Asynchronous Session Types: Exceptions and Multiparty Interactions

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PEOPLE INVOLVED

- **Kohei Honda & Nobuko Yoshida**

- Gary Brown
- Steve Ross-Talbot
- Andi Bejleri
- Dimitris Mostrous
- Ray Hu
- Joshua Guttman (security)
- ...and many more (including W3C and Robin Milner)

LECTURE PLAN

1. Asynchrony and Session Types
2. Exceptions and Session Types
3. Multiparty Session Types

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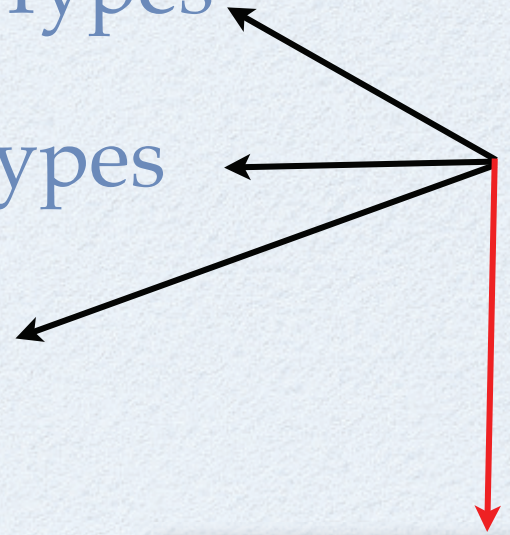

- Motivations
- Syntax and Semantics
- Typing Rules

Why types?

Types are about language primitives [...], they get associated directly with the languages.

[...]basic idea of types should be very weak: for example, if we have session-based primitives and we do not have session types, then they can have a constructor error. Same with Java types, same with ML types.

But this is not only about safety, this is about how we conceive the basic dynamics of programming languages.

-- Kohei Honda

Notation

Term	**Symbol**	**Note**
Public channels (or *shared* or *service* channels)	$a, b, \ldots$	
Session channels	$k, s, t, r \ldots$	
Polarised Session channels	κ, λ	$\kappa \in \{s^+, s^-\}$
Variables	x, y, z	
Process Term Variables	X, Y	
Public channel or Variable	$u, u', \ldots$	

SYNTAX FOR SESSIONS

P :: =	**!**a(s)**.** P	(service)
	a(s)**.** P	(request)
	s**?**(x)**.** P	(input)
	s**!**<e>**.** P	(output)
	s**?** **((**x**))**. P	(session rec)
	s**!**<<s'>>**.** P	(delegation)
	s ▷ { l_i : P_i }	(branch)
	s ◁ l**.** P	(select)
	(υ a) P / (υ s) P	(res)
	P | Q	(par)
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ASYNCHRONY&SESSIONS

Why asynchronous?

ASYNCHRONY&SESSIONS

Why asynchronous?

Reality is mostly asynchronous !!!

ASYNCHRONY&SESSIONS

- First proposed by
  - Neubauer, Thiemann [Unpublished]
- Adopted by several people:
  - Gay&Vasconcelos, Singularity, etc.
- We shall use it today!

Asynchrony in the π-calculus

Synchronous: $s?(x).\, s!\langle e\rangle.\, t!\langle e'\rangle.0$

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Asynchronous: $s?(x).\, (s!\langle e\rangle.\, 0 \;\mid\; t!\langle e'\rangle.\, 0)$

Asynchrony in the π-calculus

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Asynchrony in the π-calculus

Synchronous: $s?(x).\, s!\langle e\rangle.\, t!\langle e'\rangle.0$

Asynchronous: $s?(x).\, (s!\langle e\rangle.\, 0 \mid t!\langle e'\rangle.\, 0)$

Outputs **don't** have successors (no prefixing)

What about session (types)?

Synchronous Case.

Type : $\downarrow(int).\ \downarrow(bool).\ \uparrow(string)$

Processes :

$$a(s).\, s?(x).\, s?(y).\, s!\langle x+1\rangle$$

$$\mid$$

$$\overline{a}(s).\, s!\langle 5\rangle.\, s!\langle \texttt{true}\rangle.\, s?(z)$$

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!!! Need to preserve ordering !!!

Queues: Syntax

$$L ::= \epsilon \mid h :: L$$

(message list)

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$L ::= \epsilon \mid h :: L$ (message list)

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option 1: $s : L$ (simple channel queue)

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option 1: $s : L$ (simple channel queue)

option 2: $\kappa \hookrightarrow \overline{\kappa} : L$ (polarised queue)

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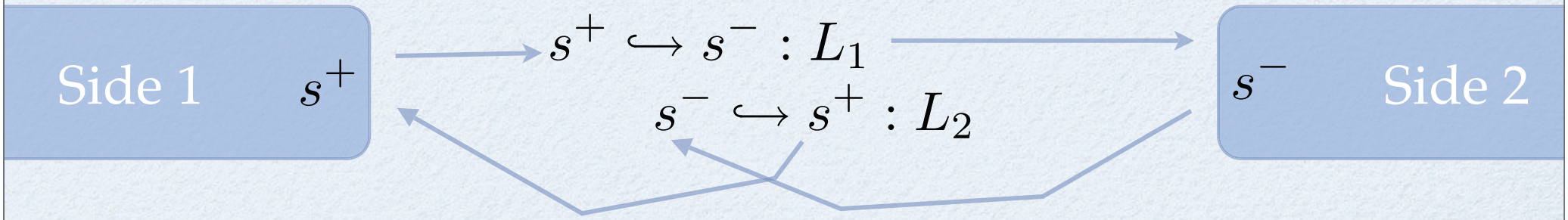

QUEUES: SEMANTICS

$$\kappa!\langle e\rangle.\,P \;\mid\; \kappa \hookrightarrow \overline{\kappa} : L \;\longrightarrow\; P \;\mid\; \kappa \hookrightarrow \overline{\kappa} : (v :: L) \qquad e \Downarrow v$$

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$$a(s).\,P \mid \overline{a}(s).\,Q \longrightarrow$$

$$(\nu s)(\,P \mid Q \mid s^- \hookrightarrow s^+ : L \mid s^+ \hookrightarrow s^- : L\,)$$

BACK TO THE EXAMPLE

Type : $\downarrow(int).\ \downarrow(bool).\ \uparrow(string)$

Processes : $a(s).\, s?(x).\, s?(y).\, s!\langle x+1\rangle$

$\mid$

$\overline{a}(s).\, s!\langle 5\rangle.\, s!\langle \texttt{true}\rangle.\, s?(z)$

BACK TO THE EXAMPLE

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$$(\nu s)\Big(\quad s^-?(x).\, s^-(y).\, s^-!\langle x+1\rangle \quad \Big)$$

$$s^+!\langle 5\rangle.\, s^+\langle \texttt{true}\rangle.\, s^+(z)$$

$$\mid\ s^- \hookrightarrow s^+ : \epsilon \ \mid\ s^+ \hookrightarrow s^- : \epsilon$$

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$$\mid\ s^- \hookrightarrow s^+ : \epsilon \ \mid\ s^+ \hookrightarrow s^- : 5$$

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EXCEPTIONS

Exceptions, literally...

- "An Exception is a person or thing that is excluded from a general statement or does not follow a rule" (Mac Dictionary)
- "Exception (handling) is a programming language construct or computer hardware mechanism designed to handle the occurrence of some condition that changes the normal flow of execution." (Wikipedia)

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EXCEPTIONS, IN GENERAL

try { /* Default Code */ }

catch { /* Handler Code */ }

If an exception is thrown by the default code then the handler is executed.

Exceptions are thrown with a special command **throw**

EXCEPTIONS & SESSIONS

What if apply this idea to *sessions*?

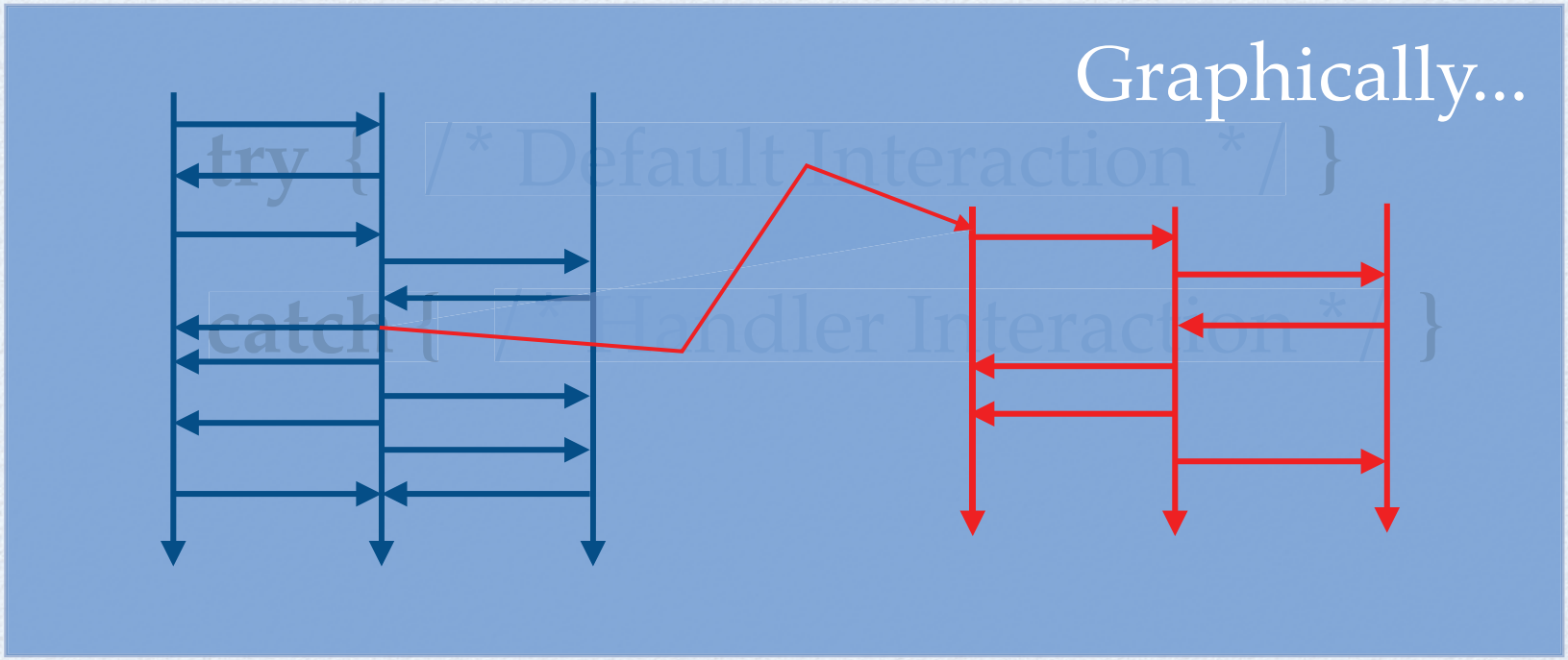
try { /* Default Interaction */ }

catch { /* Handler Interaction */ }

EXCEPTIONS & SESSIONS

What if apply this idea to *sessions*?

Graphically...



EXAMPLE: ASYNCH ESCAPE

Buyer

invoke serv@Seller(s);

try {
  $\mu X.$
  $s?(y).$
  if $(ok(y))$ **then throw else** X
}

catch $\{s!\langle \texttt{card} \rangle.\ s?(y)\ \}$

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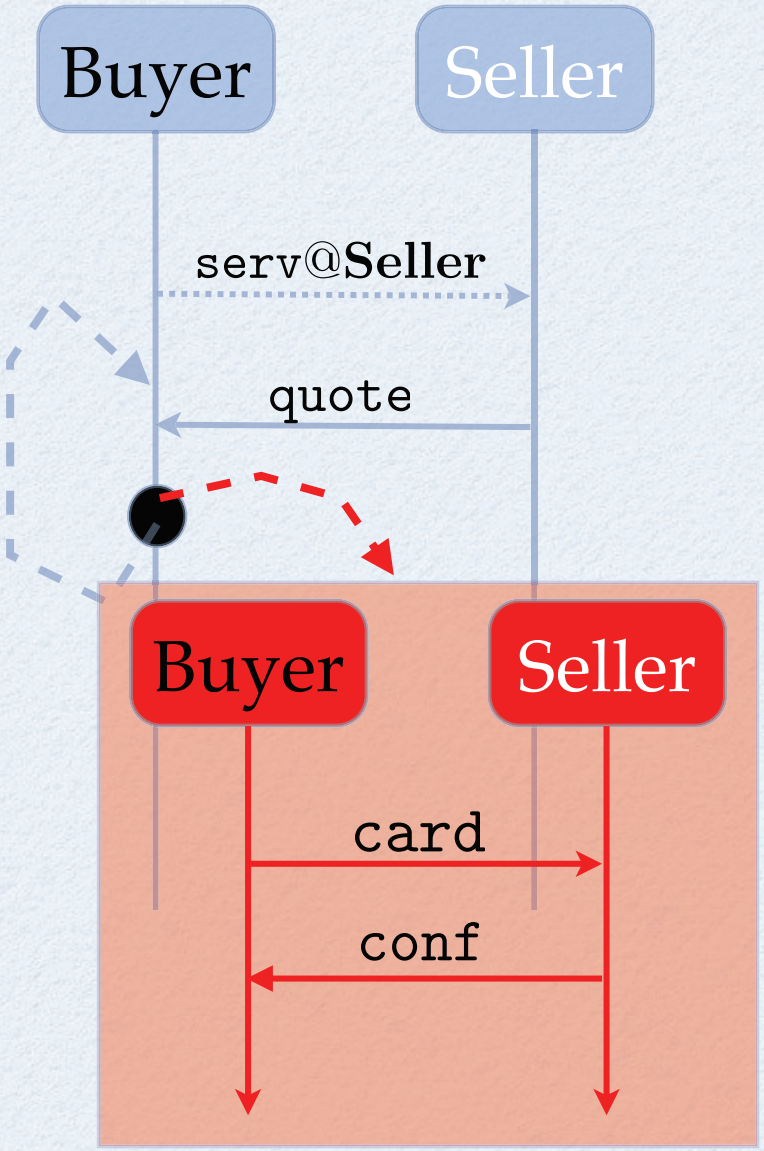

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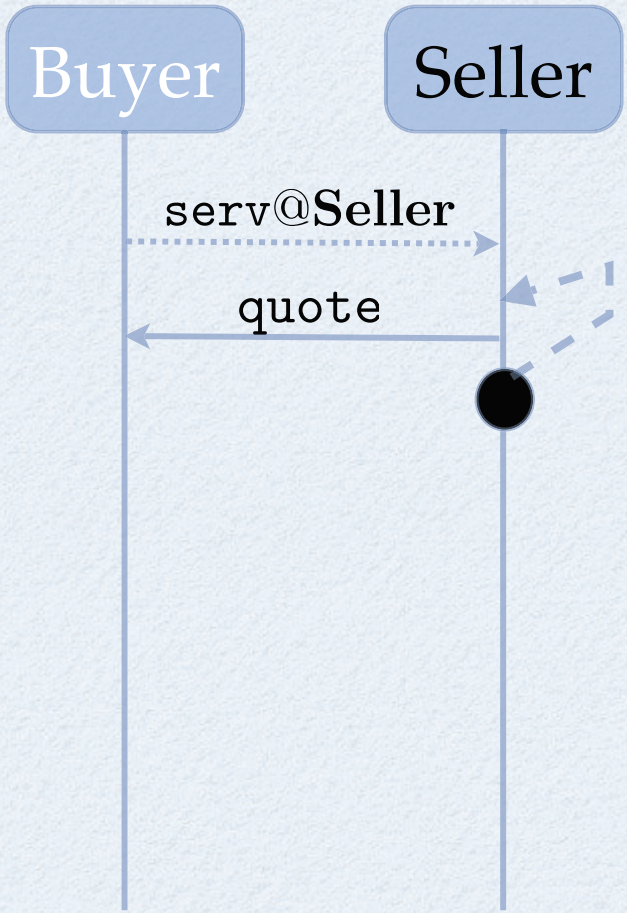

Seller

$$\textbf{service}\ \texttt{serv}(s)\ \{$$

$$\quad \textbf{try}\ \{$$

$$\quad\quad \mu X.$$

$$\quad\quad s!\langle \texttt{quote} \rangle.\ X$$

$$\quad \}$$

$$\quad \textbf{catch}\ \{s?(x).s!\langle \texttt{conf} \rangle\ \}$$

$$\}$$

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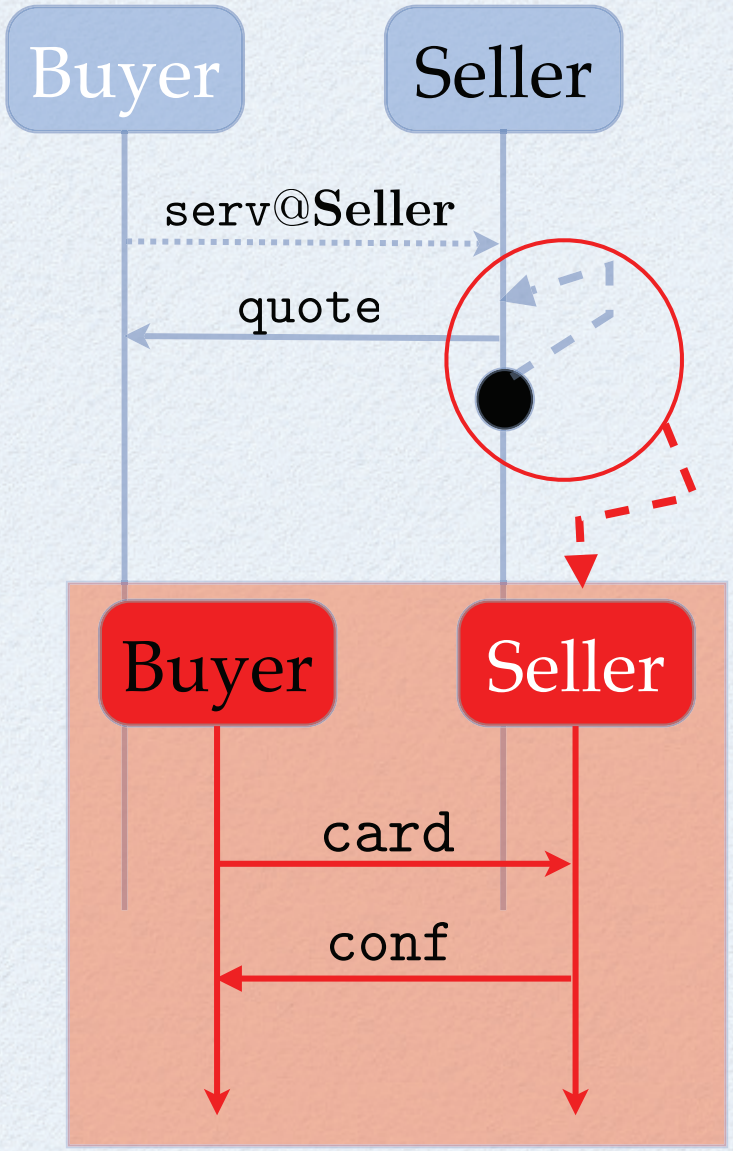

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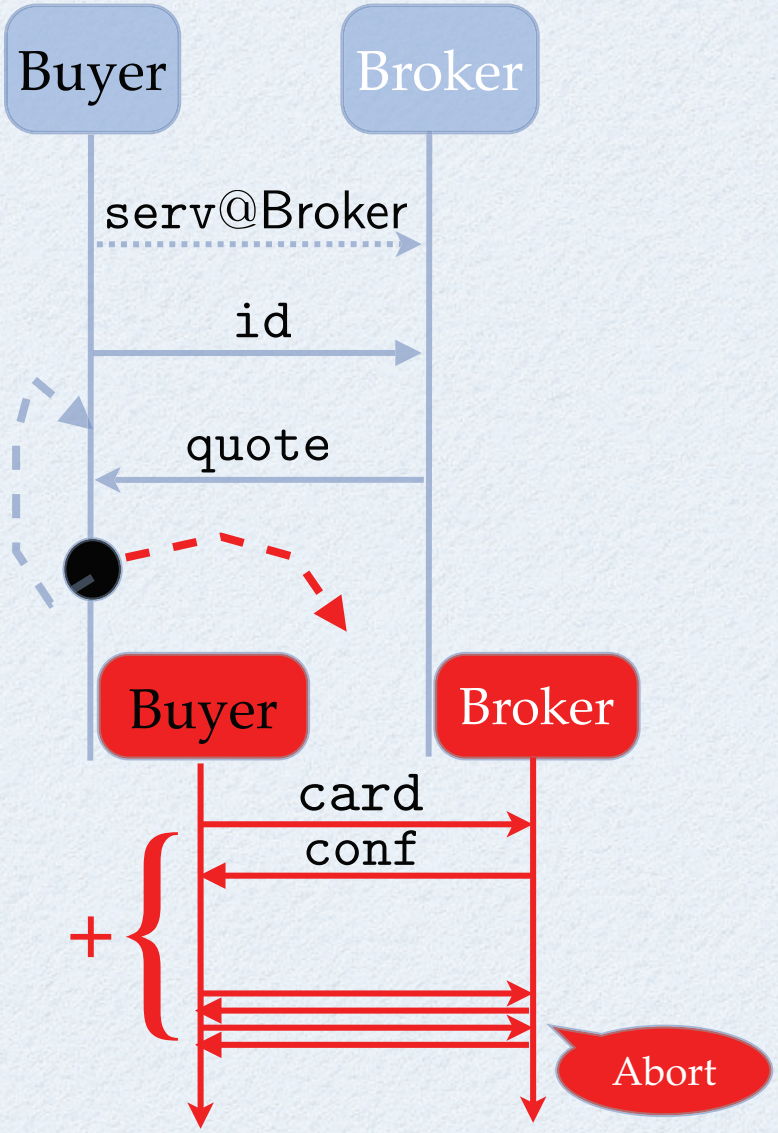
INTERACTIONAL EXCEPTIONS

- The mechanism above is called *Interactional Exception*
- It is very hard to implement with branching/ selection (especially having asynchrony)
- Is it useful? => appears in many financial protocols!

EXAMPLE: NESTED ESCAPE



Buyer

invoke $\texttt{serv@Broker}(t)$;
try {
$t!\langle \texttt{id} \rangle.$
$\mu X.$
$t?(y).$
if $(ok(y))$ **then throw else** X
}
catch {
$l_1 : \; t!\langle \texttt{card} \rangle.\; t?(y)$
$l_2 : \; P_{\texttt{abort}}$
}

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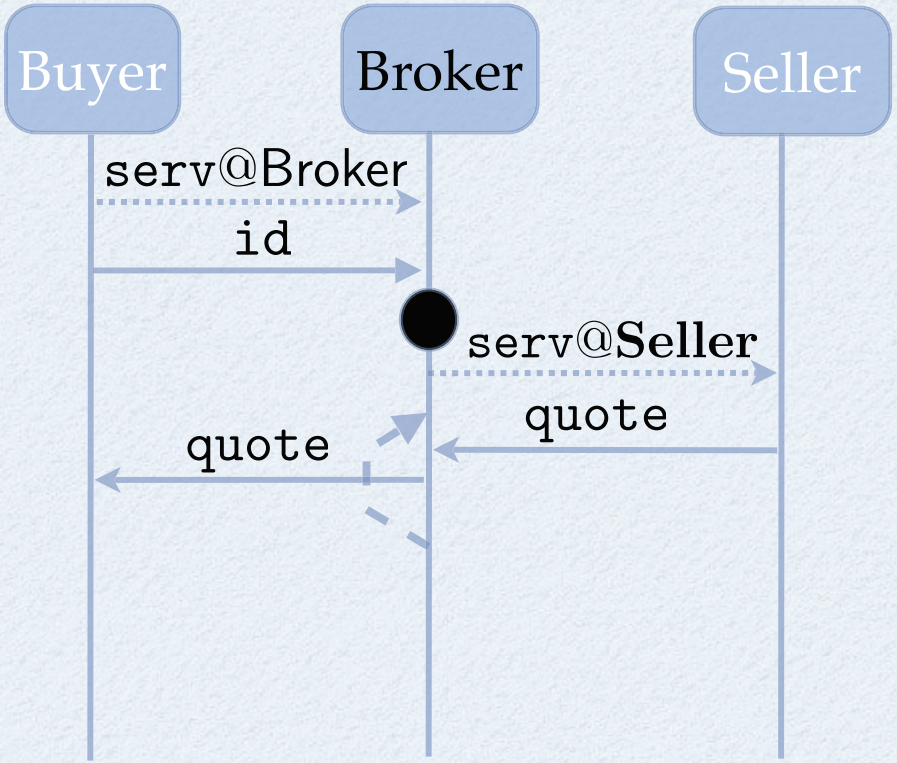


Broker

service $\mathtt{serv}(t)$ {

try (t) {

$t?(x).$

if $\mathtt{bad}(x)$ **then throw else**

invoke $\mathtt{serv@Seller}(s)$;

try (t, s) {

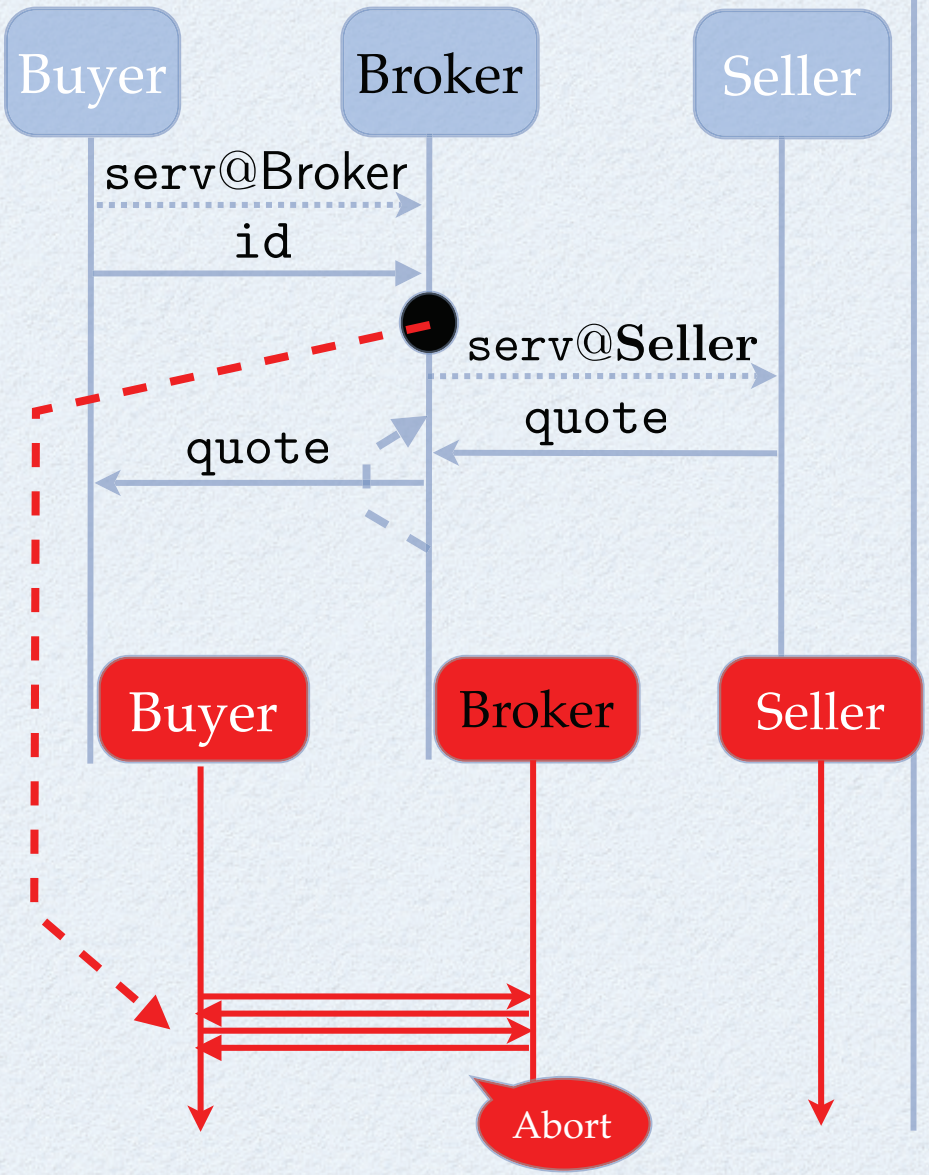
$\mu X.\ s?(x).\ t!\langle x + 10\% \rangle.\ X$

catch { $t \triangleright l_1.\ \ldots \mathrm{fwd} \ldots$ }

catch { $t \triangleright l_2.\ P'_{\mathtt{abort}}$ }

}

EXAMPLE: NESTED ESCAPE



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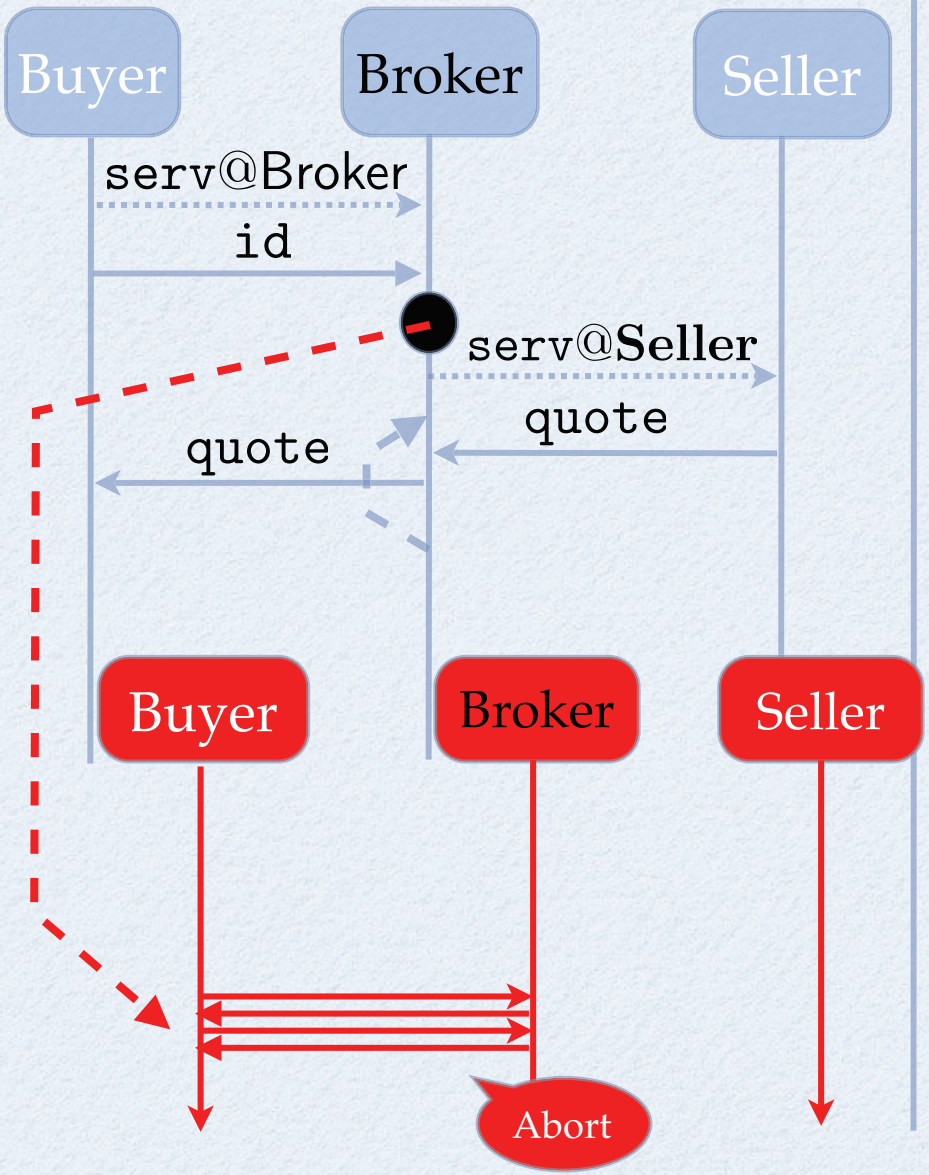

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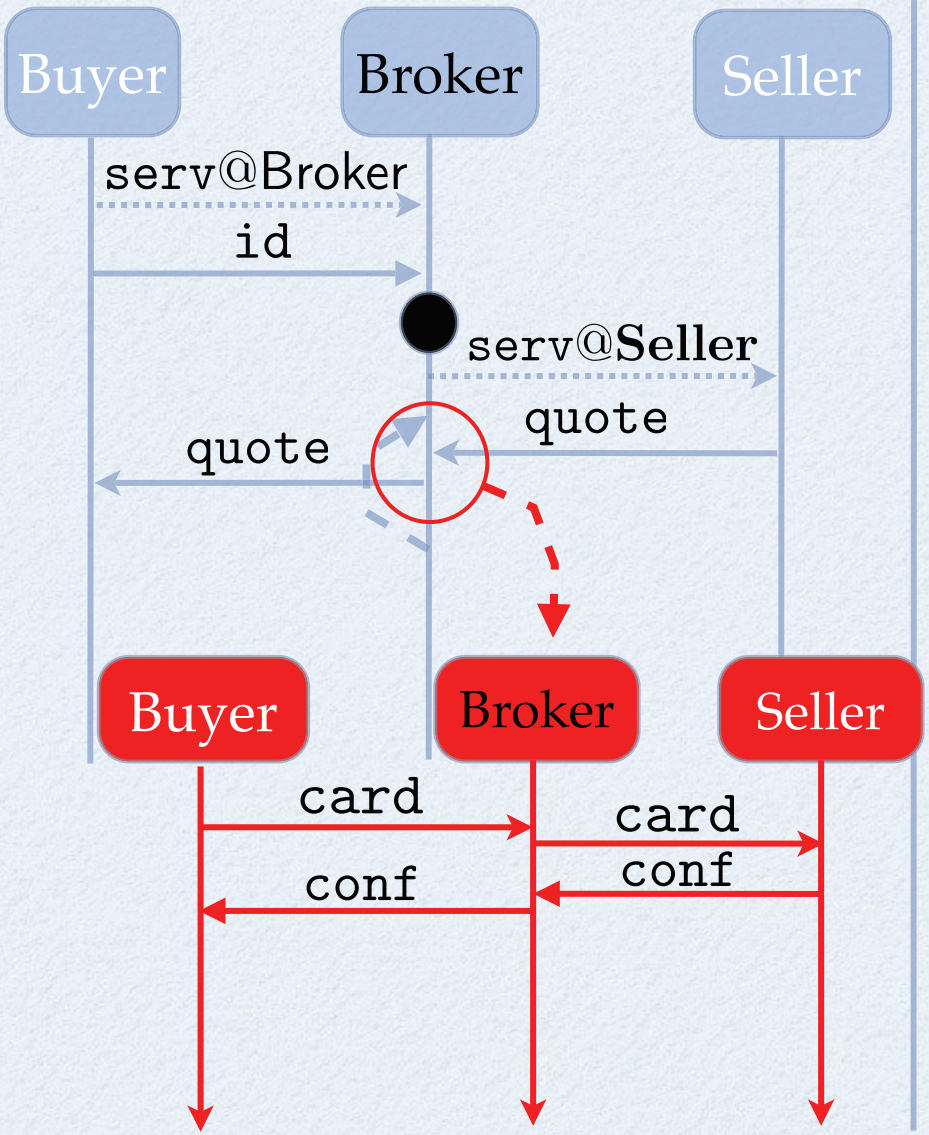

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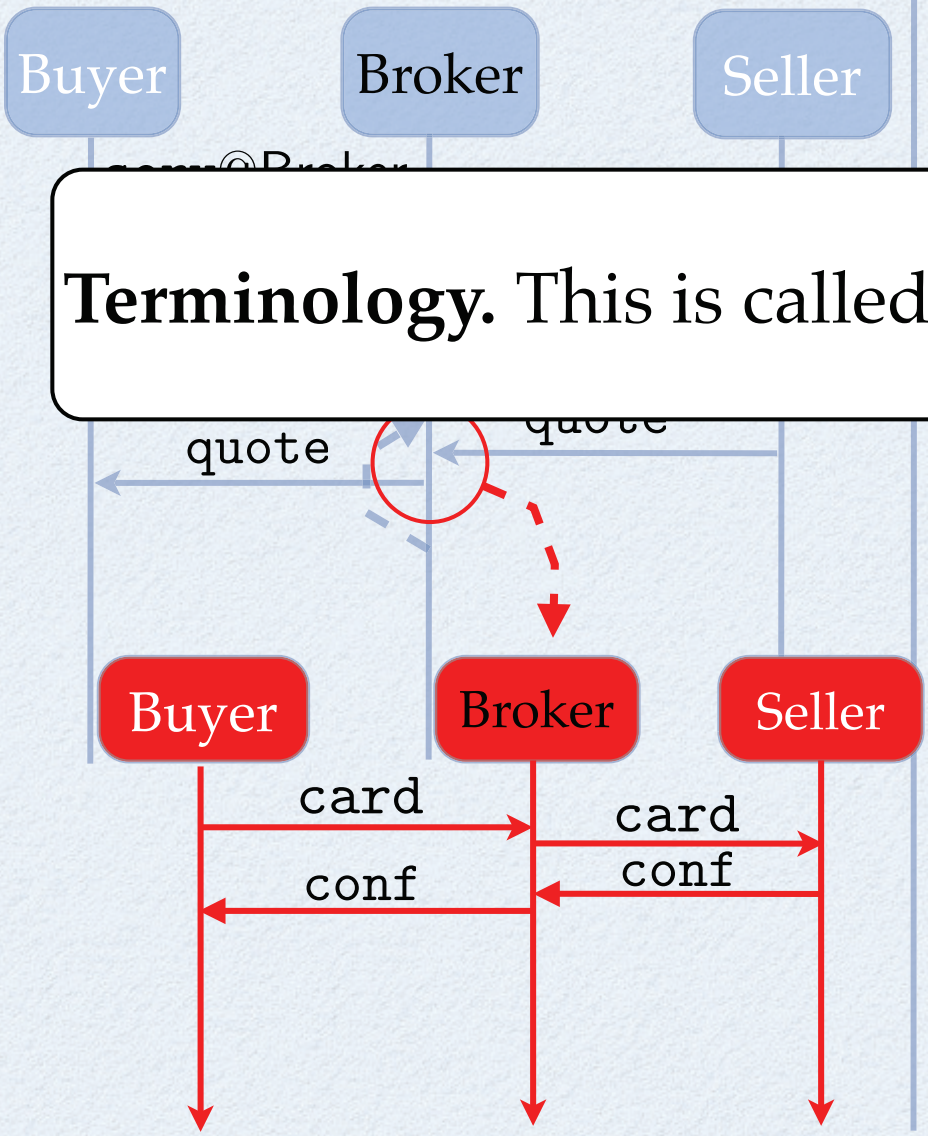

Terminology. This is called *refinement.*

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}

π-CALCULUS WITH SESSIONS

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	$P \mid Q$	(par)
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π-CALCULUS WITH SESSIONS

P :: =

~~$*a(\kappa).\ P$~~	~~(service)~~
~~$a(\kappa).\ P$~~	~~(request)~~
$!a(\kappa)[\ P, Q\ ]$	(service)
$a(\kappa')[\ \tilde{\kappa}, P, Q\ ]$	(request)
throw	(throw)
(υ a**)** P / **(υ** κ**)** P	(res)
P | Q	(par)
if e **then** P **else** P	(cond)
0	(inact)
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Refinement and other assumptions

Syntactic Assumptions. In order to have consistent operational semantics, we stipulate the following syntactic constraints:

1. *(Consistent Refinement)* given $\overline{c}(\lambda)[\tilde{\kappa}, P, Q]$, for each $\overline{c'}(\lambda')[\tilde{\kappa}', P', Q']$ occurring in P and any $\kappa_i \in \tilde{\kappa}$, we have $\kappa_i \in \tilde{\kappa}'$ implies $\tilde{\kappa} \subseteq \tilde{\kappa}'$ (for consistent refinement). Further, such a refinement never occurs inside a handler (otherwise we have ambiguity when launching a handler);

2. recursions is *guarded*, i.e. P in $\mu X.\ P$ is prefixed by an input, output, branch, select or conditional; moreover, a free term variable never occurs free in $\overline{c}(\lambda)[\tilde{\kappa}, P, Q]$;

3. the term (accept) never occurs under an input/output/recursion prefix nor inside a default process or handler thus protecting its availability from exceptions;

4. **throw** never occurs inside a handler hence preventing a handler from throwing a further exception in the same session.

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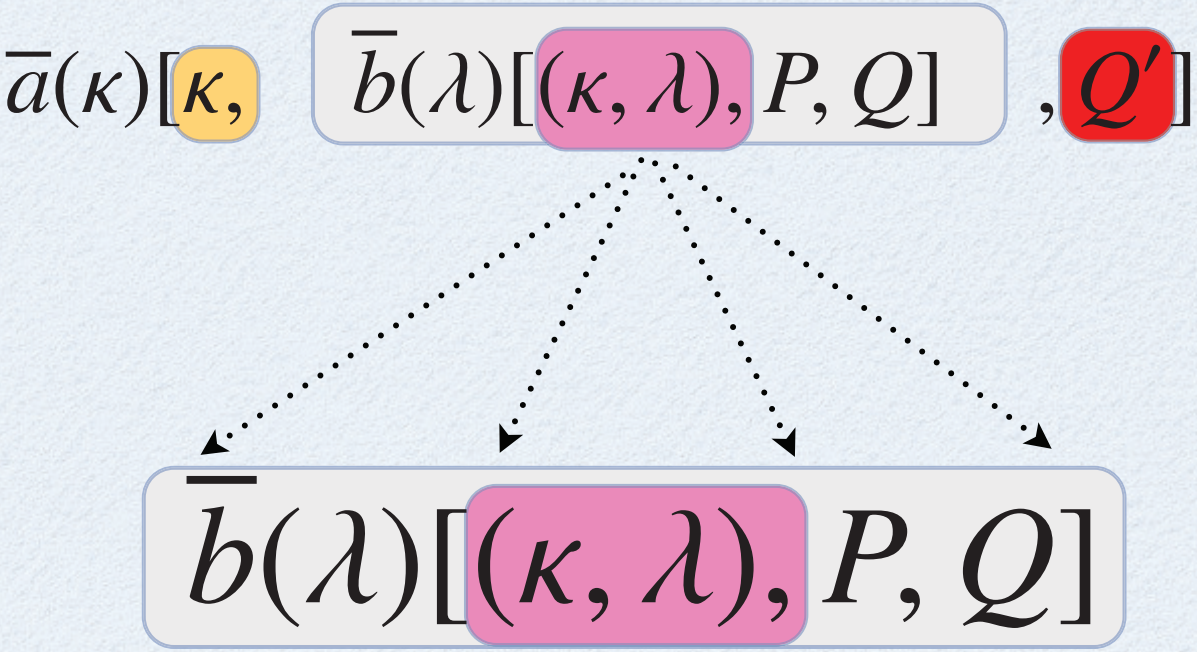
REFINEMENT

$$\overline{a}(\kappa)[\kappa,\ \overline{b}(\lambda)[(\kappa,\lambda), P, Q]\ , Q']$$

REFINEMENT


$\overline{a}(\kappa)[\kappa,$
$\overline{b}(\lambda)[(\kappa,\lambda), P, Q]$
$, Q']$
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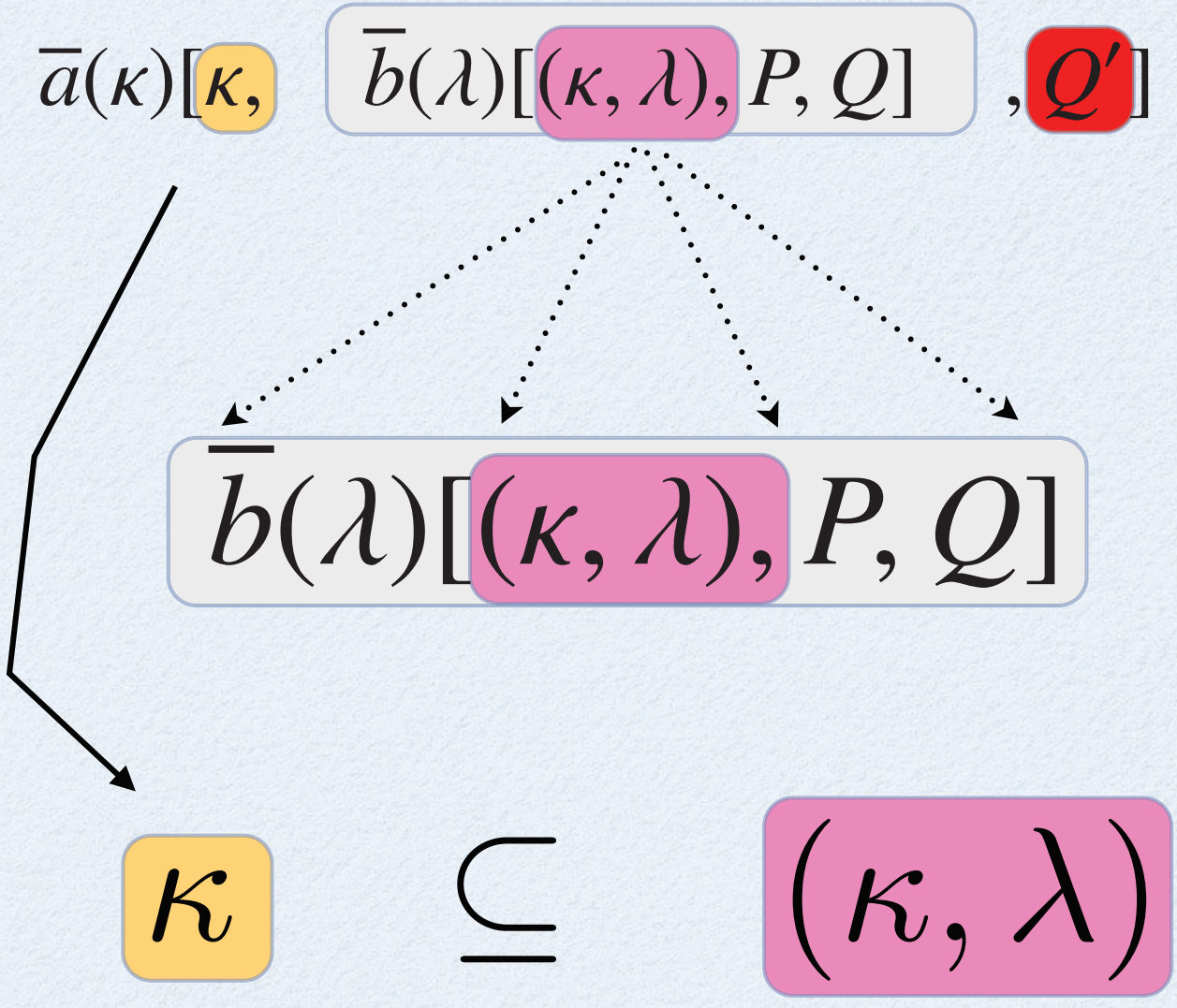
REFINEMENT


$\overline{a}(\kappa)[\kappa,$
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$, Q']$
$\overline{b}(\lambda)[(\kappa,\lambda), P, Q]$
κ
$\subseteq$
(κ,λ)

REFINEMENT (2)

$$\overline{a}(\kappa)[(\kappa, \kappa'), \quad \overline{b}(\lambda)[(\lambda, \kappa), P, Q], \quad Q']$$

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$$\overline{a}(\kappa)[(\kappa, \kappa'), \quad \overline{b}(\lambda)[(\lambda, \kappa), P, Q], \quad Q']$$

$$(\kappa, \kappa') \not\subseteq (\lambda, \kappa)$$

SEMANTICS: RUN-TIME SYNTAX

Technically challenging - hard to preserve session structure

We wish to have asynchronous ordered communication!

We need to have a *run-time* syntax which extends the static one as follows:

$$\mathbf{try}\{P\}\ \mathbf{catch}\{\tilde{\kappa} : Q\} \qquad \text{(try-catch)}$$

$$\tilde{\kappa}\{\!\{Q\}\!\} \qquad \text{(wrap)}$$

$$\kappa \hookrightarrow_{\phi} \overline{\kappa} : L \qquad \text{(queue)}$$

$$(\nu s)P \qquad \text{(res)}$$

SEMANTICS: SESSION INIT

$$!a(s)[P, Q] \quad | \quad C[\,\overline{a}(s)[\tilde{\kappa}, P', Q']\,]$$

SEMANTICS: SESSION INIT

$$!a(s)[P,Q] \quad | \quad C[\,\overline{a}(s)[\tilde{\kappa},P',Q']\,]$$

$$\longrightarrow$$

$$!a(s)[P.Q] \;|$$

SEMANTICS: SESSION INIT

$$!a(s)[P,Q] \quad | \quad C[\,\overline{a}(s)[\tilde{\kappa},P',Q']\,]$$

$$\longrightarrow$$

$$!a(s)[P.Q] \mid (\nu s)\Big(\qquad\qquad\Big)$$

SEMANTICS: SESSION INIT

$$!a(s)[P,Q] \quad | \quad C[\,\overline{a}(s)[\tilde{\kappa},P',Q']\,]$$

$$\longrightarrow$$

$$!a(s)[P.Q] \;|\; (\nu s)\left(\begin{array}{l}\mathbf{try}\{P\}\;\mathbf{catch}\{s^{-}:Q\}\;|\\ \\ \end{array}\right)$$

SEMANTICS: SESSION INIT

$$!a(s)[P, Q] \quad | \quad C[\ \overline{a}(s)[\tilde{\kappa}, P', Q']\ ]$$

$$\longrightarrow$$

$$!a(s)[P.Q] \ | \ (\nu s)\left(\begin{array}{l} \mathbf{try}\{P\}\ \mathbf{catch}\{s^- : Q\} \ | \\ C[\ \mathbf{try}\{P'\}\ \mathbf{catch}\{\tilde{\kappa} : Q'\}\ ] \ | \end{array}\right)$$

SEMANTICS: SESSION INIT

$$!a(s)[P,Q] \quad | \quad C[\ \overline{a}(s)[\tilde{\kappa},P',Q']\ ]$$

$$\longrightarrow$$

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SEMANTICS: SESSION INIT

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WHY WRAPS?

$$\mathbf{try}\{\ \ P' \mid \mathbf{try}\{\ P\ \}\ \mathbf{catch}\ \{\ \lambda :\ Q\ \}\ \ \}\ \ \mathbf{catch}\ \ \{\ \kappa :\ Q'\ \}$$

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$$\mathbf{try}\{\quad P' \quad|\quad \mathbf{try}\{\ P\ \}\ \mathbf{catch}\ \{\ \lambda:\ Q\ \}\quad \}\quad \mathbf{catch}\quad \{\ \kappa:\ Q'\ \}$$

- P throws an exception

$$\mathbf{try}\{\quad P' \quad|\quad Q \quad \}\quad \mathbf{catch}\quad \{\ \kappa:\ Q'\ \}$$

WHY WRAPS?

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- P throws an exception

$$\mathbf{try}\{\ \ P' \mid Q\ \ \}\ \ \mathbf{catch}\ \ \{\ \kappa :\ Q'\ \}$$

- P' throws an exception

- *What happens to Q?*

WHY WRAPS?

$$\mathbf{try}\{\quad P' \quad|\quad \mathbf{try}\{\ P\ \}\ \mathbf{catch}\ \{\ \lambda:\ Q\ \}\quad \}\quad \mathbf{catch}\quad \{\ \kappa:\ Q'\ \}$$

- P throws an exception

$$\mathbf{try}\{\quad P' \quad|\quad \lambda\{\{Q\}\}\quad \}\quad \mathbf{catch}\quad \{\ \kappa:\ Q'\ \}$$

META REDUCTION

When a process must be suddenly terminated, *Meta Reduction will* take care of:

i) propagation (partly)
ii) wrapped processes

Meta reduction is a relation on processes:

$$P \searrow (P', S)$$

where

i) *S* is a set of session names (keep track of propagation)
ii) *P'* contains wraps only (in parallel and/or nested)

Meta Reduction (2)

Some rules:

$$P \searrow (P', S)$$

META REDUCTION (2)

Some rules:

$$\frac{P \searrow (P', S)}{\mathbf{try}\{P\}\ \mathbf{catch}\{\tilde{\kappa} : Q\} \ \searrow \ (\tilde{\kappa}\{\!\{Q\}\!\} \mid P' \ , \ S \cup \tilde{\kappa})}$$

META REDUCTION (2)

Some rules:

$$\frac{P \searrow (P', S)}{\mathbf{try}\{P\}\ \mathbf{catch}\{\tilde{\kappa} : Q\} \ \searrow\ (\tilde{\kappa}\{\!\{Q\}\!\} \mid P'\ ,\ S \cup \tilde{\kappa})}$$

$$\tilde{\kappa}\{\!\{Q\}\!\} \ \searrow\ (\tilde{\kappa}\{\!\{Q\}\!\}\ ,\ \emptyset)$$

EXAMPLE OF META REDUCTION

$$\mathbf{try}\{\quad \mathbf{throw} \;\mid\; \mathbf{try}\{\, P \,\}\, \mathbf{catch}\, \{\, \lambda : \; Q \,\} \quad \}\quad \mathbf{catch}\, \{\, \kappa : \; Q' \,\}$$

EXAMPLE OF META REDUCTION

$$\mathbf{try}\{\quad \mathbf{throw} \quad|\quad \mathbf{try}\{\ P\ \}\ \mathbf{catch}\ \{\ \lambda :\ Q\ \}\quad \}\quad \mathbf{catch}\ \{\ \kappa :\ Q'\ \}$$

- if **throw** is exec then we must deal with

EXAMPLE OF META REDUCTION

$$\mathbf{try}\{\ \ \mathbf{throw}\ \mid\ \mathbf{try}\{\ P\ \}\ \mathbf{catch}\ \{\ \lambda:\ Q\ \}\ \ \}\ \ \mathbf{catch}\ \{\ \kappa:\ Q'\ \}$$

- if **throw** is exec then we must deal with

$$\mathbf{throw}\ \mid\ \mathbf{try}\{\ P\ \}\ \mathbf{catch}\ \{\ \lambda:\ Q\ \}$$

EXAMPLE OF META REDUCTION

$$\mathbf{try}\{\quad \mathbf{throw} \;\mid\; \mathbf{try}\{\, P \,\}\, \mathbf{catch}\, \{\, \lambda :\; Q \,\} \quad \}\quad \mathbf{catch}\, \{\, \kappa :\; Q' \,\}$$

- if **throw** is exec then we must deal with

$$\mathbf{throw} \;\mid\; \mathbf{try}\{\, P \,\}\, \mathbf{catch}\, \{\, \lambda :\; Q \,\}$$

- we get (i) something from P and $\lambda\{\{Q\}\}$
- and (ii) we must propagate over λ

$$(P'' \;\mid\; \lambda\{\{Q\}\},\quad \{\lambda\})$$

SEMANTICS (3): THROW

$$\mathbf{try}\{\ \mathbf{throw} \mid P\ \}\ \mathbf{catch}\ \{\kappa, \kappa' : Q\} \mid \kappa \hookrightarrow_0 \overline{\kappa} : L \mid \kappa' \hookrightarrow_0 \overline{\kappa}' : L'$$

SEMANTICS (3): THROW

$$\mathbf{try}\{\ P\ \}\ \mathbf{catch}\ \{\kappa, \kappa' : Q\}\ \searrow\ (R, S)$$

$$\mathbf{try}\{\ \mathbf{throw} \mid P\ \}\ \mathbf{catch}\ \{\kappa, \kappa' : Q\}\ |\ \kappa \hookrightarrow_0 \overline{\kappa} : L \mid \kappa' \hookrightarrow_0 \overline{\kappa}' : L'$$

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$$\longrightarrow$$

$$R \mid \kappa \hookrightarrow_0 \overline{\kappa} : (\dagger :: L) \mid \kappa' \hookrightarrow_0 \overline{\kappa}' : (\dagger :: L')$$

SEMANTICS (3): THROW

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$$\kappa_1 \hookrightarrow \overline{\kappa_1} : L_1 \mid \ \ldots\ \mid \kappa_n \hookrightarrow \overline{\kappa_n} : L_n$$

$$\longrightarrow$$

for all $\kappa_i \in S$

$$R \mid \kappa \hookrightarrow_0 \overline{\kappa} : (\dagger :: L) \mid \kappa' \hookrightarrow_0 \overline{\kappa}' : (\dagger :: L')$$

$$\kappa_1 \hookrightarrow \overline{\kappa_1} : (\dagger :: L_1) \mid \ \ldots\ \mid \kappa_n \hookrightarrow \overline{\kappa_n} : (\dagger :: L_n)$$

Process P may contain other try catch blocks!

SEMANTICS (4): RTHROW

$$\mathbf{try}\{P\}\ \mathbf{catch}\{\kappa, \kappa' : Q\} \mid \overline{\kappa} \hookrightarrow_0 \kappa : (L'' :: \dagger) \mid \kappa \hookrightarrow_0 \overline{\kappa} : L \mid \kappa' \hookrightarrow_0 \overline{\kappa}' : L'$$

SEMANTICS (4): RTHROW

$$\mathbf{try}\{P\}\ \mathbf{catch}\ \{\kappa, \kappa' : Q\} \searrow (R, S)$$

$$\mathbf{try}\{P\}\ \mathbf{catch}\{\kappa, \kappa' : Q\} \mid \overline{\kappa} \hookrightarrow_0 \kappa : (L'' :: \dagger) \mid \kappa \hookrightarrow_0 \overline{\kappa} : L \mid \kappa' \hookrightarrow_0 \overline{\kappa}' : L'$$

SEMANTICS (4): RTHROW

$$\mathbf{try}\{P\}\ \mathbf{catch}\ \{\kappa, \kappa' : Q\} \searrow (R, S)$$

$$\mathbf{try}\{P\}\ \mathbf{catch}\{\kappa, \kappa' : Q\} \mid \overline{\kappa} \hookrightarrow_0 \kappa : (L'' :: \dagger) \mid \kappa \hookrightarrow_0 \overline{\kappa} : L \mid \kappa' \hookrightarrow_0 \overline{\kappa}' : L'$$

$$\longrightarrow$$

$$R \mid \overline{\kappa} \hookrightarrow_1 \kappa : L'' \mid \kappa \hookrightarrow_0 \overline{\kappa} : (\dagger :: L) \mid \kappa' \hookrightarrow_0 \overline{\kappa}' : (\dagger :: L')$$

SEMANTICS (4): RTHROW

$$\mathbf{try}\{P\}\ \mathbf{catch}\ \{\kappa, \kappa' : Q\} \searrow (R, S)$$

$$\mathbf{try}\{P\}\ \mathbf{catch}\{\kappa, \kappa' : Q\} \mid \overline{\kappa} \hookrightarrow_0 \kappa : (L'' :: \dagger) \mid \kappa \hookrightarrow_0 \overline{\kappa} : L \mid \kappa' \hookrightarrow_0 \overline{\kappa}' : L'$$

$$\kappa_1 \hookrightarrow \overline{\kappa_1} : L_1 \mid \ldots \mid \kappa_n \hookrightarrow \overline{\kappa_n} : L_n$$

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for all $\kappa_i \in S$

$$R \mid \overline{\kappa} \hookrightarrow_1 \kappa : L'' \mid \kappa \hookrightarrow_0 \overline{\kappa} : (\dagger :: L) \mid \kappa' \hookrightarrow_0 \overline{\kappa}' : (\dagger :: L')$$

$$\kappa_1 \hookrightarrow \overline{\kappa_1} : (\dagger :: L_1) \mid \ldots \mid \kappa_n \hookrightarrow \overline{\kappa_n} : (\dagger :: L_n)$$

STRUCTURAL CONGR.

$$(\lambda \in \tilde{\kappa} \Rightarrow \dagger \notin L)$$

$$\textbf{try}\{\, P \mid \lambda \hookrightarrow_{\phi} \overline{\lambda} : L \,\}\ \textbf{catch}\ \{\, \tilde{\kappa} : Q \,\} \ \equiv\ \textbf{try}\{\, P \,\}\ \textbf{catch}\ \{\, \tilde{\kappa} : Q \,\} \mid \lambda \hookrightarrow_{\phi} \overline{\lambda} : L$$

$$\tilde{\kappa}\{\!|P \mid \overline{\lambda} \hookrightarrow_{\phi} \lambda : L|\!\} \ \equiv\ \tilde{\kappa}\{\!|P|\!\} \mid \overline{\lambda} \hookrightarrow_{\phi} \lambda : L \qquad (\lambda \notin \tilde{\kappa})$$

$$\tilde{\kappa}\{\!|P|\!\} \mid \overline{\kappa}_i \hookrightarrow_{\phi} \kappa_i : L \ \equiv\ \tilde{\kappa}\{\!|P \mid \overline{\kappa}_i \hookrightarrow_{\phi-1} \kappa_i : L|\!\}$$

$$\textbf{try}\{\, (\nu a)\, P \,\}\ \textbf{catch}\ \{\, \tilde{\kappa} : Q \,\} \ \equiv\ (\nu a)\ \textbf{try}\{\, P \,\}\ \textbf{catch}\ \{\, \tilde{\kappa} : Q \,\} \qquad (a \notin fn(Q))$$

$$\tilde{\kappa}\{\!|(\nu a)\, P|\!\} \ \equiv\ (\nu a)\, \tilde{\kappa}\{\!|P|\!\}$$

EXAMPLE

$$\mathbf{try}\{\ \mathbf{throw} \mid \kappa!\langle 5\rangle\ \}\ \mathbf{catch}\ \{\kappa : \kappa!\langle \mathtt{tt}\rangle\} \mid \kappa \hookrightarrow_0 \overline{\kappa} : \epsilon \mid$$
$$\mathbf{try}\{\ \mathbf{throw} \mid \overline{\kappa}?(x)\ \}\ \mathbf{catch}\ \{\overline{\kappa} : \overline{\kappa}?(x)\} \mid \overline{\kappa} \hookrightarrow_0 \kappa : \epsilon$$

$$\longrightarrow$$

$$\kappa\{\!|\kappa!\langle \mathtt{tt}\rangle|\!\} \mid \kappa \hookrightarrow_0 \overline{\kappa} : \dagger \mid$$
$$\mathbf{try}\{\ \mathbf{throw} \mid \overline{\kappa}?(x)\ \}\ \mathbf{catch}\ \{\overline{\kappa} : \overline{\kappa}?(x)\} \mid \overline{\kappa} \hookrightarrow_0 \kappa : \epsilon$$

$$\longrightarrow$$

$$\kappa\{\!|\mathbf{0}|\!\} \mid \kappa \hookrightarrow_0 \overline{\kappa} : (\mathtt{tt} :: \dagger) \mid$$
$$\mathbf{try}\{\ \mathbf{throw} \mid \overline{\kappa}?(x)\ \}\ \mathbf{catch}\ \{\overline{\kappa} : \overline{\kappa}?(x)\} \mid \overline{\kappa} \hookrightarrow_0 \kappa : \epsilon$$

EXAMPLE

$$\kappa\{[\![\mathbf{0}]\!] \mid \kappa \hookrightarrow_0 \overline{\kappa} : (\mathtt{tt} :: \dagger) \mid \mathbf{try}\{\, \mathbf{throw} \mid \overline{\kappa}?(x) \,\}\, \mathbf{catch}\, \{\, \overline{\kappa} : \overline{\kappa}?(x) \,\} \mid \overline{\kappa} \hookrightarrow_0 \kappa : \epsilon$$

$$\longrightarrow$$

$$\kappa\{[\![\mathbf{0}]\!] \mid \kappa \hookrightarrow_1 \overline{\kappa} : \mathtt{tt} \mid \overline{\kappa}\{[\![\overline{\kappa}?(x)]\!]\} \mid \overline{\kappa} \hookrightarrow_0 \kappa : \dagger$$

$$\longrightarrow$$

$$\kappa\{[\![\mathbf{0}]\!] \mid \overline{\kappa} \hookrightarrow_1 \kappa : \epsilon \mid \kappa \hookrightarrow_1 \overline{\kappa} : \mathtt{tt} \mid \overline{\kappa}\{[\![\overline{\kappa}?(x)]\!]\}$$

OTHER RULES

$$\tilde{\kappa}\{\!\{Q\}\!\} \mid \overline{\kappa}_i \hookrightarrow_0 \kappa_i : (L :: v) \longrightarrow \tilde{\kappa}\{\!\{Q\}\!\} \mid \overline{\kappa}_i \hookrightarrow_0 \kappa_i : L$$

$$\tilde{\kappa}\{\!\{Q\}\!\} \mid \overline{\kappa}_i \hookrightarrow_0 \kappa_i : (L :: \dagger) \longrightarrow \tilde{\kappa}\{\!\{Q\}\!\} \mid \overline{\kappa}_i \hookrightarrow_1 \kappa_i : L$$

SESSION TYPES

The grammar extends standard session types:

$$\begin{array}{rcl} \alpha, \beta & ::= & \downarrow(\theta).\,\alpha \mid \uparrow(\theta).\,\alpha \mid \oplus\{l_i : \alpha_i\}_{i \in I} \mid \&\{l_i : \alpha_i\}_{i \in I} \mid \alpha[\![\beta]\!] \mid \mathbf{end} \mid \mu \mathbf{t}.\,\alpha \mid \mathbf{t} \\ \theta & ::= & \langle \alpha[\![\beta]\!] \rangle \mid \mathtt{bool} \mid \ldots \end{array}$$

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try-catch type

$$\alpha\{\!\![\beta]\!\!\}$$

$$\mathbf{try}\{P\}\ \mathbf{catch}\{\tilde{\kappa} : Q\}$$

SESSION TYPES (2)

Judgements

$$\Gamma \vdash P \triangleright \Delta$$

where

- Γ contains shared types
- Δ contains session/queue types

SESSION TYPES (2)

Judgements

$$\Gamma \vdash P \triangleright \Delta$$

where

(Session Typing) $\Delta ::= \emptyset \mid \Delta,\ \kappa : \alpha \mid \Delta,\ (\kappa, \overline{\kappa}) : \alpha \mid \Delta,\ (\kappa, \overline{\kappa}) : \bot$

(Service Typing) $\Gamma ::= \emptyset \mid \Gamma,\ c : \langle \alpha \{\!| \beta |\!\} \rangle \mid c : \mathtt{bool} \mid \Gamma,\ X : \Delta$

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$$\Gamma \vdash \mathbf{try}\{P\}\ \mathbf{catch}\{\kappa : Q\} \triangleright \Delta \cdot \kappa :_p \alpha\{\{\beta\}\}$$

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$$\Gamma \vdash Q \triangleright \kappa :_u \beta$$

$$\Gamma \vdash \mathbf{try}\{P\}\ \mathbf{catch}\{\kappa : Q\} \triangleright \Delta \cdot \kappa :_p \alpha\{\{\beta\}\}$$

SESSION TYPES (2)

Judgements

$$\Gamma \vdash P \triangleright \Delta$$

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$$\frac{\Gamma \vdash Q \triangleright \kappa :_u \beta \qquad \Gamma \vdash P \triangleright \Delta \cdot \kappa :_\rho \alpha\{\{\beta\}\}}{\Gamma \vdash \mathbf{try}\{P\}\ \mathbf{catch}\{\kappa : Q\} \triangleright \Delta \cdot \kappa :_p \alpha\{\{\beta\}\}}$$

all protected ("p") in Δ - $\varrho \in \{p,u\}$

TYPING RULES

$$\frac{\mathsf{fv}(\Gamma) = \emptyset \quad \alpha_i \in \{\mathsf{end}, \mathsf{end}\{\![\beta_i]\!\}\}}{\Gamma \vdash \mathbf{0} \triangleright \prod_i \kappa_i : \alpha_i}$$

$$\frac{\Gamma, a : \langle \alpha\{\![\beta]\!\} \rangle \vdash P \triangleright \Delta}{\Gamma \vdash (\nu a)\, P \triangleright \Delta}$$

TYPING RULES: INIT

$$\frac{\begin{array}{l}\Gamma \vdash P \triangleright \prod_i \kappa_i : \overline{\alpha}_i\{\![\overline{\beta}_i]\!\} \\ \Gamma' \vdash Q \triangleright \prod_i \kappa_i : \overline{\beta}_i \qquad s^+ = \kappa_j \\ \Gamma \vdash c : \langle \alpha_j\{\![\beta_j]\!\}\rangle \qquad \Gamma' \subseteq \Gamma,\ \mathsf{fv}(\Gamma') = \emptyset\end{array}}{\Gamma \vdash \overline{c}(s^+)[\tilde{\kappa}, P, Q] \triangleright \prod_{i \neq j} \kappa_i : \overline{\alpha}_i\{\![\overline{\beta}_i]\!\}}$$

$$\frac{\begin{array}{l}\Gamma \vdash P \triangleright s^- : \alpha\{\![\beta]\!\} \\ \Gamma \vdash Q \triangleright s^- : \beta \qquad \mathsf{fv}(\Gamma) = \emptyset\end{array}}{\Gamma,\ a : \langle \alpha\{\![\beta]\!\}\rangle \vdash *a(s^-)[P, Q] \triangleright \emptyset}$$

Typing Rules: In/Out

$$\frac{\Gamma, x : \theta \vdash P \triangleright \Delta \cdot \kappa : \alpha}{\Gamma \vdash \kappa?(x).\, P \triangleright \Delta \cdot \kappa : \downarrow(\theta).\, \alpha}$$

$$\frac{\Gamma \vdash e : \theta \qquad \Gamma \vdash P \triangleright \Delta \cdot \kappa : \alpha}{\Gamma \vdash \kappa!\langle e \rangle.\, P \triangleright \Delta \cdot \kappa : \uparrow(\theta).\, \alpha}$$

Typing Rules: Branching/ Selection

$$\frac{\Gamma \vdash P_i \triangleright \Delta \cdot \kappa : \alpha_i \quad \forall i \in I}{\Gamma \vdash \kappa \rhd \{l_i : P_i\}_{i \in I} \triangleright \Delta \cdot \kappa : \&\{l_i : \alpha_i\}_{i \in I}}$$

$$\frac{\Gamma \vdash P \triangleright \Delta \cdot \kappa : \alpha_j}{\Gamma \vdash \kappa \lhd l_j . P \triangleright \Delta \cdot \kappa : \oplus\{l_i : \alpha_i\}_{i \in I}}$$

TYPING EXAMPLE

Let's show that:

$$\Gamma \vdash \begin{array}{l} *\texttt{chSeller}(s^-)[\\ \quad s^-!\langle \texttt{quote} \rangle, \\ \quad s^-?(y_2).\, s^-!\langle \texttt{time} \rangle\ ] \end{array} \triangleright \emptyset$$

where:

$$\Gamma = \mathsf{chSeller} : \uparrow (int).\, \texttt{end} \quad \{[\downarrow (cardt).\ \uparrow (time).\, \texttt{end}]\}$$

TYPING EXAMPLE (2)

$$\Gamma \vdash \begin{array}{l} *\texttt{chSeller}(s^-)[\\ \quad s^-!\langle \texttt{quote} \rangle, \\ \quad s^-?(y_2).\, s^-!\langle \texttt{time} \rangle\ ] \end{array} \triangleright \emptyset$$

$$\frac{\mathsf{fv}(\Gamma) = \emptyset \quad \alpha_i \in \{\mathsf{end}, \mathsf{end}\{\![\beta_i]\!\}\}}{\Gamma \vdash \mathbf{0} \triangleright \prod_i \kappa_i : \alpha_i}$$

$$\Gamma \vdash \mathbf{0} \triangleright s^- : \mathsf{end}$$

TYPING EXAMPLE (2)

$$\Gamma \vdash \begin{array}{l} *\texttt{chSeller}(s^-)[\\ \quad s^-!\langle \texttt{quote} \rangle, \\ \quad s^-?(y_2).\, s^-!\langle \texttt{time} \rangle\ ] \end{array} \triangleright \emptyset$$

$$\frac{\Gamma \vdash e : \theta \qquad \Gamma \vdash P \triangleright \Delta \cdot \kappa : \alpha}{\Gamma \vdash \kappa!\langle e \rangle.\, P \triangleright \Delta \cdot \kappa : \uparrow(\theta).\, \alpha}$$

$$\Gamma \vdash \mathbf{0} \triangleright s^- : \mathsf{end}$$

$$\Gamma \vdash s^-!\langle \texttt{quote} \rangle.\, \mathbf{0} \triangleright s^- : \uparrow(int).\, \mathsf{end}$$

TYPING EXAMPLE (2)

$$\Gamma \vdash \begin{array}{l} *\texttt{chSeller}(s^-)[\\ \quad s^-!\langle \texttt{quote} \rangle, \\ \quad s^-?(y_2).\, s^-!\langle \texttt{time} \rangle\ ] \end{array} \triangleright \emptyset$$

$$\frac{\Gamma, x : \theta \vdash P \triangleright \Delta \cdot \kappa : \alpha}{\Gamma \vdash \kappa?(x).\, P \triangleright \Delta \cdot \kappa : \downarrow(\theta).\, \alpha}$$

$$\Gamma \vdash \mathbf{0} \triangleright s^- : \mathsf{end}$$

$$\Gamma \vdash s^-!\langle \texttt{quote} \rangle.\, \mathbf{0} \triangleright s^- :\uparrow (int).\, \mathsf{end}$$

$$\Gamma \vdash s^-?(y_2).\, s^-!\langle \texttt{time} \rangle.\, \mathbf{0} \triangleright s^- :\downarrow (cardt).\, \uparrow (time)$$

TYPING EXAMPLE (3)

$$\Gamma \vdash \begin{array}{l} *\texttt{chSeller}(s^-)[\\ \quad s^-!\langle \texttt{quote} \rangle, \\ \quad s^-?(y_2).\, s^-!\langle \texttt{time} \rangle\ ] \end{array} \triangleright \emptyset$$

$$\frac{\Gamma \vdash P \triangleright s^- : \alpha\{\!|\beta|\!\} \qquad \Gamma \vdash Q \triangleright s^- : \beta \qquad \mathsf{fv}(\Gamma) = \emptyset}{\Gamma,\ a : \langle \alpha\{\!|\beta|\!\} \rangle \vdash *a(s^-)[P, Q] \triangleright \emptyset}$$

$$\Gamma \vdash \quad s^-!\langle \texttt{quote} \rangle.\, \mathbf{0} \quad \triangleright \ s^- :\uparrow (int).\, \mathsf{end}$$

$$\Gamma \vdash \ s^-?(y_2).\, s^-!\langle \texttt{time} \rangle.\, \mathbf{0} \ \triangleright \ s^- :\downarrow (cardt).\ \uparrow (time)$$

TYPING EXAMPLE (3)

$$\Gamma \vdash \begin{array}{l} *\texttt{chSeller}(s^-)[\\ \quad s^-!\langle \texttt{quote} \rangle, \\ \quad s^-?(y_2).\, s^-!\langle \texttt{time} \rangle\ ] \end{array} \triangleright \emptyset$$

$$\frac{\Gamma \vdash P \triangleright s^- : \alpha\{\!|\beta|\!\} \qquad \Gamma \vdash Q \triangleright s^- : \beta \qquad \mathsf{fv}(\Gamma) = \emptyset}{\Gamma,\ a : \langle \alpha\{\!|\beta|\!\} \rangle \vdash *a(s^-)[P, Q] \triangleright \emptyset}$$

$$\Gamma \vdash \quad s^-!\langle \texttt{quote} \rangle.\, \mathbf{0} \quad \triangleright \ s^- :\uparrow (int).\, \mathsf{end}$$

$$\Gamma \vdash \ s^-?(y_2).\, s^-!\langle \texttt{time} \rangle.\, \mathbf{0} \ \triangleright \ s^- :\downarrow (cardt).\ \uparrow (time)$$

Now we can apply the rule and finally get:

$$\Gamma \vdash \begin{array}{l} *\texttt{chSeller}(s^-)[\\ \quad s^-!\langle \texttt{quote} \rangle, \\ \quad s^-?(y_2).\, s^-!\langle \texttt{time} \rangle\ ] \end{array} \triangleright \emptyset$$

RESULTS

Theorem (Subject Reduction).

If $\Gamma \vdash P \triangleright \emptyset$ and $P \longrightarrow^* Q$ then $\Gamma \vdash Q \triangleright \emptyset$

Theorem (Communication Safety).

Communication Error Example.

$$\kappa(x).\ P \mid \kappa(x).\ P' \mid \overline{\kappa} \hookrightarrow_0 \kappa : 5$$

RESULTS

Theorem (Subject Reduction).

If $\Gamma \vdash P \triangleright \emptyset$ and $P \longrightarrow^* Q$ then $\Gamma \vdash Q \triangleright \emptyset$

Theorem (Communication Safety).

Communication Safe Example.

$$\mathbf{try}\{\ \mathbf{throw}\ |(\kappa!\langle 5\rangle)\ \}\ \mathbf{catch}\ \{\kappa : (\kappa!\langle \mathtt{tt}\rangle)\}$$
$$\mathbf{try}\{\ \ \ \ \ \ \ \ \ |(\overline{\kappa}?(x))\ \}\ \mathbf{catch}\ \{\overline{\kappa} : (\overline{\kappa}?(x))\}$$

Exceptions: Summary

- Interactional exceptions
- Asynchrony and Exceptions
- Very “wild” mechanism that can be tamed with session types
- Still missing bits e.g. delegation (name passing)

EXCEPTIONS, FUTURE WORK

- (extension) a re-try operation in the handler e.g.

  try{P}**catch** { k: **try**{P'}**catch**{k:Q} }

we would then have nested wrap as

  k{{ k{{Q}} }}

- (extension) multiparty sessions
- (extension) Termination ==> Progress (Concur08)

- integration into WS-CDL ([PLACES08])

- (future work) introduce delegation (very hard):
  - liveness?, termination?, etc.
- (future work) exception kinds (e.g. in Java)

EXERCISES

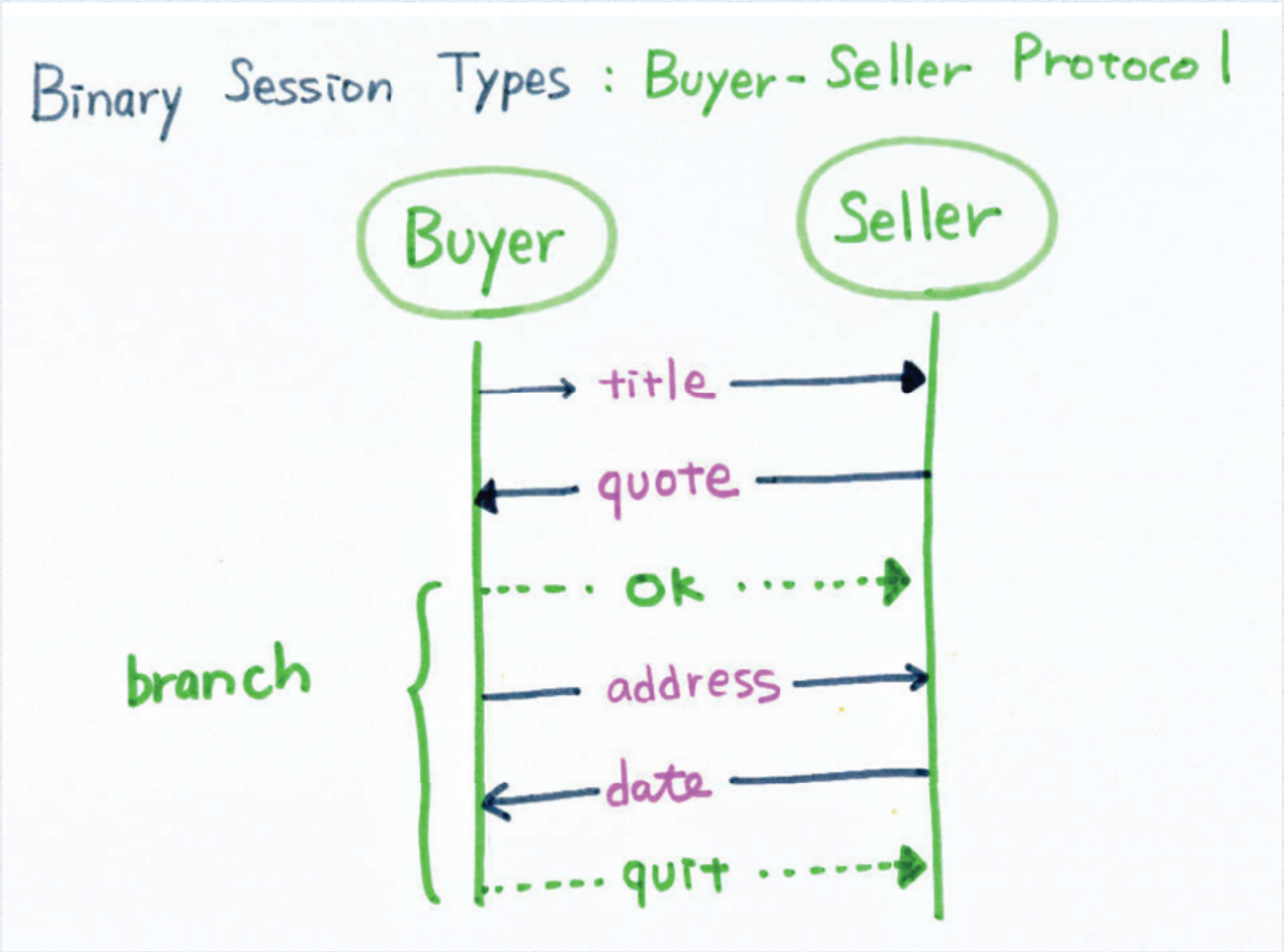
- Apply the typing rules to examples in the note.

- Prove subject reduction ;-)

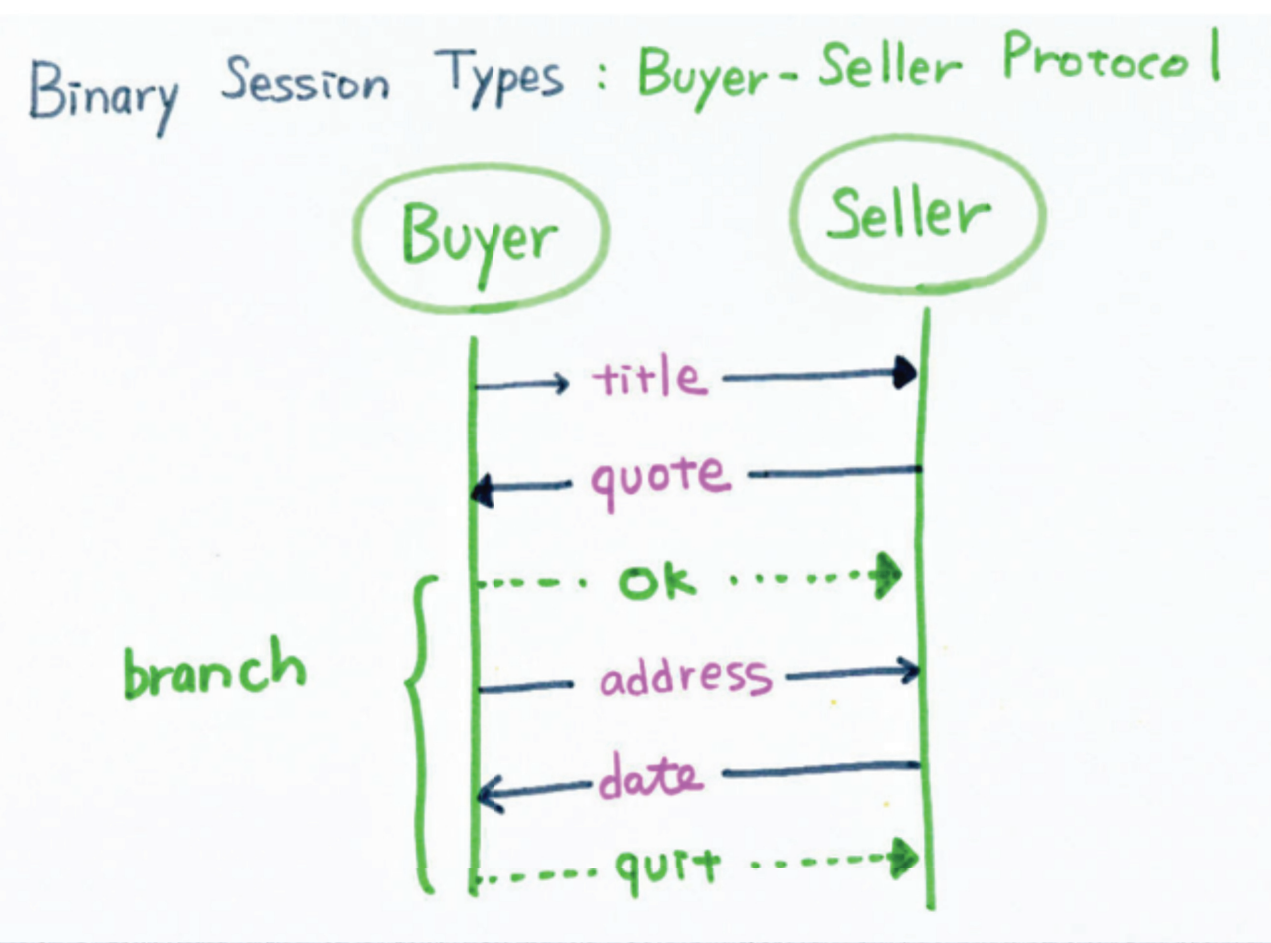
I'm here until Sunday!

Multiparty

IS BINARY ENOUGH?

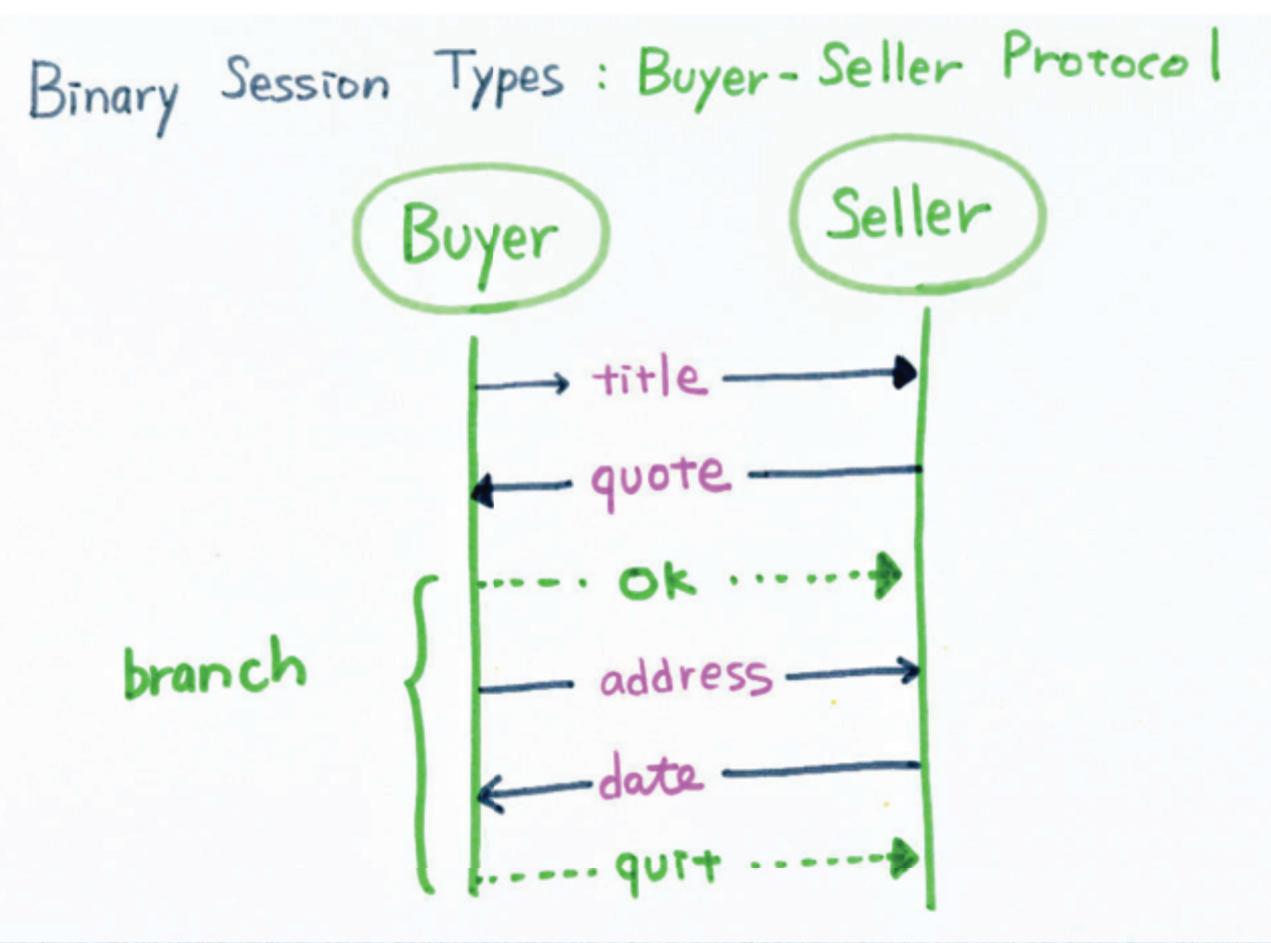


IS BINARY ENOUGH?



$$\uparrow String.\ \downarrow Int.\ \oplus \{\mathsf{ok} :\uparrow String.\ \downarrow Date.\ \mathsf{end};\quad \mathsf{quit} : \mathsf{end}\}$$

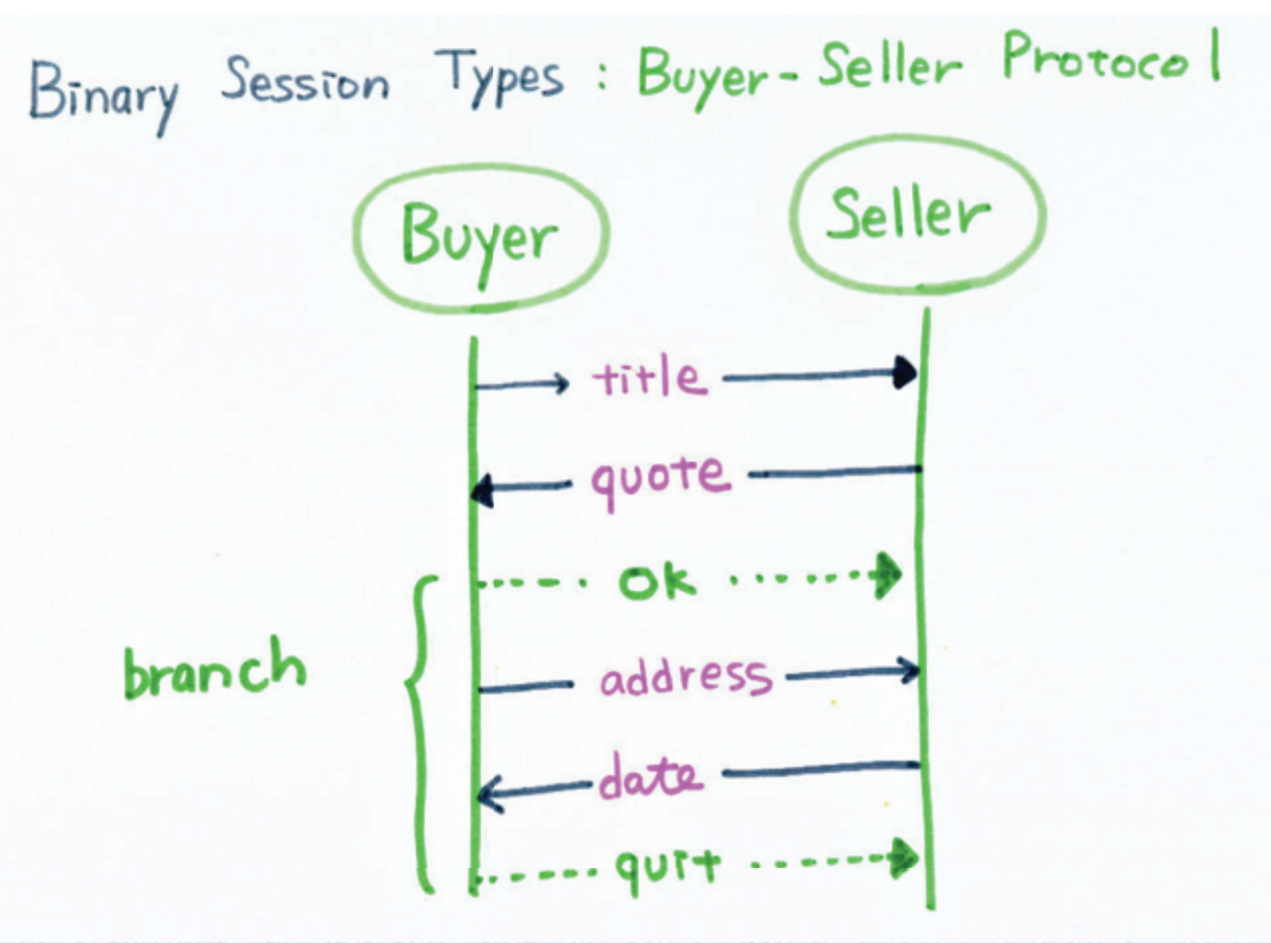
IS BINARY ENOUGH?



$\uparrow String.\ \downarrow Int.\ \oplus \{\mathsf{ok} :\uparrow String.\ \downarrow Date.\ \mathsf{end};\quad \mathsf{quit} : \mathsf{end}\}$

$\downarrow String.\ \uparrow Int.\ \& \{\mathsf{ok} :\downarrow String.\ \uparrow Date.\ \mathsf{end};\quad \mathsf{quit} : \mathsf{end}\}$

IS BINARY ENOUGH?



P has type α

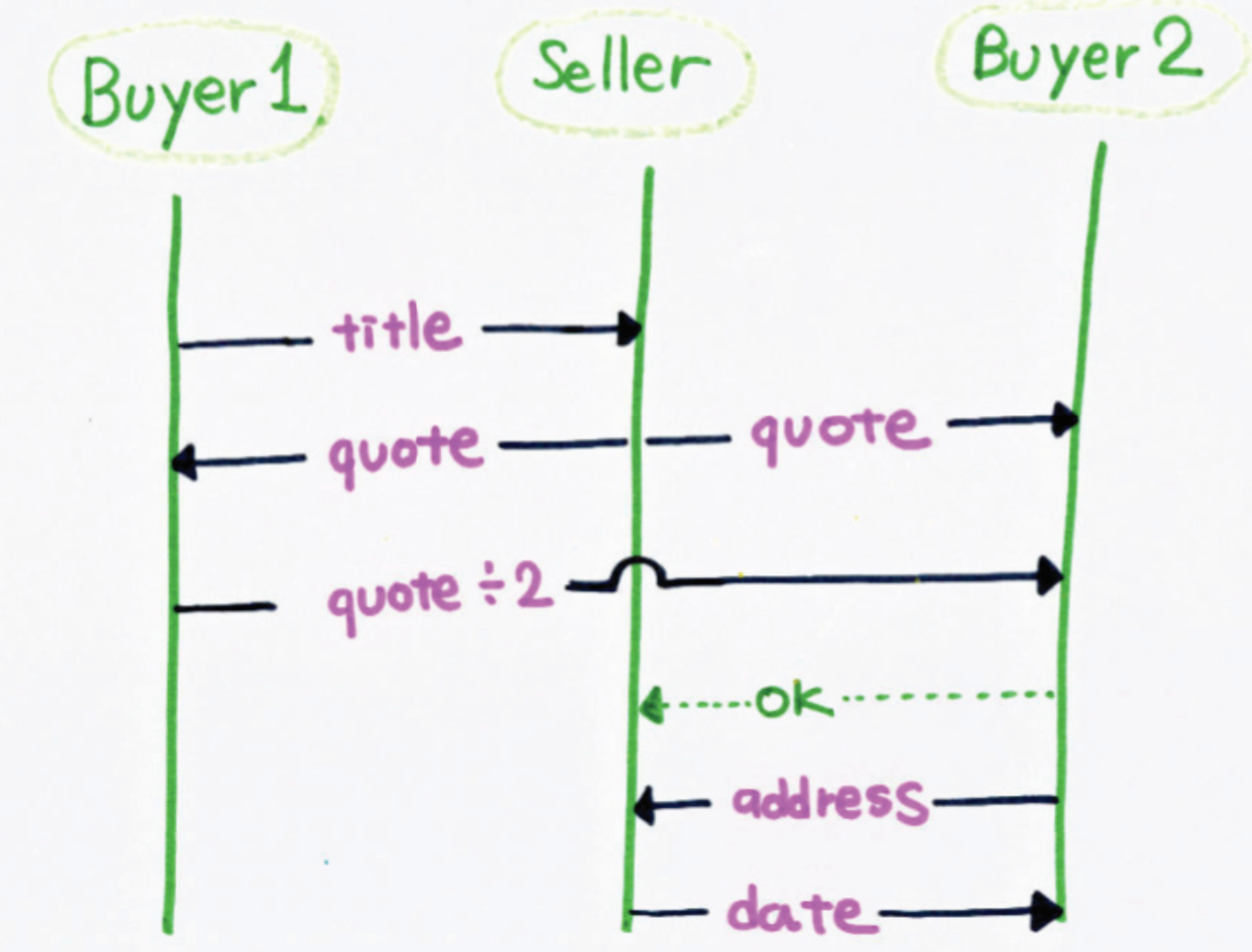
Q has type $\overline{\alpha}$

then $P \mid Q$ is typable

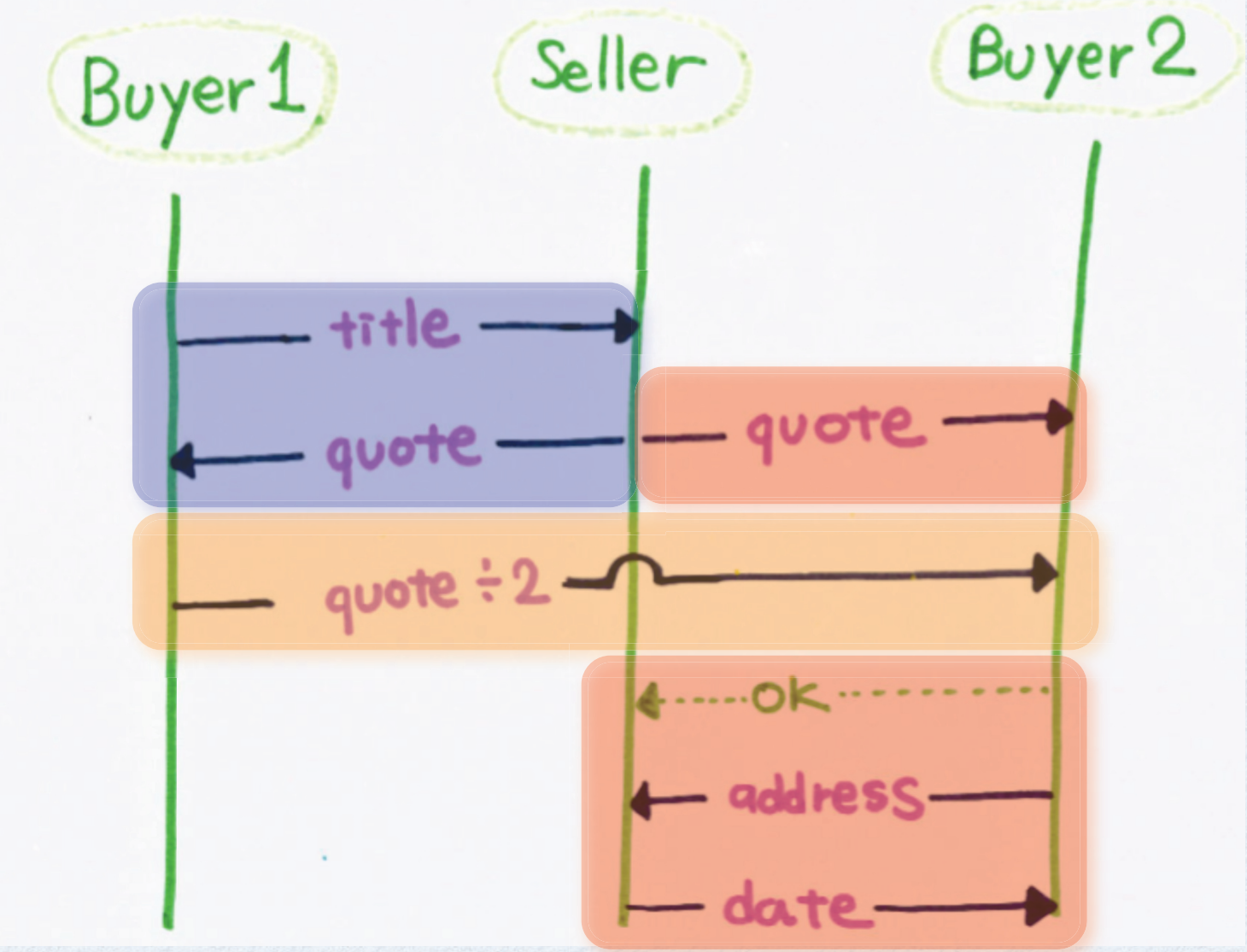
$\uparrow String.\ \downarrow Int.\ \oplus \{\mathsf{ok} :\uparrow String.\ \downarrow Date.\ \mathsf{end};\quad \mathsf{quit} : \mathsf{end}\}$

$\downarrow String.\ \uparrow Int.\ \& \{\mathsf{ok} :\downarrow String.\ \uparrow Date.\ \mathsf{end};\quad \mathsf{quit} : \mathsf{end}\}$

A MULTIPARTY SESSION



A MULTIPARTY SESSION

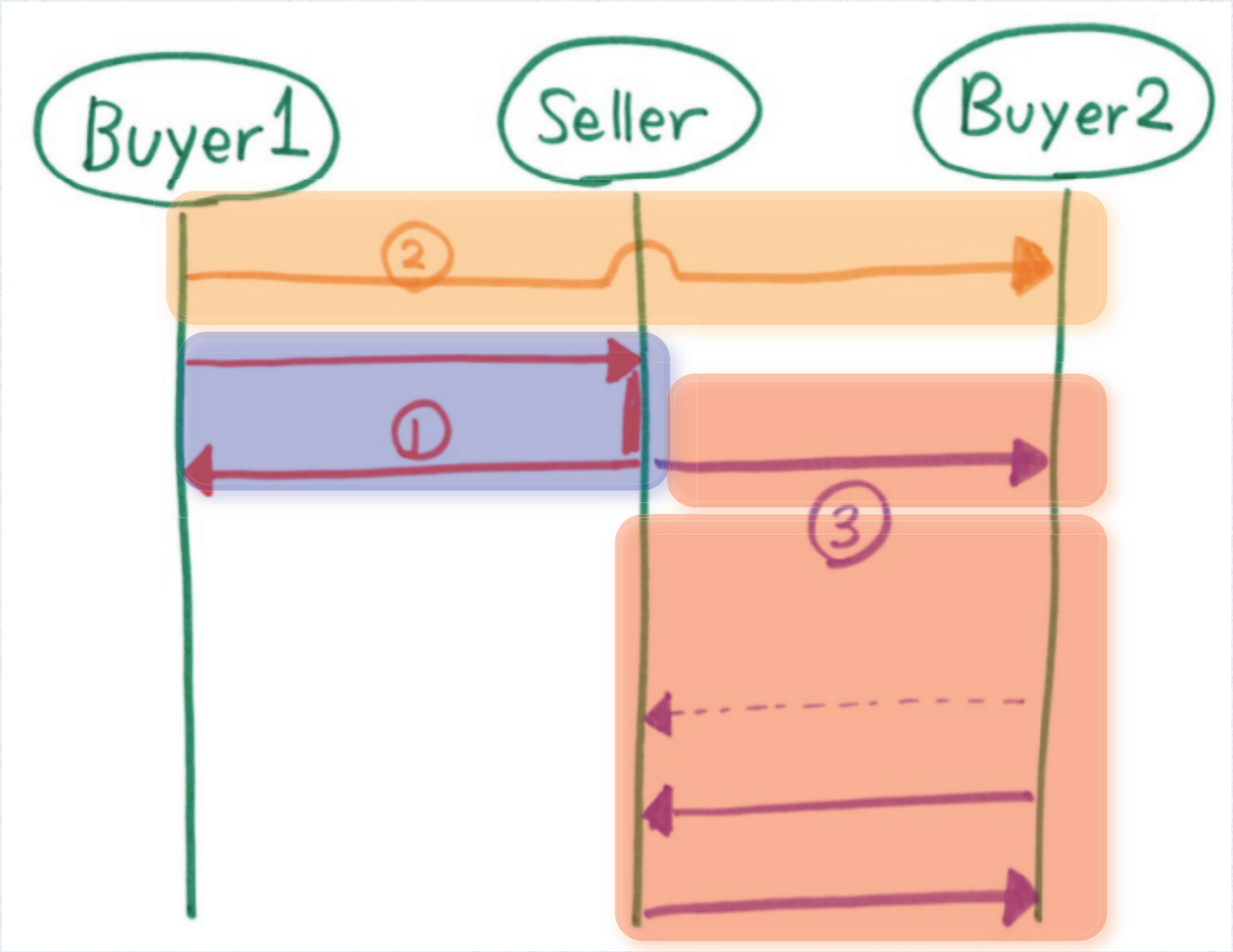


1

2

3

A MULTIPARTY SESSION

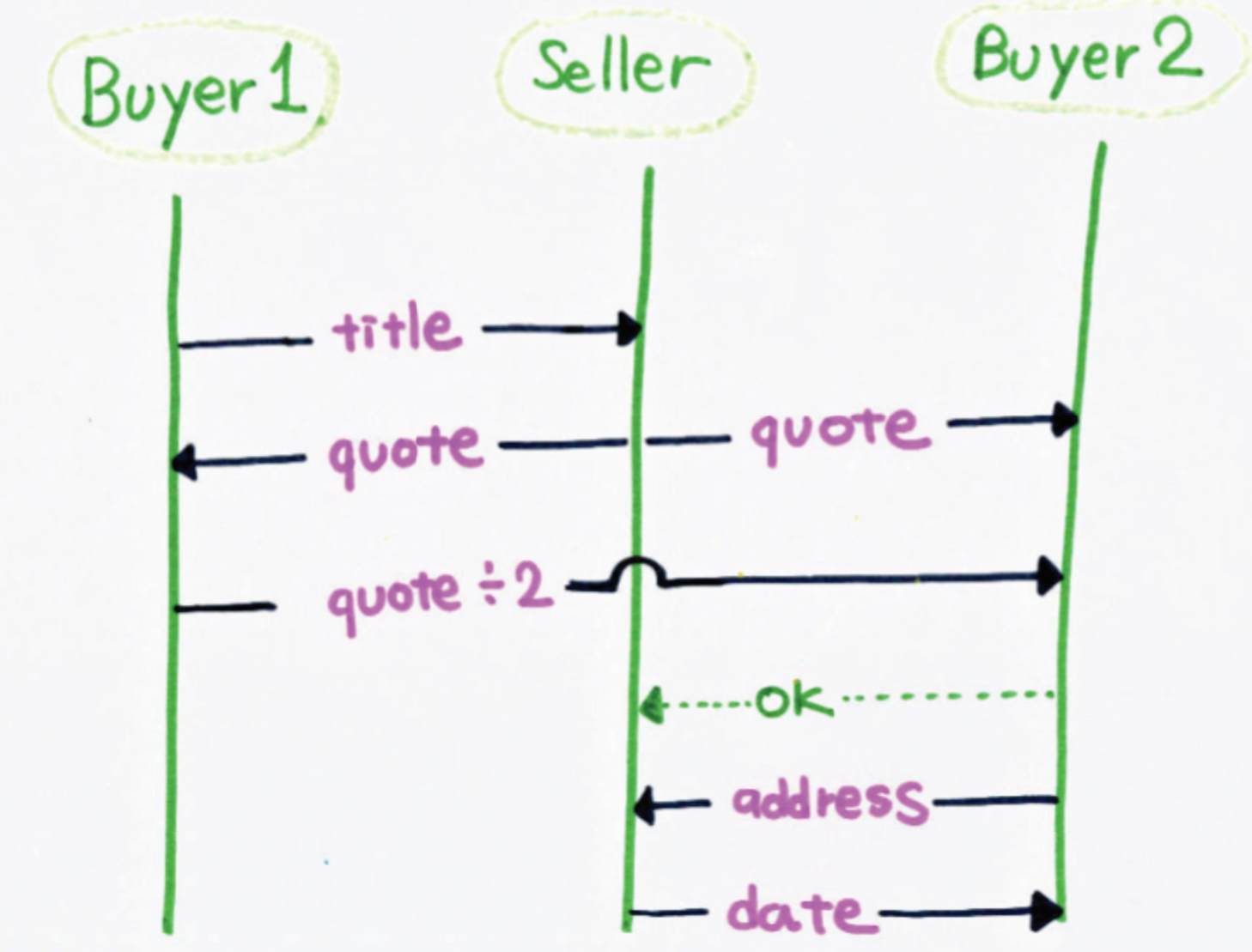


1

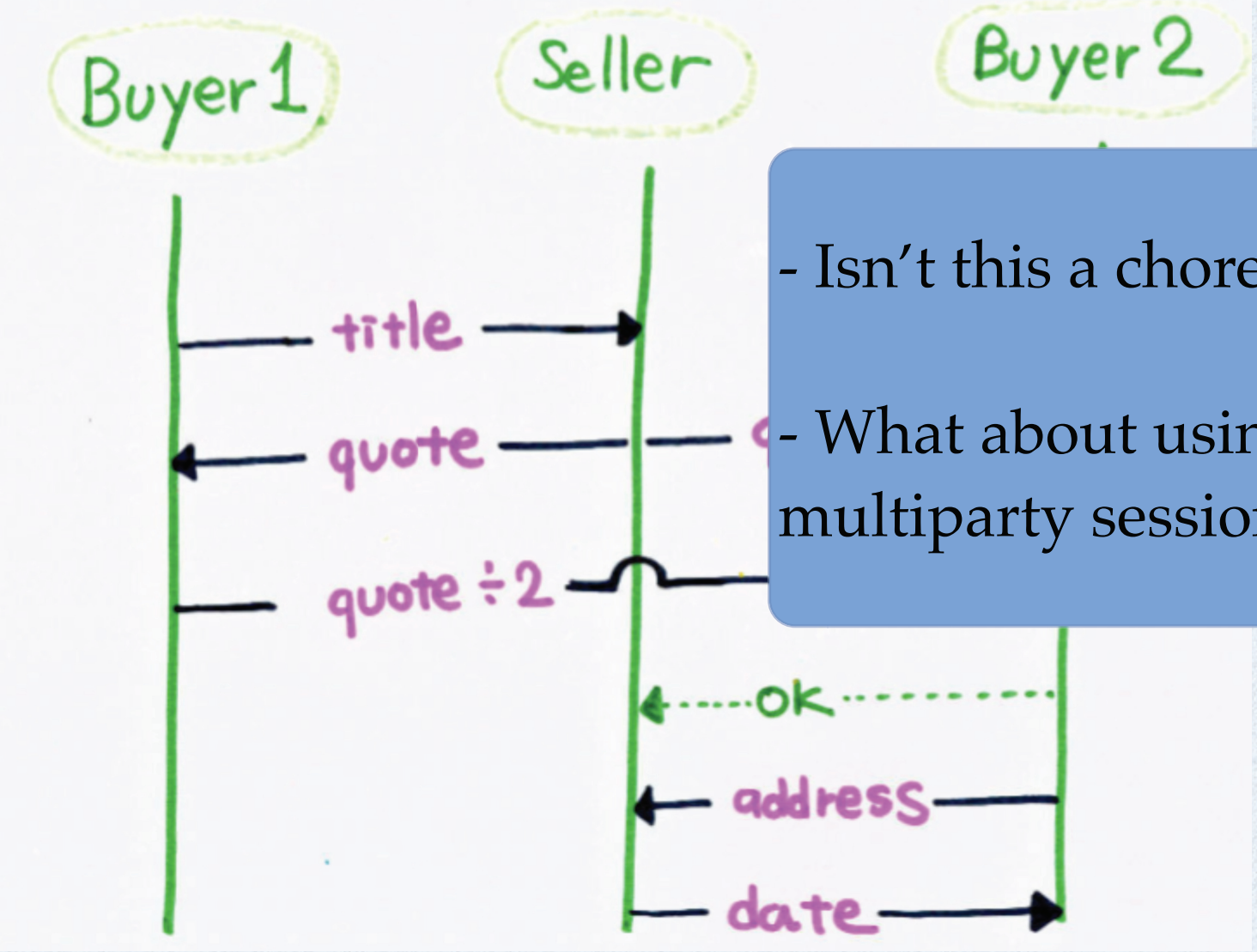
2

3

A MULTIPARTY SESSION



A MULTIPARTY SESSION



- Isn't this a choreography?

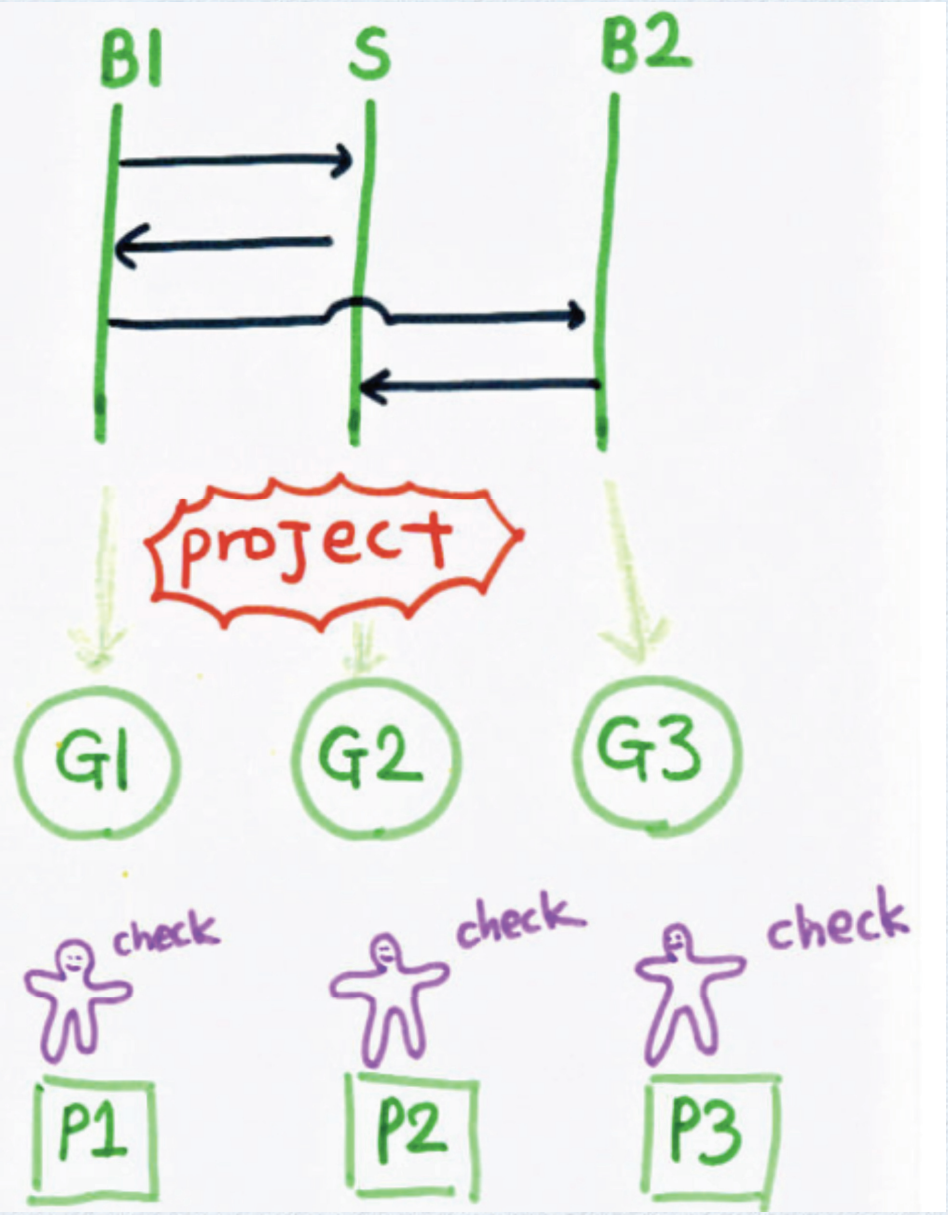
- What about using it as a multiparty session type?

A DESIGN APPROACH

1. Write a Global Type

2. Check Global Type is Good

3. Project and Local Type-Check



π-CALCULUS WITH SESSIONS

P :: =	$*a(\kappa).\ P$	(service)
	$a(\kappa).\ P$	(request)
	$\kappa?(x).\ P$	(input)
	$\kappa!\langle e\rangle.\ P$	(output)
	$\kappa \triangleright \{ l_i : P_i \}$	(branch)
	$\kappa \triangleleft l.\ P$	(select)
	$(\upsilon\ a)\ P$ / $(\upsilon\ \kappa)\ P$	(res)
	$P \mid Q$	(par)
	if e **then** P **else** P	(cond)
	0	(inact)
	μX. P	(rec)
	X	(recVar)

π-CALCULUS WITH SESSIONS

P :: =	~~$*a(\kappa).\ P$~~	~~(service)~~
	~~$a(\kappa).\ P$~~	~~(request)~~
	$\kappa?(x).\ P$	(input)
	$\kappa!\langle e \rangle.\ P$	(output)
	$\kappa \triangleright \{ l_i : P_i \}$	(branch)
	$\kappa \triangleleft l.\ P$	(select)
	$(\nu\ a)\ P$ / $(\nu\ \kappa)\ P$	(res)
	$P \mid Q$	(par)
	if e **then** P **else** P	(cond)
	0	(inact)
	$\mu X.\ P$	(rec)
	X	(recVar)

π-CALCULUS WITH SESSIONS

P :: = ~~$*a(k).\ P$~~ (service)

~~$a(k).\ P$~~ (request)

$a_{[p]}(\tilde{s}).\ P$ (accept)

$\overline{a}_{[2..n]}(\tilde{s}).\ P$ (request)

$s?((\tilde{s})).\ P$ (session reception)

$s!\langle\langle \tilde{s} \rangle\rangle.\ P$ (delegation)

X (recVar)

SEMANTICS

$$\overline{a}[2..n]\,(\tilde{s}).\ P_1 \mid a[2]\,(\tilde{s}).\ P_2 \mid \cdots \mid a[n]\,(\tilde{s}).\ P_n$$

$$\rightarrow\ (\nu\tilde{s})\,(P_1 \mid P_2 \mid \ldots \mid P_n \mid s_1 : \epsilon \mid \cdots \mid s_m : \epsilon)$$

SEMANTICS

$$\overline{a}[2..n]\,(\tilde{s}).\ P_1 \mid a[2]\,(\tilde{s}).\ P_2 \mid \cdots \mid a[n]\,(\tilde{s}).\ P_n$$

$$\rightarrow\ (\nu\tilde{s})\,(P_1 \mid P_2 \mid \ldots \mid P_n \mid s_1 : \epsilon \mid \cdots \mid s_m : \epsilon)$$

In/Out/Branch/Select/etc. are similar to the exception case

$$s!\langle \tilde{e} \rangle.\ P \mid s : L \rightarrow P \mid s : (\tilde{v} :: L)$$
$$s!\langle\langle \tilde{t} \rangle\rangle.\ P \mid s : L \rightarrow P \mid s : (\tilde{t} :: L)$$

Assumption. One session channel per each (ordered) pair of roles.

REDUCTION EXAMPLE

Buyer1 $\stackrel{\text{def}}{=} \overline{a}[2,3](b_1, b_2, b_2', s_1, s_2).\ s_1!\langle\text{“War and Peace”}\rangle.$

$b_1?(quote).\ \ b_2'!\langle quote \text{ div } 2\rangle.\ P_1$

Buyer2 $\stackrel{\text{def}}{=} a[2](b_1, b_2, b_2', s_1, s_2).\ \ b_2?(quote).\ \ b_2'?(contrib).$

$\textbf{if}\ (quote - contrib \leq 99)$

$\textbf{then}\quad s_2 \lhd \text{ok}.\ s_2!\langle address\rangle; b_2?(x).\ P_2$

$\textbf{else}\quad s_2 \lhd \text{quit}.\ \mathbf{0}$

Seller $\stackrel{\text{def}}{=} a[3](b_1, b_2, b_2', s_1, s_2).\ \ s_1?(title).\ \ \ b_1, b_2!\langle quote\rangle.$

$s_2 \rhd \{\text{ok}:\ s_2?(x).\ b_2!\langle date\rangle; Q,\quad \text{quit}:\ \mathbf{0}\}$

REDUCTION EXAMPLE

Buyer1 $\stackrel{\text{def}}{=}$ $s_1!\langle\text{“War and Peace”}\rangle.$

$b_1?(quote).\ \ b_2'!\langle quote \text{ div } 2\rangle.\ P_1$

Buyer2 $\stackrel{\text{def}}{=}$ $b_2?(quote).\ \ b_2'?(contrib).$

$\textbf{if}\ (quote - contrib \leq 99)$

$\textbf{then}\quad s_2 \lhd \text{ok}.\ s_2!\langle address\rangle; b_2?(x).\ P_2$

$\textbf{else}\quad s_2 \lhd \text{quit}.\ \mathbf{0}$

Seller $\stackrel{\text{def}}{=}$ $s_1?(title).\ \ b_1, b_2!\langle quote\rangle.$

$s_2 \rhd \{\text{ok}:\ s_2?(x).\ b_2!\langle date\rangle; Q,\quad \text{quit}:\ \mathbf{0}\}$

$b_1 : \epsilon$

$b_2 : \epsilon$

$b_2' : \epsilon$

$s_1 : \epsilon$

$s_2 : \epsilon$

REDUCTION EXAMPLE

Buyer1 $\stackrel{\text{def}}{=}$

$b_1?(quote).\ b_2'!\langle quote \text{ div } 2\rangle.\ P_1$

Buyer2 $\stackrel{\text{def}}{=}$

$b_2?(quote).\ b_2'?(contrib).$

$\textbf{if}\ (quote - contrib \leq 99)$

$\textbf{then}\quad s_2 \lhd \text{ok}.\ s_2!\langle address\rangle; b_2?(x).\ P_2$

$\textbf{else}\quad s_2 \lhd \text{quit}.\ \mathbf{0}$

Seller $\stackrel{\text{def}}{=}$

$s_1?(title).\ b_1, b_2!\langle quote\rangle.$

$s_2 \rhd \{\text{ok}:\ s_2?(x).\ b_2!\langle date\rangle; Q,\quad \text{quit}:\ \mathbf{0}\}$

$b_1 : \epsilon$

$b_2 : \epsilon$

$b_2' : \epsilon$

$s_1 : w\&p$

$s_2 : \epsilon$

REDUCTION EXAMPLE

Buyer1 $\stackrel{\text{def}}{=}$

$b_1?(quote).\ b_2'!\langle quote \text{ div } 2\rangle.\ P_1$

Buyer2 $\stackrel{\text{def}}{=}$

$b_2?(quote).\ b_2'?(contrib).$

if $(quote - contrib \leq 99)$

then $s_2 \lhd \text{ok}.\ s_2!\langle address\rangle; b_2?(x).\ P_2$

else $s_2 \lhd \text{quit}.\ \mathbf{0}$

Seller $\stackrel{\text{def}}{=}$

$b_1, b_2!\langle quote\rangle.$

$s_2 \rhd \{\text{ok}:\ s_2?(x).\ b_2!\langle date\rangle; Q,\quad \text{quit}:\ \mathbf{0}\}$

$b_1 : \epsilon$

$b_2 : \epsilon$

$b_2' : \epsilon$

$s_1 : \epsilon$

$s_2 : \epsilon$

REDUCTION EXAMPLE

Buyer1 $\stackrel{\text{def}}{=}$

$$b_1?(quote).\ b_2'!\langle quote \text{ div } 2\rangle.\ P_1$$

Buyer2 $\stackrel{\text{def}}{=}$

$$b_2?(quote).\ b_2'?(contrib).$$

$$\textbf{if } (quote - contrib \leq 99)$$

$$\textbf{then} \quad s_2 \lhd \text{ok}.\ s_2!\langle address\rangle; b_2?(x).\ P_2$$

$$\textbf{else} \quad s_2 \lhd \text{quit}.\ \mathbf{0}$$

Seller $\stackrel{\text{def}}{=}$

$$s_2 \rhd \{\text{ok}:\ s_2?(x).\ b_2!\langle date\rangle; Q,\quad \text{quit}:\ \mathbf{0}\}$$

$b_1 : quote$

$b_2 : quote$

$b_2' : \epsilon$

$s_1 : \epsilon$

$s_2 : \epsilon$

REDUCTION EXAMPLE

Buyer1 $\stackrel{\text{def}}{=}$

$b_1?(quote).\ b_2'!\langle quote \text{ div } 2\rangle.\ P_1$

Buyer2 $\stackrel{\text{def}}{=}$

$b_2'?(contrib).$

$\textbf{if}\ (quote - contrib \leq 99)$

$\textbf{then}\quad s_2 \lhd \text{ok}.\ s_2!\langle address\rangle; b_2?(x).\ P_2$

$\textbf{else}\quad s_2 \lhd \text{quit}.\ \mathbf{0}$

Seller $\stackrel{\text{def}}{=}$

$s_2 \rhd \{\text{ok}:\ s_2?(x).\ b_2!\langle date\rangle; Q,\quad \text{quit}:\ \mathbf{0}\}$

$b_1 : \epsilon$

$b_2 : quote$

$b_2' : \epsilon$

$s_1 : \epsilon$

$s_2 : \epsilon$

REDUCTION EXAMPLE

Buyer1 $\stackrel{\text{def}}{=}$

$b_2'!\langle quote \text{ div } 2\rangle.\ P_1$

Buyer2 $\stackrel{\text{def}}{=}$

$b_2'?(contrib).$

$\textbf{if}\ (quote - contrib \leq 99)$

$\textbf{then}\quad s_2 \lhd \text{ok}.\ s_2!\langle address\rangle; b_2?(x).\ P_2$

$\textbf{else}\quad s_2 \lhd \text{quit}.\ \mathbf{0}$

Seller $\stackrel{\text{def}}{=}$

$s_2 \rhd \{\text{ok}:\ s_2?(x).\ b_2!\langle date\rangle; Q,\quad \text{quit}:\ \mathbf{0}\}$

$b_1 : \epsilon$

$b_2 : \epsilon$

$b_2' : \epsilon$

$s_1 : \epsilon$

$s_2 : \epsilon$

REDUCTION EXAMPLE

Buyer1 $\stackrel{\text{def}}{=}$

$$P_1$$

Buyer2 $\stackrel{\text{def}}{=}$ $b_2'?(contrib).$

$$\textbf{if } (quote - contrib \leq 99)$$

$$\textbf{then} \quad s_2 \lhd \mathsf{ok}.\ s_2!\langle address\rangle; b_2?(x).\ P_2$$

$$\textbf{else} \quad s_2 \lhd \mathsf{quit}.\ \mathbf{0}$$

Seller $\stackrel{\text{def}}{=}$

$$s_2 \rhd \{\mathsf{ok}:\ s_2?(x).\ b_2!\langle date\rangle; Q,\quad \mathsf{quit}:\ \mathbf{0}\}$$

$b_1 : \epsilon$

$b_2 : \epsilon$

$b_2' : quote/2$

$s_1 : \epsilon$

$s_2 : \epsilon$

REDUCTION EXAMPLE

Buyer1 $\stackrel{\text{def}}{=}$

P_1

Buyer2 $\stackrel{\text{def}}{=}$

if $(quote - contrib \leq 99)$

then $s_2 \lhd \mathsf{ok}.\ s_2!\langle address\rangle; b_2?(x).\ P_2$

else $s_2 \lhd \mathsf{quit}.\ \mathbf{0}$

Seller $\stackrel{\text{def}}{=}$

$s_2 \rhd \{\mathsf{ok}:\ s_2?(x).\ b_2!\langle date\rangle; Q,\quad \mathsf{quit}:\ \mathbf{0}\}$

$b_1 : \epsilon$

$b_2 : \epsilon$

$b_2' : \epsilon$

$s_1 : \epsilon$

$s_2 : \epsilon$

REDUCTION EXAMPLE

Buyer1 $\stackrel{\text{def}}{=}$

$$P_1$$

Buyer2 $\stackrel{\text{def}}{=}$

$$\textbf{if } (quote - contrib \leq 99)$$

$$\textbf{then } s_2 \lhd \text{ok}.\, s_2!\langle address\rangle;\, b_2?(x).\, P_2$$

$$\textbf{else } s_2 \lhd \text{quit}.\, \mathbf{0}$$

Seller $\stackrel{\text{def}}{=}$

$$s_2 \rhd \{\text{ok}: s_2?(x).\, b_2!\langle date\rangle;\, Q,\quad \text{quit}: \mathbf{0}\}$$

$b_1 : \epsilon$

$b_2 : \epsilon$

$b_2' : \epsilon$

$s_1 : \epsilon$

$s_2 : \epsilon$

GLOBAL TYPES

$$
\begin{array}{rcll}
G & ::= & \mathtt{p} \to \mathtt{p}' : k\,\langle\theta\rangle.G' & \text{(values)} \\
 & | & \mathtt{p} \to \mathtt{p}' : k\,\{l_j : G_j\}_{j\in J} & \text{(branching)} \\
 & | & G, G' & \text{(parallel)} \\
 & | & \mu \mathbf{t}.G & \text{(recursive)} \\
 & | & \mathbf{t} & \text{(variable)} \\
 & | & \mathsf{end} & \text{(end)}
\end{array}
$$

$$
\theta \;::=\; \tilde{S} \;\mid\; \alpha@\mathtt{p} \qquad S \;::=\; \mathsf{bool} \mid \mathsf{nat} \mid \ldots \mid \langle G \rangle
$$

Global Types: Examples

Two-Buyers Protocol

$$\mathtt{B1} \to \mathtt{S} : s_1\langle \mathsf{string}\rangle.\quad \mathtt{S} \to \mathtt{B1} : b_1\langle \mathsf{int}\rangle.\quad \mathtt{S} \to \mathtt{B2} : b_2\langle \mathsf{int}\rangle.\quad \mathtt{B1} \to \mathtt{B2} : b_2'\langle \mathsf{int}\rangle.$$

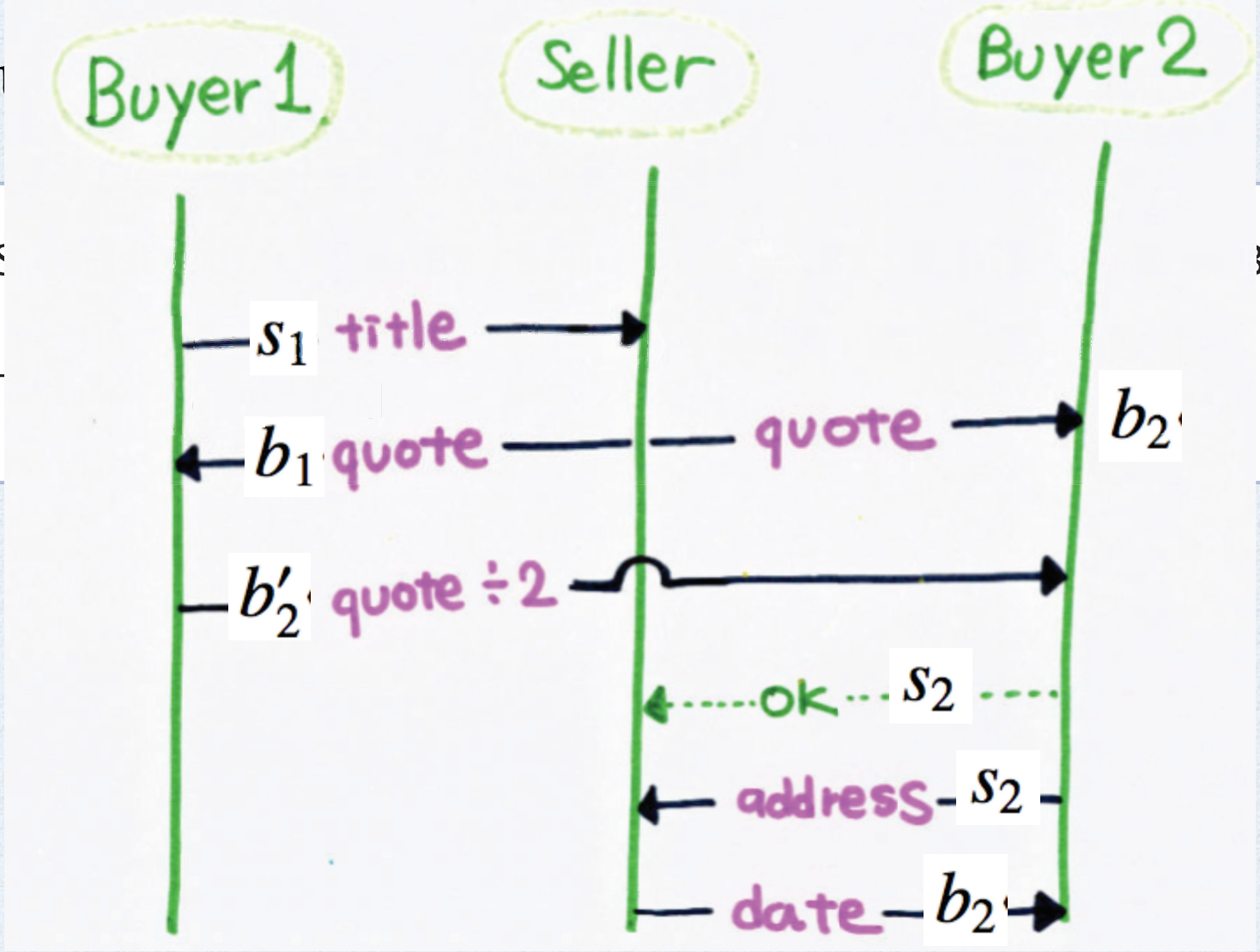
$$\mathtt{B2} \to \mathtt{S} : s_2 \left\{ \begin{array}{l} \mathsf{ok} : \mathtt{B2} \to \mathtt{S} : s_2\langle \mathsf{string}\rangle.\mathtt{S} \to \mathtt{B2} : b_2\langle \mathsf{date}\rangle.\mathsf{end}, \\ \mathsf{quit} : \mathsf{end} \} \end{array} \right\}$$

GLOBAL TYPES: EXAMPLES

Two-Bu

B1 → S

B2 –



$2 : b'_2\langle\text{int}\rangle$.

LOCAL TYPES

$$\alpha ::= k!\langle\theta\rangle.\,\alpha \mid k?\langle\theta\rangle.\,\alpha \mid k \oplus \{l_i : \alpha_i\}_{i\in I}$$

$$\mid k \,\&\, \{l_i : \alpha_i\}_{i\in I} \mid \mu \mathbf{t}.\,\alpha \mid \mathbf{t} \mid \mathsf{end}$$

- (almost) **standard**
- actions labelled with channel to be used.

EXAMPLE

Buyer1 $\stackrel{\text{def}}{=} \overline{a}_{[2,3]}(b_1, b_2, b_2', s_1, s_2).\ \ s_1!\langle\text{“War and Peace”}\rangle.$

$b_1?(quote).\ \ b_2'!\langle quote \text{ div } 2\rangle.\ P_1$

In the two buyer protocol, write Buyer1 as

$\overline{a}_{[2,3]}(b_1, b_2, b_2', s_1, s_2).\ Q_1$ and Buyer2 as $a_{[2]}(b_1, b_2, b_2', s_1, s_2).\ Q_2$. Then

$\Gamma \vdash Q_1 \triangleright \tilde{s} : s_1!\langle\text{string}\rangle.\ b_1?\langle\text{int}\rangle.\ b_2'!\langle\text{int}\rangle@\text{B1}$

$\Gamma \vdash Q_2 \triangleright \tilde{s} : b_2?\langle\text{int}\rangle.\ b_2'?\langle\text{int}\rangle.\ s_2 \oplus \{\text{ok} : s_2!\langle\text{string}\rangle.\ b_2?\langle\text{date}\rangle.\ \text{end},\ \text{quit} : \text{end}\}@\text{B2}$

EXAMPLE

$$\text{Buyer2} \stackrel{\text{def}}{=} a_{[2]}(b_1, b_2, b_2', s_1, s_2).\ \ b_2?(quote).\ \ b_2'?(contrib).$$
$$\textbf{if}\ (quote - contrib \leq 99)$$
$$\textbf{then}\quad s_2 \triangleleft \text{ok}.\ s_2!\langle address\rangle; b_2?(x).\ P_2$$
$$\textbf{else}\quad s_2 \triangleleft \text{quit}.\ \mathbf{0}$$

In the two buyer protocol, write Buyer1 as $\overline{a}_{[2,3]}(b_1, b_2, b_2', s_1, s_2).\ Q_1$ and Buyer2 as $a_{[2]}(b_1, b_2, b_2', s_1, s_2).\ Q_2$. Then

$$\Gamma \vdash Q_1 \triangleright \tilde{s} : s_1!\langle \text{string}\rangle.\ b_1?\langle \text{int}\rangle.\ b_2'!\langle \text{int}\rangle@\text{B1}$$
$$\Gamma \vdash Q_2 \triangleright \tilde{s} : b_2?\langle \text{int}\rangle.\ b_2'?\langle \text{int}\rangle.\ s_2 \oplus \{\text{ok} : s_2!\langle \text{string}\rangle.\ b_2?\langle \text{date}\rangle.\ \text{end},\ \text{quit} : \text{end}\}@\text{B2}$$

PROJECTION

$$(\mathtt{p}_1 \to \mathtt{p}_2 : k\,\langle\theta\rangle.G')\upharpoonright \mathtt{p} = \begin{cases} k!\,\langle\theta\rangle.(G'\upharpoonright \mathtt{p}) & \text{if } \mathtt{p} = \mathtt{p}_1 \neq \mathtt{p}_2 \\ k?\,\langle\theta\rangle.(G'\upharpoonright \mathtt{p}) & \text{if } \mathtt{p} = \mathtt{p}_2 \neq \mathtt{p}_1 \\ (G'\upharpoonright \mathtt{p}) & \text{if } \mathtt{p} \neq \mathtt{p}_2 \wedge \mathtt{p} \neq \mathtt{p}_1 \end{cases}$$

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$$(G_1, G_2) \upharpoonright \mathtt{p} = \begin{cases} G_i \upharpoonright \mathtt{p} & \text{if } \mathtt{p} \in G_i \text{ and } \mathtt{p} \notin G_j,\ i \neq j \in \{1,2\} \\ \mathsf{end} & \text{if } \mathtt{p} \notin G_1 \text{ and } \mathtt{p} \notin G_2 \end{cases}$$

PROJECTION

$$(\mathtt{p}_1 \to \mathtt{p}_2 : k\,\langle\theta\rangle.G')\upharpoonright \mathtt{p} = \begin{cases} k!\,\langle\theta\rangle.(G'\upharpoonright \mathtt{p}) & \text{if } \mathtt{p} = \mathtt{p}_1 \neq \mathtt{p}_2 \\ k?\,\langle\theta\rangle.(G'\upharpoonright \mathtt{p}) & \text{if } \mathtt{p} = \mathtt{p}_2 \neq \mathtt{p}_1 \\ (G'\upharpoonright \mathtt{p}) & \text{if } \mathtt{p} \neq \mathtt{p}_2 \wedge \mathtt{p} \neq \mathtt{p}_1 \end{cases}$$

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$$(\mathtt{p}_1 \to \mathtt{p}_2 : k\,\{l_j : G_j\}_{j\in J})\upharpoonright \mathtt{p} = \begin{cases} k \oplus \{l_j : (G_j\upharpoonright \mathtt{p})\}_{j\in J} & \text{if } \mathtt{p} = \mathtt{p}_1 \neq \mathtt{p}_2 \\ k\&\{l_j : (G_j\upharpoonright \mathtt{p})\}_{j\in J} & \text{if } \mathtt{p} = \mathtt{p}_2 \neq \mathtt{p}_1 \\ (\bigsqcup_{i\in I} G_i\upharpoonright \mathtt{p}) & \text{if } \mathtt{p} \neq \mathtt{p}_2 \wedge \mathtt{p} \neq \mathtt{p}_1 \\ & \text{and } \forall i, j \in I.G_i\upharpoonright \mathtt{p} \bowtie G_j\upharpoonright \mathtt{p} \end{cases}$$

PROJECTION (2)

Two-Buyers Protocol

$$\mathtt{B1} \to \mathtt{S} : s_1\langle \mathsf{string}\rangle.\quad \mathtt{S} \to \mathtt{B1} : b_1\langle \mathsf{int}\rangle.\quad \mathtt{S} \to \mathtt{B2} : b_2\langle \mathsf{int}\rangle.\quad \mathtt{B1} \to \mathtt{B2} : b_2'\langle \mathsf{int}\rangle.$$

$$\mathtt{B2} \to \mathtt{S} : s_2 \left\{ \begin{array}{l} \mathsf{ok} : \mathtt{B2} \to \mathtt{S} : s_2\,\langle \mathsf{string}\rangle.\mathtt{S} \to \mathtt{B2} : b_2\,\langle \mathsf{date}\rangle.\mathsf{end}, \\ \mathsf{quit} : \mathsf{end}\} \end{array} \right\}$$

For Buyer2:

$$b_2?\langle \mathsf{int}\rangle.\ b_2'?\langle \mathsf{int}\rangle.\ s_2 \oplus \{\mathsf{ok} : s_2!\langle \mathsf{string}\rangle.\ b_2?\langle \mathsf{date}\rangle.\ \mathsf{end},\ \mathsf{quit} : \mathsf{end}\}@\mathtt{B2}$$

MERGEABILITY&PROJECTION

Mergeability.

Smallest equiv. on local types closed under contexts and s.t.:

$$\frac{\forall i \in (I \cap J).\ \alpha_i \bowtie \beta_i \qquad \forall i \in I \setminus J.\ \forall j \in J \setminus I.\ l_i \neq l_j}{k \,\&\, \{l_i : \alpha_i\}_{i \in I} \bowtie k \,\&\, \{l_j : \beta_j\}_{j \in J}}$$

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Merge.
Partial (on mergeability) operator. It is the identity except from:

$$k \,\&\, \{l_i : \alpha_i\}_{i \in I} \sqcup k \,\&\, \{l_j : \beta_j\}_{j \in J} =$$

$$k \& (\{l_i : \alpha_i \sqcup \beta_i\}_{i \in I \cap J} \cup \{l_i : \alpha_i\}_{i \in I} \cup \{l_j : \beta_j\}_{j \in J})$$

MERGING EXAMPLE

$p_1 \to p_2 : k \{$
ok : $p_2 \to p_3 : k'$ {paymore : . . .},
quit : $p_2 \to p_3 : k'$ {refund : . . .}
$\}$

p_1 $k \oplus \{\text{ok}: \ldots, \text{quit}: \ldots,\}$

p_2 $k\&\{ \quad \text{ok}: k' \oplus \{\text{paymore}: \ldots\}, \quad \text{quit}: k' \oplus \{\text{reject}: \ldots\} \quad \}$

p_3 $k'\&\{ \text{paymore}: \ldots, \text{refund}: \ldots \}$

TYPING

Environments.

$$\Gamma ::= \emptyset \mid \Gamma, u : S \mid \Gamma, X : \Delta$$
$$\Delta ::= \emptyset \mid \Delta \cdot \tilde{s} : \{\alpha @ \mathrm{p}\}_{\mathrm{p} \in I}$$

Judgements.

$$\Gamma \vdash P \triangleright \Delta$$

Typing Rules: Init

$$\frac{\Gamma \vdash a : \langle G \rangle \quad \Gamma \vdash P \triangleright \Delta, \tilde{s} : (G \upharpoonright 1)@1 \quad |\tilde{s}| = \max(\mathsf{sid}(G))}{\Gamma \vdash \overline{a}_{[2..n]}(\tilde{s}).\, P \triangleright \Delta}$$

$$\frac{\Gamma \vdash a : \langle G \rangle \quad \Gamma \vdash P \triangleright \Delta, \tilde{s} : (G \upharpoonright \mathtt{p})@\mathtt{p} \quad |\tilde{s}| = \max(\mathsf{sid}(G))}{\Gamma \vdash a_{[\mathtt{p}]}(\tilde{s}).\, P \triangleright \Delta}$$

TYPING RULES: IN/OUT

$$\frac{\Gamma, x : \tilde{S} \vdash P \triangleright \Delta \cdot \tilde{s} : \alpha @ \mathtt{p}}{\Gamma \vdash s_k ?(\tilde{x}).\, P \triangleright \Delta \cdot \tilde{s} : k ?\langle \tilde{S} \rangle.\, \alpha @ \mathtt{p}}$$

$$\frac{\forall j.\, \Gamma \vdash e_j : S_j \qquad \Gamma \vdash P \triangleright \Delta \cdot \tilde{s} : \alpha @ \mathtt{p}}{\Gamma \vdash s_k !\langle \tilde{e} \rangle.\, P \triangleright \Delta \cdot \tilde{s} : k !\langle \tilde{S} \rangle.\, \alpha @ \mathtt{p}}$$

TYPING RULES: DELEGATION

$$\frac{\Gamma \vdash P \triangleright \Delta \cdot \tilde{s} : \alpha @ \mathrm{p} \cdot \tilde{t} : \alpha' @ \mathrm{p}'}{\Gamma \vdash s_k\ ?((\tilde{t}))\mathbf{.}\ P \triangleright \Delta \cdot \tilde{s} : k?\langle \alpha' @ \mathrm{p}' \rangle\mathbf{.}\ \alpha @ \mathrm{p}}$$

$$\frac{\Gamma \vdash P \triangleright \Delta \cdot \tilde{s} : \alpha @ \mathrm{p}}{\Gamma \vdash s_k !\langle\langle \tilde{t} \rangle\rangle\mathbf{.}\ P \triangleright \Delta \cdot \tilde{s} : k!\langle \alpha' @ \mathrm{p}' \rangle\mathbf{.}\ \alpha @ \mathrm{p} \cdot \tilde{t} : \alpha' @ \mathrm{p}'}$$

Typing rules: branching/selection

$$\frac{\Gamma \vdash P_j \triangleright \Delta \cdot \tilde{s} : \alpha_j @ \mathtt{p} \qquad \forall j \in J \qquad I \subseteq J}{\Gamma \vdash s_k \rhd \{l_j : P_j\}_{i \in J} \triangleright \Delta \cdot \tilde{s} : k \,\&\, \{l_i : \alpha_i\}_{i \in I} @ \mathtt{p}}$$

$$\frac{\Gamma \vdash P \triangleright \Delta \cdot \tilde{s} : \alpha_j @ \mathtt{p} \qquad j \in I}{\Gamma \vdash s_k \lhd l_j . P \triangleright \Delta \cdot \tilde{s} : k \oplus \{l_i : \alpha_i\}_{i \in I} @ \mathtt{p}}$$

RESULTS

Theorem (Subject Reduction).

If $\Gamma \vdash P \triangleright \emptyset$ and $P \longrightarrow^* Q$ then $\Gamma \vdash Q \triangleright \emptyset$

Theorem (Communication Safety).

MULTIPARTY: SUMMARY

- Multiparty Session Types
  - Global Types + Causal Analysis + Projection
  - Simple · Concise Syntax
  - Communication Error Freedom
  - Session Fidelity, Progress in a session

Related Work

Bonelli & Compagnoni TGC'07

Ongoing Work

Java with Session Types

W3C CDL (Choreography Description Language)

ISO UNIFI (UNIversal Financial Industry message scheme)

THE END

TERMINATION

- We may wish to terminate a try-catch block. This can be useful in many situation, e.g. when having the sequence operator ";":

$$\mathbf{try}\ \{\ \mathbf{0}\ \}\ \mathbf{catch}\ \{\ \kappa : Q\ \};\ R \longrightarrow R$$

TERMINATION

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$$\mathbf{try}\ \{\ \mathbf{0}\ \}\ \mathbf{catch}\ \{\ \kappa : Q\ \};\ R \longrightarrow R$$

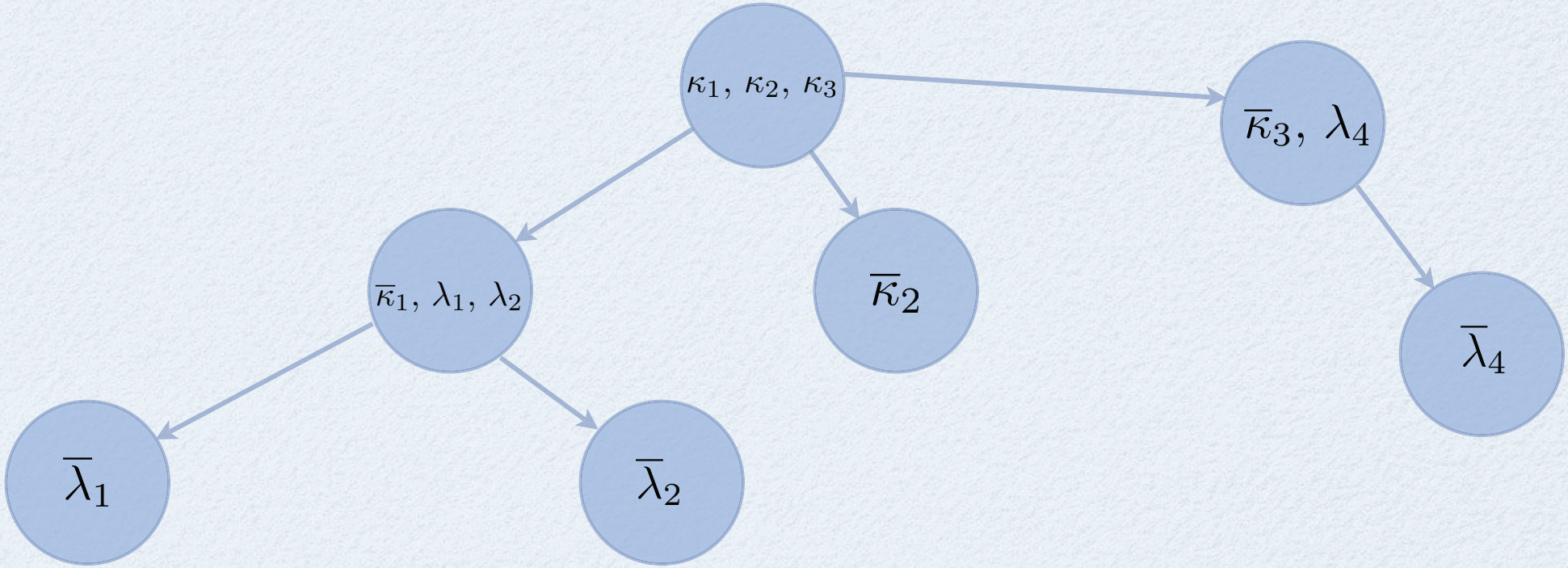
- But...there is a problem...

$$\mathbf{try}\ \{\ \mathbf{0}\ \}\ \mathbf{catch}\ \{\ \kappa : Q\ \} \mid \mathbf{try}\ \{\ \mathbf{throw}\ \}\ \mathbf{catch}\ \{\ \overline{\kappa} : Q'\ \}$$

- If the rhs disappears before the exception is thrown, how can we guarantee propagation?

TERMINATION PROTOCOL

Consider the caller-callee relation on services (instances)



TERMINATION PROTOCOL

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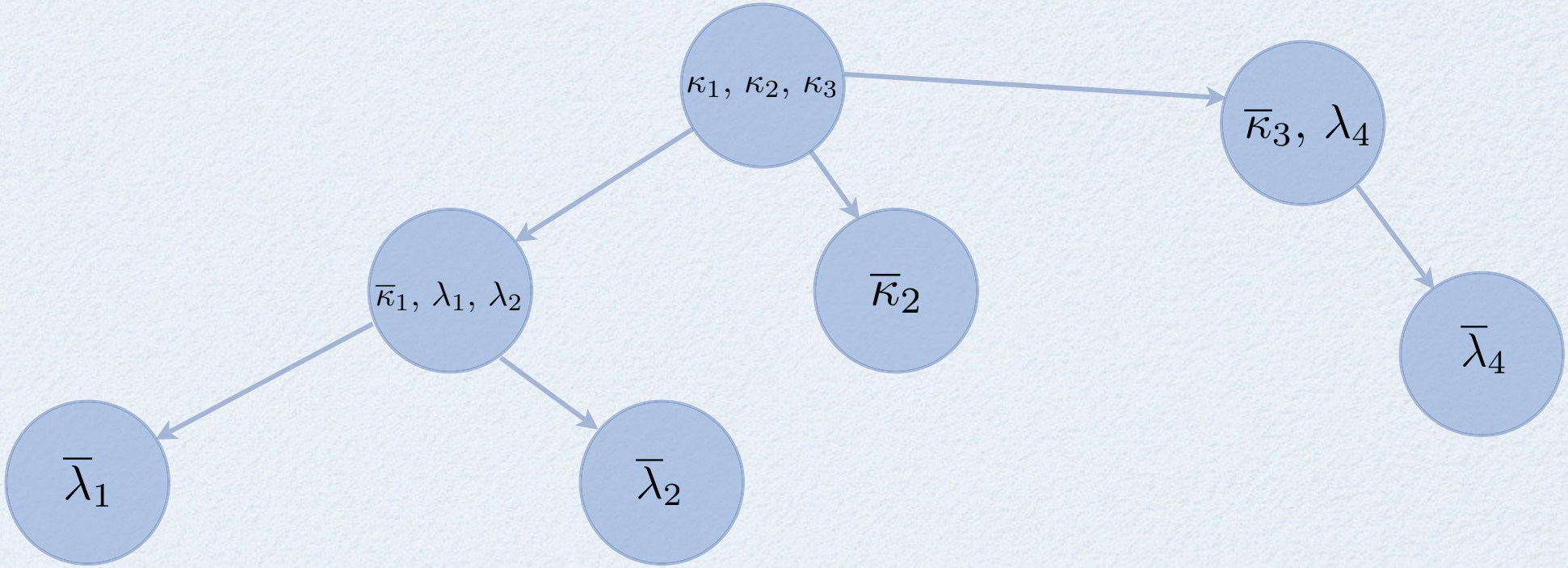
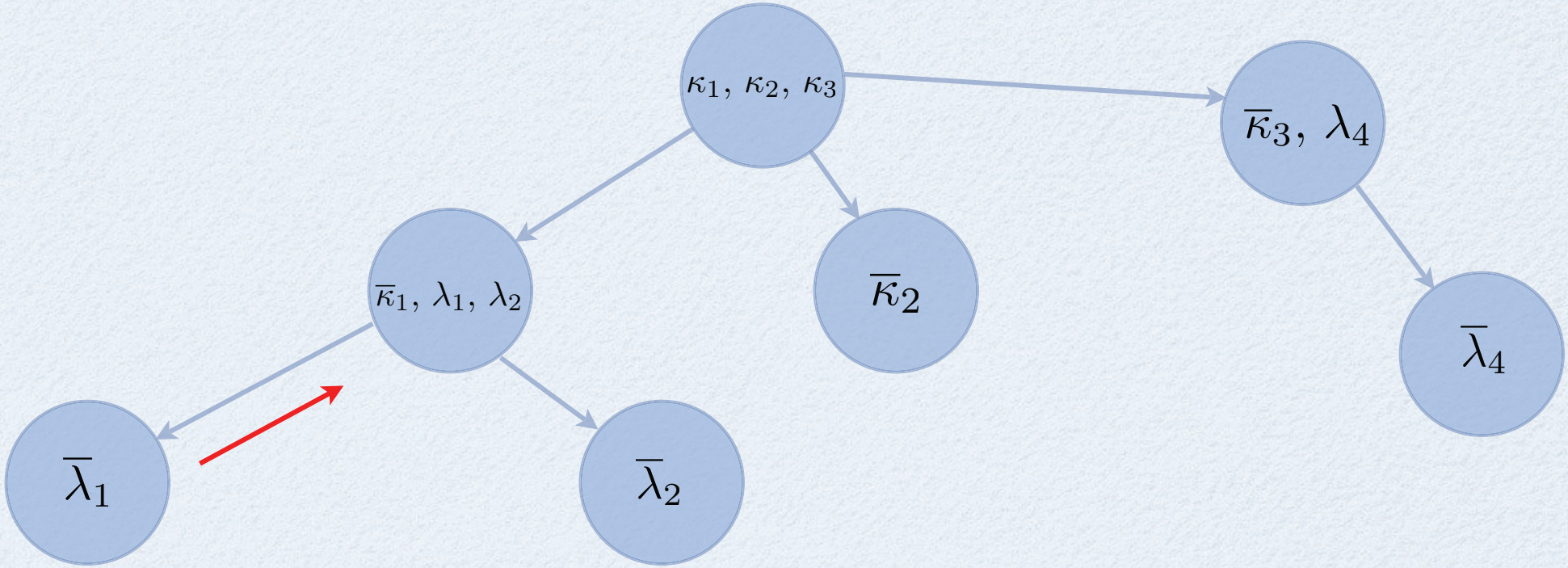


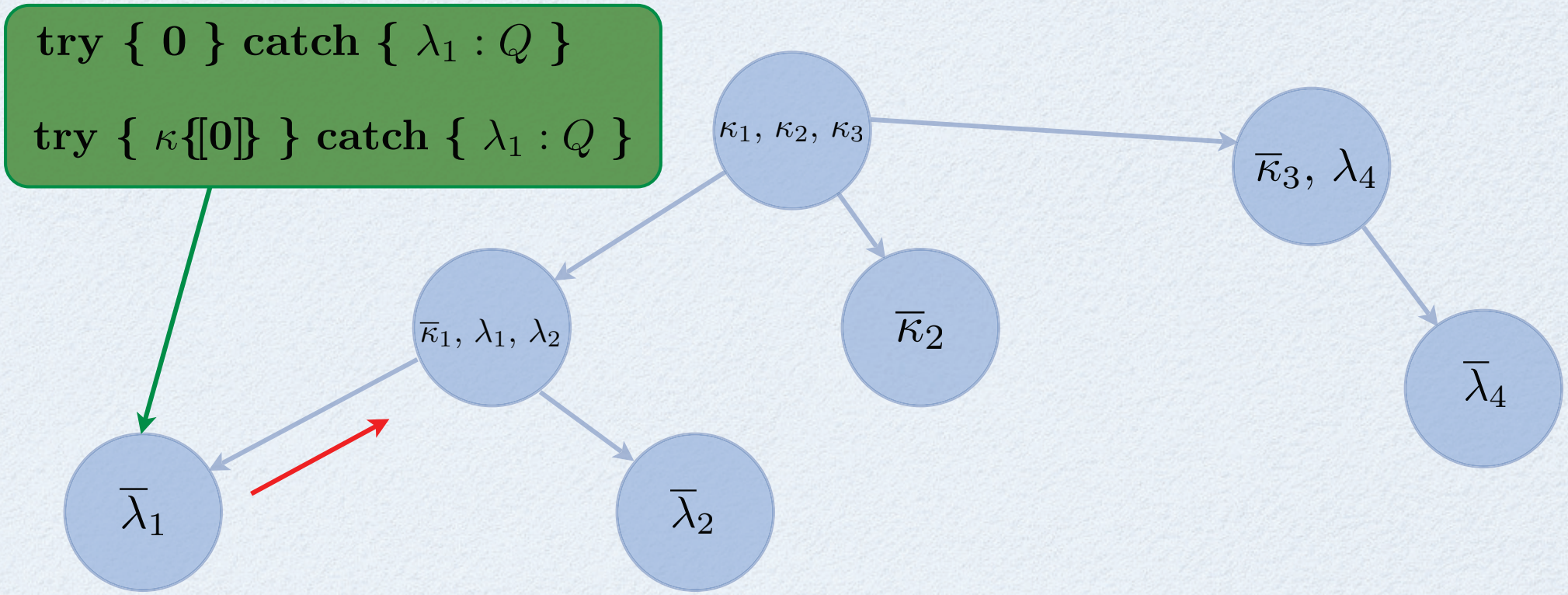
Theorem. The graph above is always a tree (for well-typed processes)

TERMINATION PROTOCOL



1) the leaves communicate their will to terminate to their father i.e. there are no active processes

TERMINATION PROTOCOL



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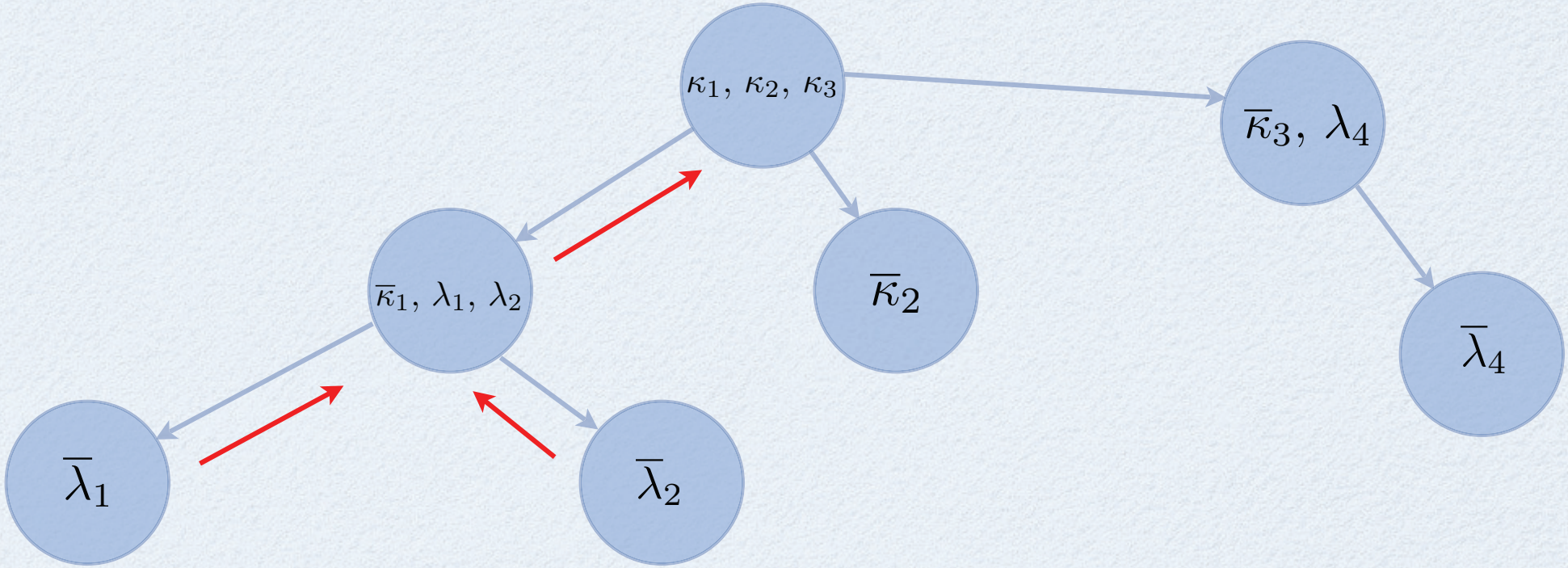
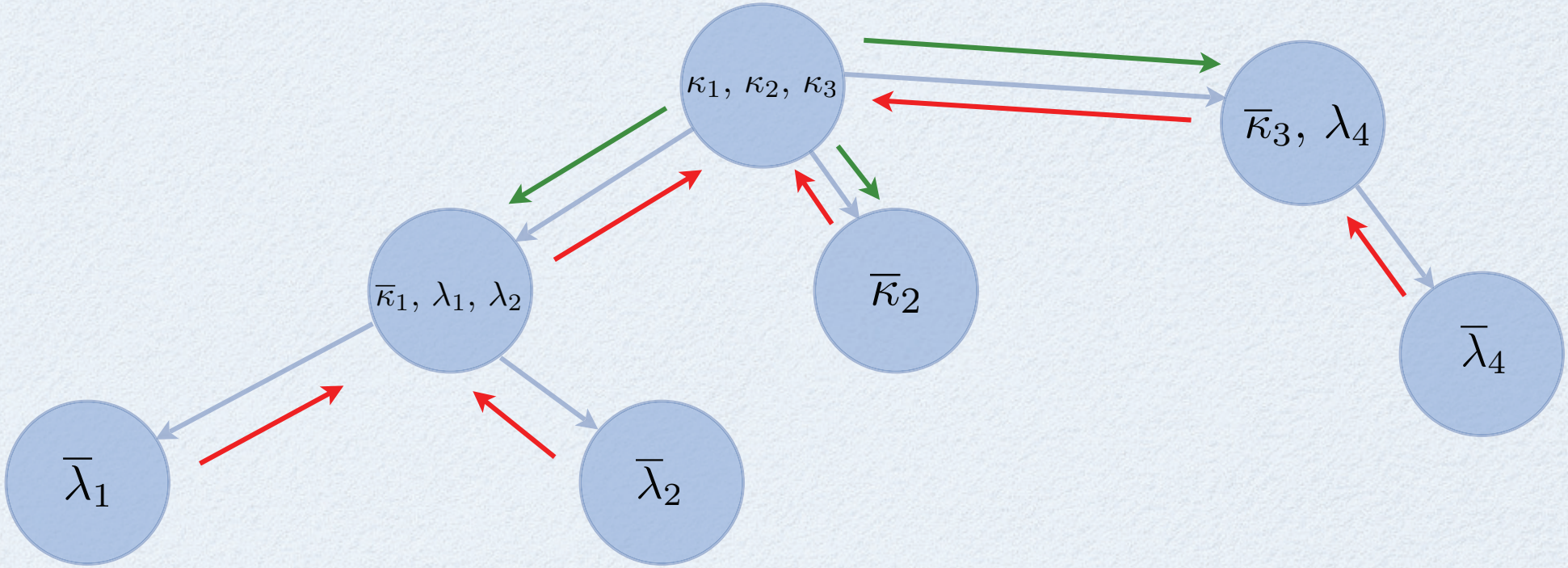


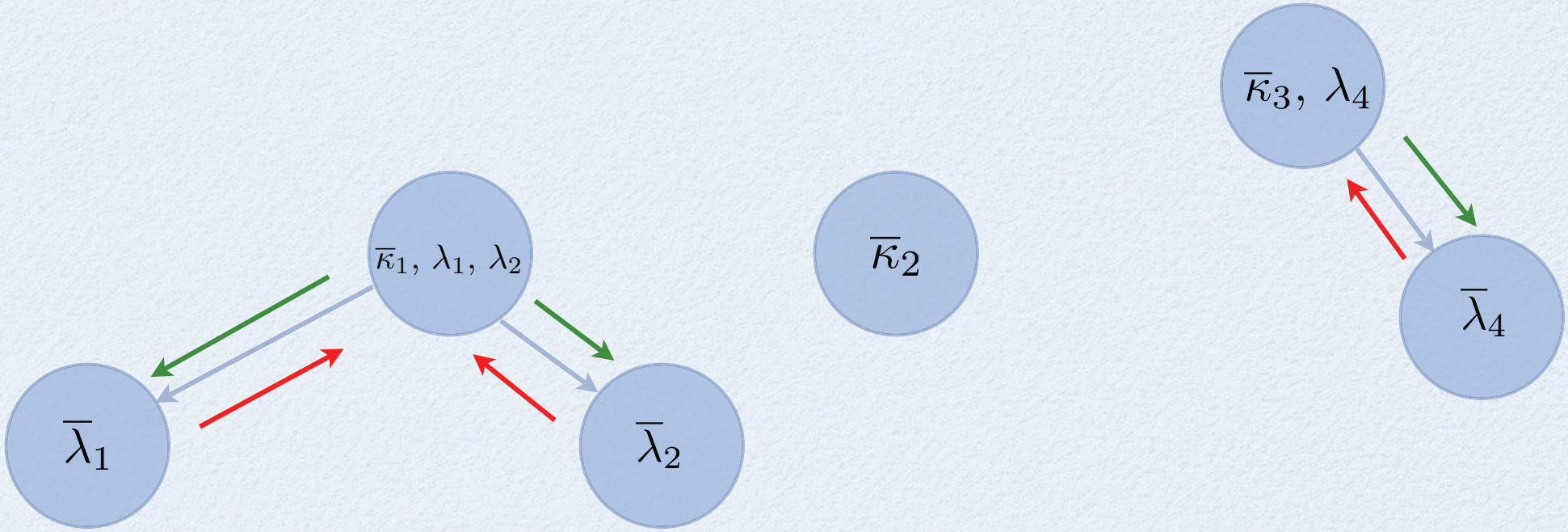
2) when a node receives a request from all children (and it is willing to terminate), it propagates upwards

TERMINATION PROTOCOL



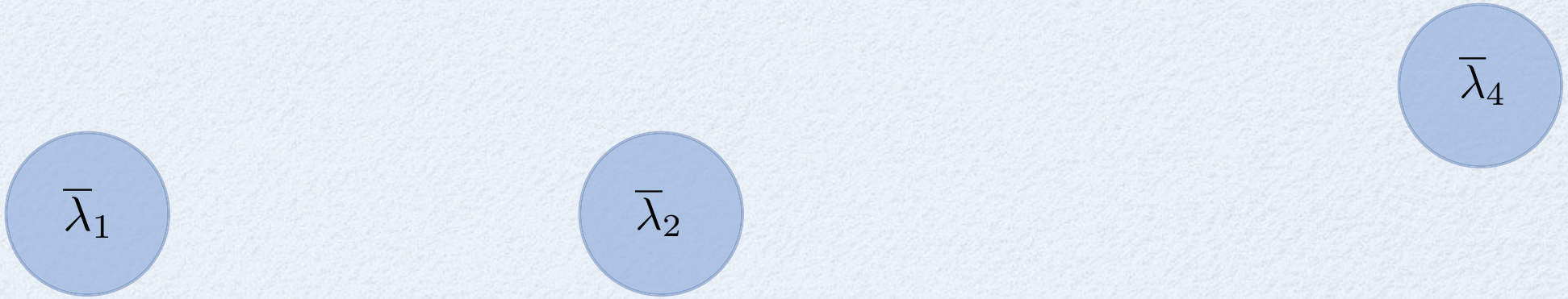
3) when the root collects a request from all its children then it terminates and sends ack to children

TERMINATION PROTOCOL



4) Each node sends acks and terminates.

TERMINATION PROTOCOL



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TERMINATION PROTOCOL

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Liveness

Theorem (Liveness).
If P is typable and $P \rightarrow^* Q$ then either Q is stable or $Q \rightarrow Q'$

*a process is stable whenever there are no pending outputs, inputs or try-catch blocks