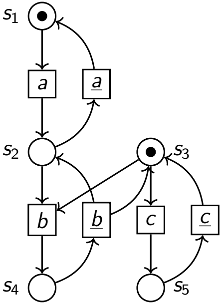

reverse causality and prevention are added using the structure of the net

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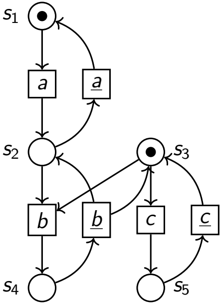

The structure of the net is the limitation: it works with some reversible ES with suitable conditions

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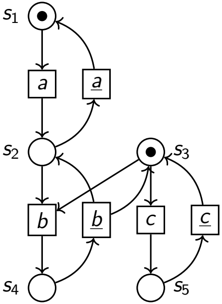

ES where to undo an event you should guarantee that no event causally depending on it is present

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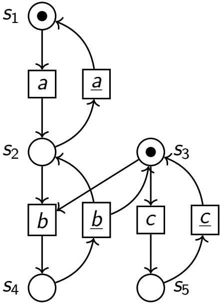

ES where each event is reversible

Back to dependencies in nets

Observe: dependencies via shared places can be modeled differently

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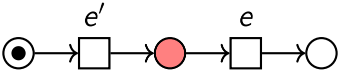

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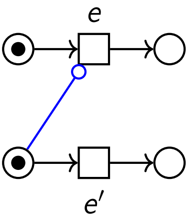

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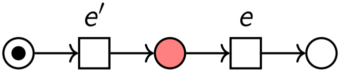
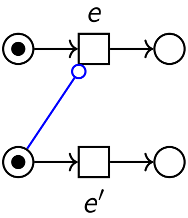
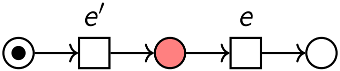
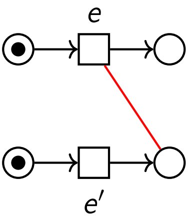
from 

to 

or from 

to 

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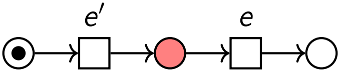
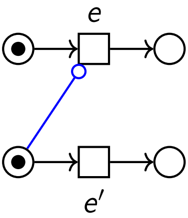
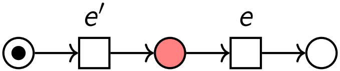
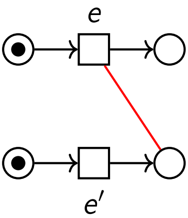
from 

to 

or from 

to 

dependencies can be all modeled using contextual arcs

Pushing this idea further

Inhibitor arcs can be used to model causal dependencies

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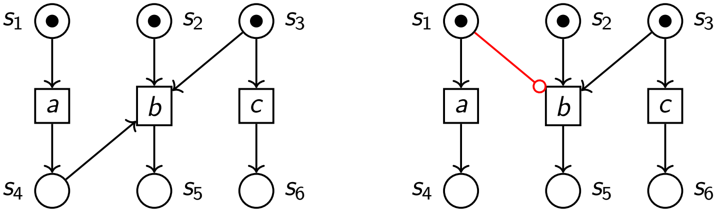

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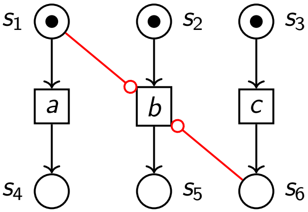

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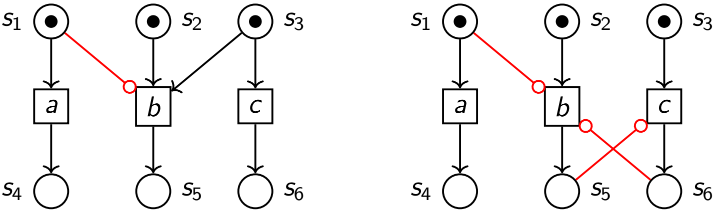

$$a < b,\ b \nearrow c \text{ and } c \nearrow b$$

$b \nearrow c$ and $c \nearrow b$ model $b \# c$

Reversing transitions again

Can we apply the same intuition for the reversing transitions?

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reverse causality and prevention may be modeled again with the same intuition: inhibitor arcs

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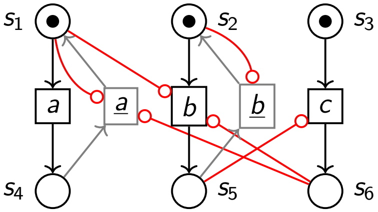

here just a and b are reversed

What we have done

On the net side we have defined nets where all the dependencies among transitions are modeled using inhibitor arcs (called Causal Nets, CN)

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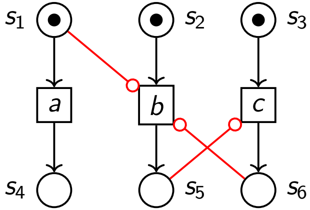

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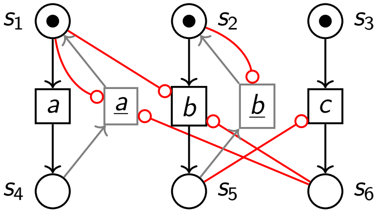

the reverse causality and prevention are on the reversing transitions

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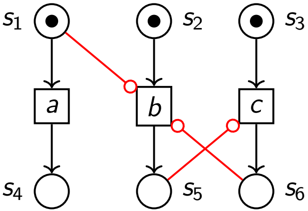
forgetting the reversing transitions we get a



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We have done it for reversible asymmetric ES and reversible prime ES

Do we lose something?

No: each ON can be transformed into a CN

if the CN has suitable characteristics we can retrive an ON

Categorical view

reversible event structures can be turned into categories

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and we have an adjuntion between these categories

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