SFM-QUANTICOL 2016, Bertinoro, Italy

Spatio-Temporal Model-Checking

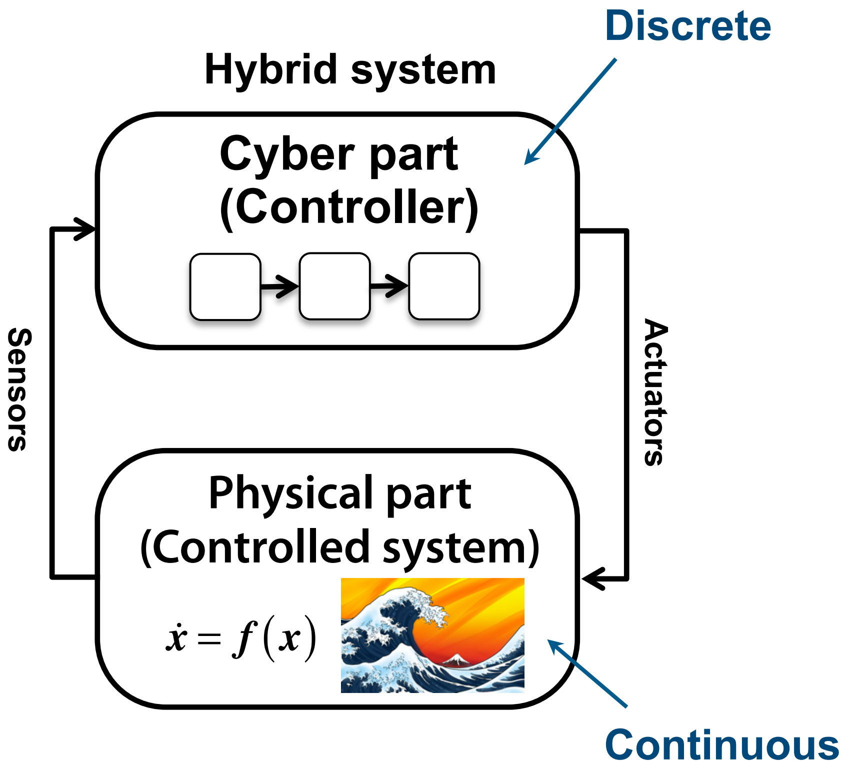
Ezio Bartocci, Radu Grosu

Outline

- **Motivation**
  - ***Verifying Cyber-Physical Systems***
- **Temporal Logics**
  - ***Linear and Signal Temporal Logics***
- **Spatial Superposition Logics**
  - ***Linear Spatial Superposition Logics***
- **Spatio-Temporal Logics**
  - ***SpaTel and Signal Spatio-Temporal Logics***

Motivation

Cyber-Physical Systems (CPS)

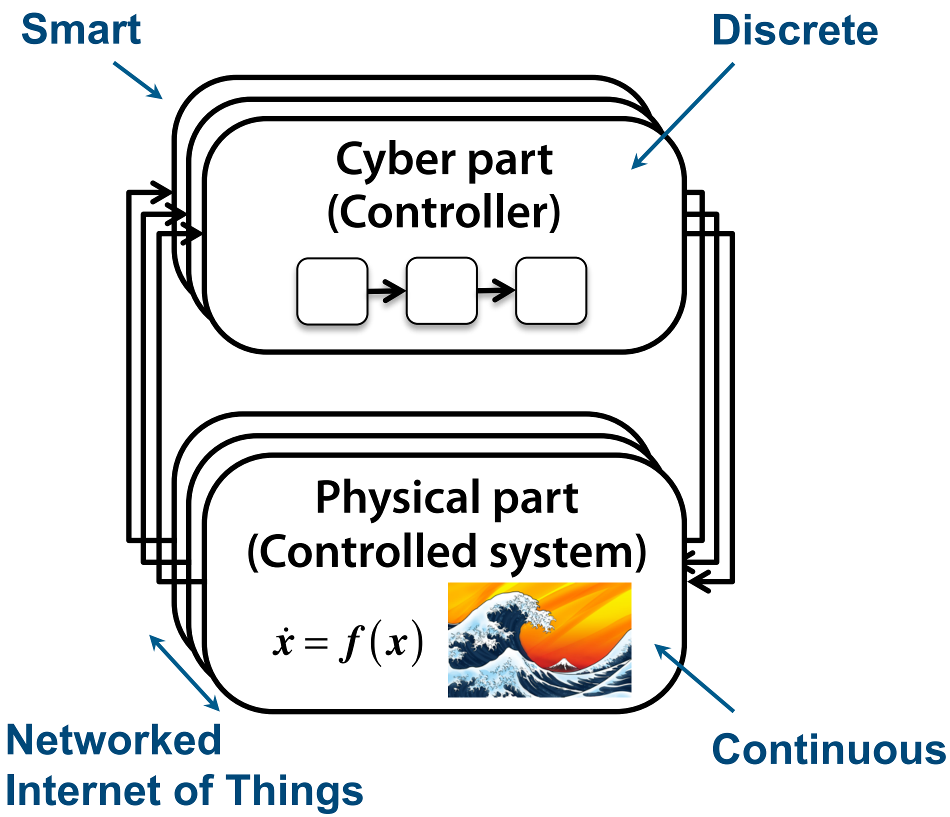


Amazon drone

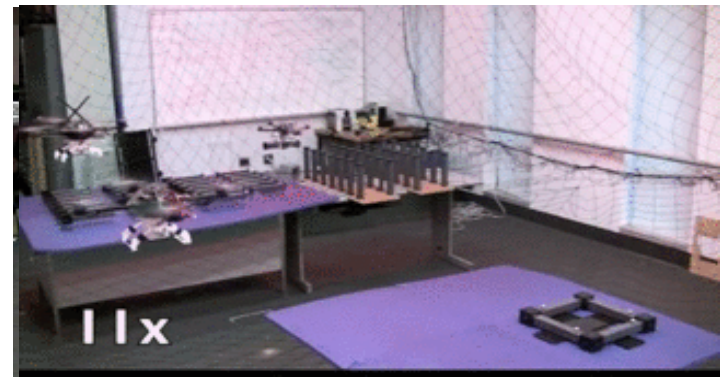
Kiva robots

Google self-driving car

Cyber-Physical Systems (CPS)



CPS collaborate to achieve a common goal !!

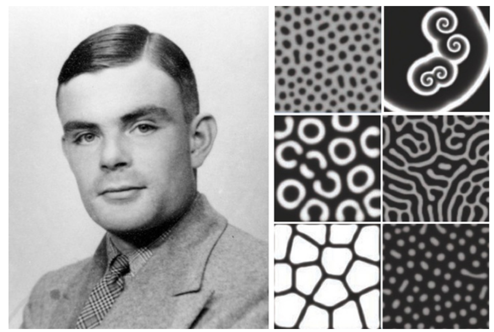


Construction with a swarm of drones

Amazon Kiva Warehouse Automation

Biological CPS

Reaction Diffusion Examples



Parameters

$$\dot{u} = F(u,v) - d_u v + D_u \nabla u$$
$$\dot{v} = G(u,v) - d_v v + D_v \nabla v$$

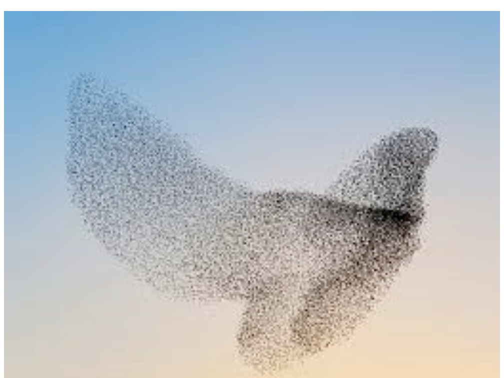
REACTION

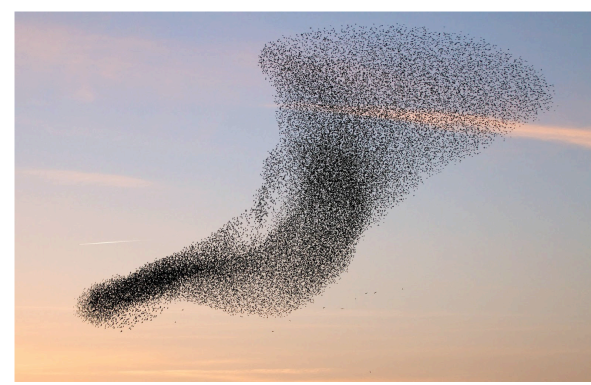
DEGRADATION

DIFFUSION

Turing Diffusion Model

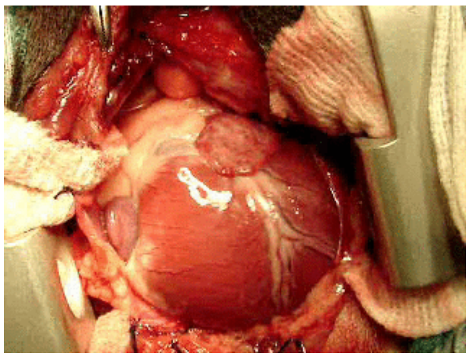
Bird flocking

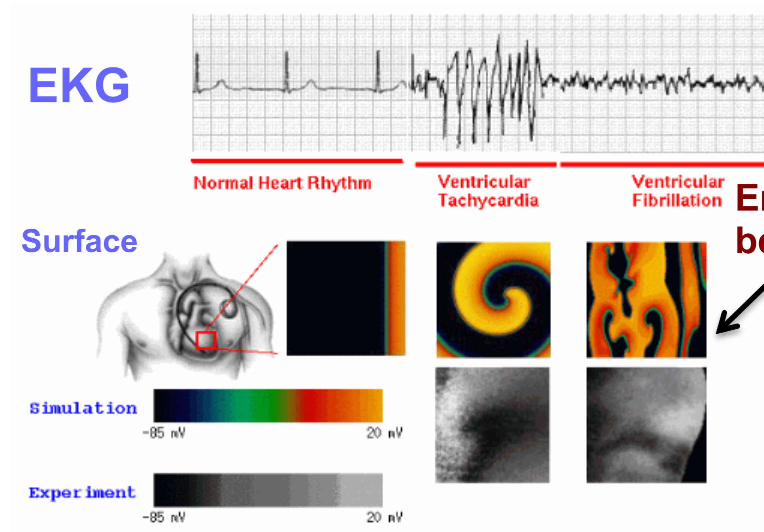




Spatio-temporal behaviors in the heart

A smart electro-mechanical pump engineered by nature



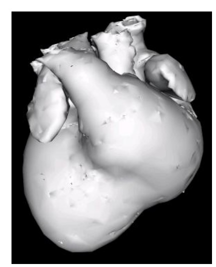


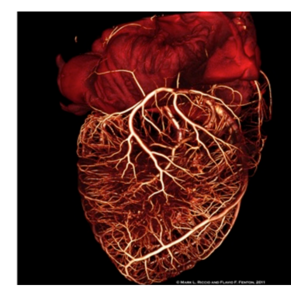
Emergent behaviors

5 billions of cells (nodes):

- communicating over a complex structure
- synchronizing to contract the muscle
- fault-tolerant, self-stabilizing

Anatomy **Fibers** **Vessels**





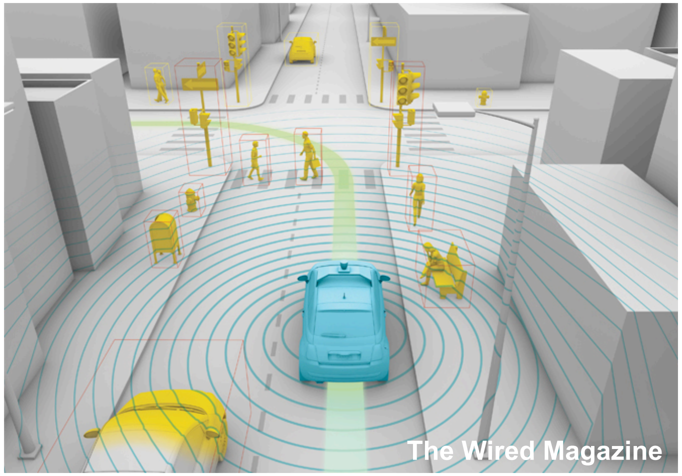
Pittsburgh NMR Center **MicroCT Cornell**

Engineering Safe CPS

How to automatically ensure safety-critical requirements in CPS ?

Exhaustive verification of CPS is increasingly intractable:

- **Openness, environmental change**
- **Uncertainty, spatial distribution**
- **Emergent behaviors resulting from the local interactions are not predictable by the analysis of system's individual parts**
- **Classic state-space explosion problem**



Google Cars

Open Hot Topics:

- **Apply CS methods for optimization & control**
- **Predicting emergent behaviors**

Temporal Logics

Temporal logics in a nutshell

- **Temporal logics**
  - ***Concise and intuitive formal specification languages to specify temporal behaviors***
- **Example: Linear Temporal Logic (LTL)**
  - ***LTL deals with discrete sequences of states***
  - ***Classical logical operators (not, and, or) + temporal operators: "next", "always" (G), "eventually" (F) and "until" (U)***

Linear Temporal Logic (LTL)

A. Pnueli, 1977

Syntax:

$$\varphi := T \mid p \mid \neg\varphi \mid \varphi_1 \vee \varphi_2 \mid \bigcirc\varphi \mid \varphi_1 U \varphi_2$$

Derived operators

$$F\ \varphi = T\ U\ \varphi$$

$$G\ \varphi = \neg F \neg \varphi$$

An LTL formula φ is evaluated on a sequence of events,

e.g.: $w = aaabbaaa\ldots$

At each step of w, we can define a truth value of φ, noted $\chi^{\varphi}(w,i)$

An LTL atoms are symbols: a, b

$$\begin{array}{rcccccccccc} i = & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & \ldots \\ w = & a & a & a & b & b & a & a & a & \ldots \\ \chi^a(w,i) = & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & \ldots \\ \chi^b(w,i) = & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & \ldots \end{array}$$

LTL, Temporal Operators

$\bigcirc$ (next), G (always), F (eventually), U (until)

They are evaluated at each step w.r.t. the future of sequences

	Trace	$w =$	a	a	a	b	b	a	a	a	$\dots$
$\bigcirc b$	(next)	$\chi^{\bigcirc b}(w,i) =$	0	0	1	1	0	0	0	?	$\dots$
G a	(always)	$\chi^{\mathsf{G}a}(w,i) =$	0	0	0	0	0	1?	1?	1?	$\dots$
F b	(eventually)	$\chi^{\mathsf{F}b}(w,i) =$	1	1	1	1	1	0?	0?	0?	$\dots$
a $\mathbf{U}$ b	(until)	$\chi^{a\mathbf{U}b}(w,i) =$	1	1	1	0	0	0?	0?	0?	$\dots$

LTL, Temporal Operators

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$a\ \mathbf{U}\ b$	(until)	$\chi^{a\mathbf{U}b}(w,i) =$	1	1	1	0	0	0?	0?	0?	$\dots$

χ is acasual: it depends on future events

Finite sequences semantics allows to define a unique value $\forall(w,i)$

Notation: $w \models \varphi \Leftrightarrow \chi^{\varphi}(w,0) = 1$

Verification

Suppose w are execution traces of a system M

System M $\rightarrow$ $aaaabbbaa\ldots$ $\rightarrow$ Property φ $\rightarrow$ $111000\ldots$

Model-checking: proving that $M \models \varphi$

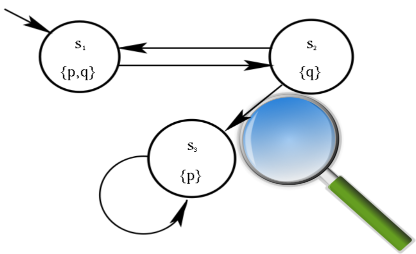
where $M \models \varphi \Leftrightarrow \forall w \in traces(M),\ \chi^{\varphi}(w,0) = 1$

Monitoring: computing $\chi^{\varphi}(w,0)$ for finite sets of w

Statistical Model-Checking

Computing statistics on $\chi^{\varphi}(w,0)$ for population of w

Model Checking and Monitoring



- *White-box Systems:*
  - ✓ ***We need a system model***
- *It deals with infinite words:*
  - ✓ ***It is exhaustive***
- *Very computational expensive:*
  - ✓ **State Explosion Problem**
- *It can be used for certification*
  - ✓ **It return a counterexample**

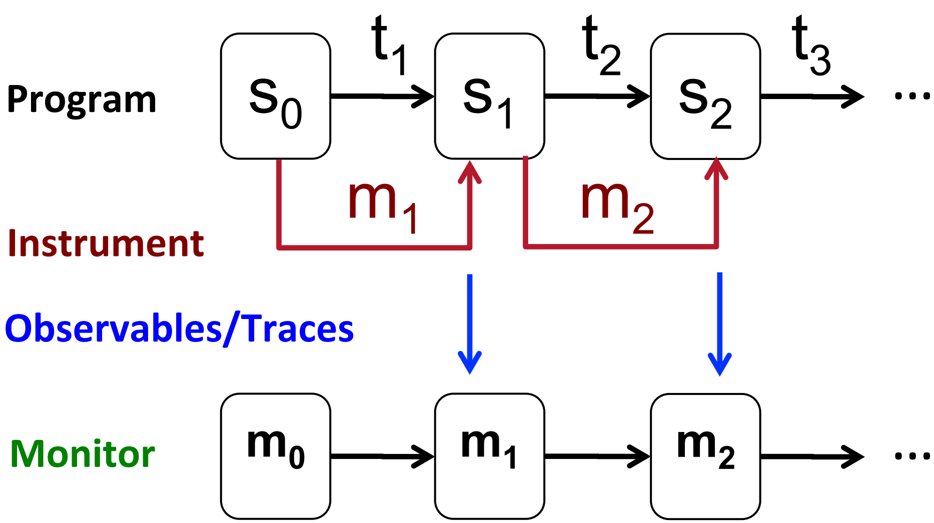
- *Black-box Systems:*
  - ✓ ***We just need the system running !!***
  - ✓ ***No legacy issues***
- *It deals with finite (expanding) words*
  - ✓ ***It is not exhaustive***
- *Lightweight*
  - ✓ ***The complexity of monitor generation is less important than the complexity of monitoring***
- *It can be used both for testing and to trigger safe mechanisms*

Monitoring

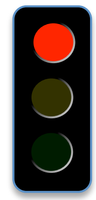
Problem definition:

Given a program P, an execution trace τ of P, and a property ϕ, decide whether τ satisfies ϕ.

Monitoring Process



A monitor reads a finite trace and return a verdict (True, False, Not known yet)



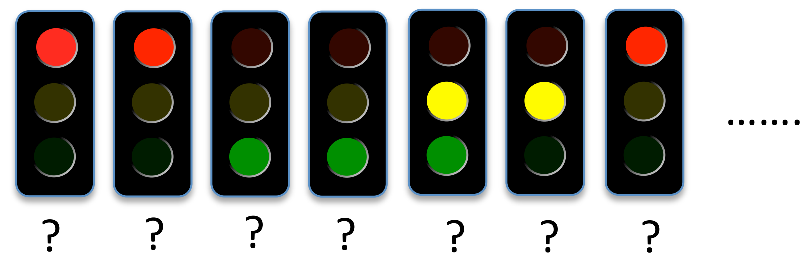
Example: Traffic Light

Property:

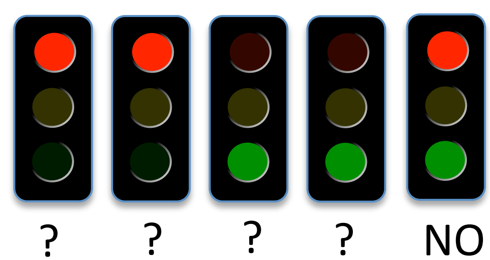
Always if the light is green implies no red light until yellow

$$\varphi := \Box(green \rightarrow \neg red \ \mathbf{U} \ \mathbf{yellow})$$

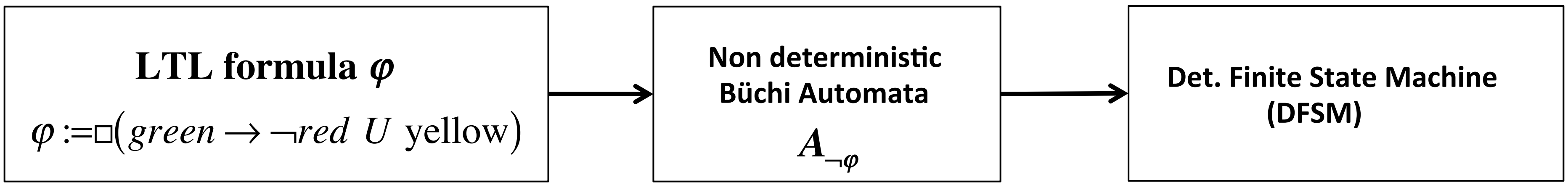
Safe Behavior:



Unsafe Behavior:



From LTL to DFSM



Wolper, Vardi 1986

Complexity: size of monitor $|M| \leq 2^{|\varphi|}$

Literature

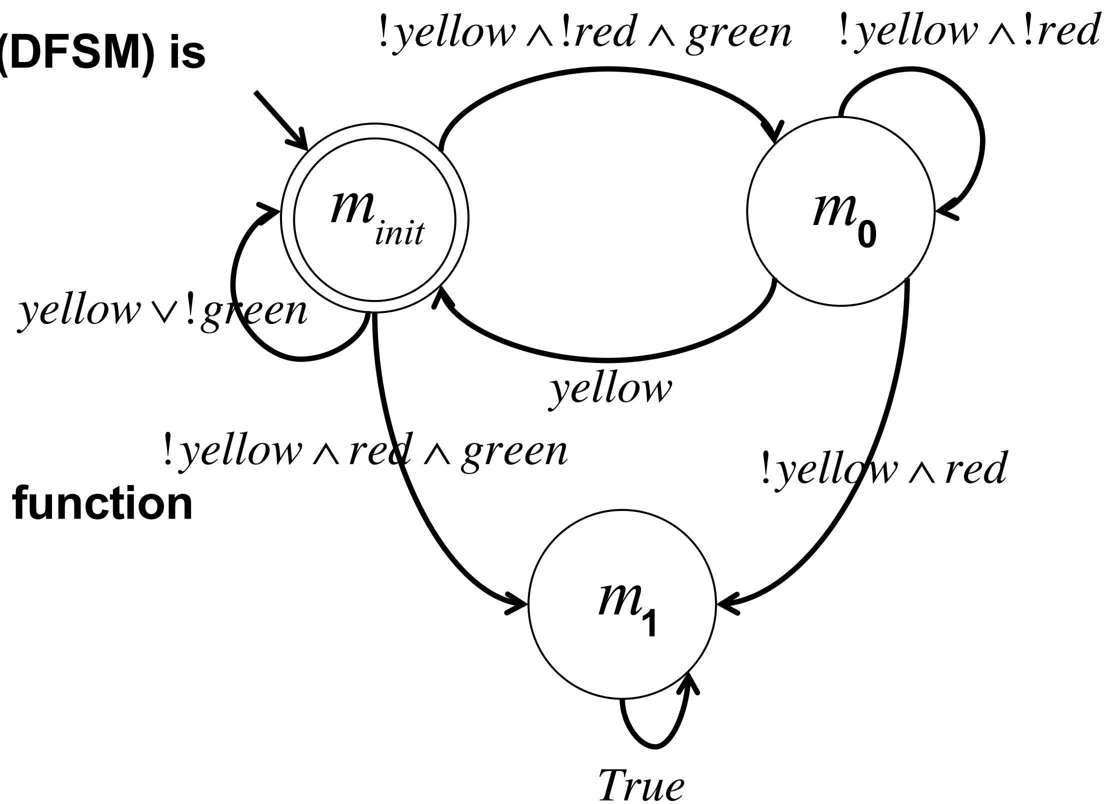
P. Wolper (2001): **Constructing Automata from Temporal Logic Formulas: A Tutorial,** Lectures on formal methods and performance analysis, LNCS 2090.

M. Geilen (2001)**: On the Construction of Monitors for Temporal Logic Properties**, Electr. Notes Theor. Comput. Sci., 55(2), pp. 181–199,

Deterministic Finite State Machine

A deterministic finite state machine (DFSM) is a tuple $M = \langle S_M, m_{init}, V, \delta, F \rangle$ where:

- S_M **is the set of states**
- $m_{init} \in S_M$ **is the initial state**
- **V is the alphabet**
- $\delta : S_M \times V \rightarrow S_M$ **is the transition function**
- **F is the set of accepting states**



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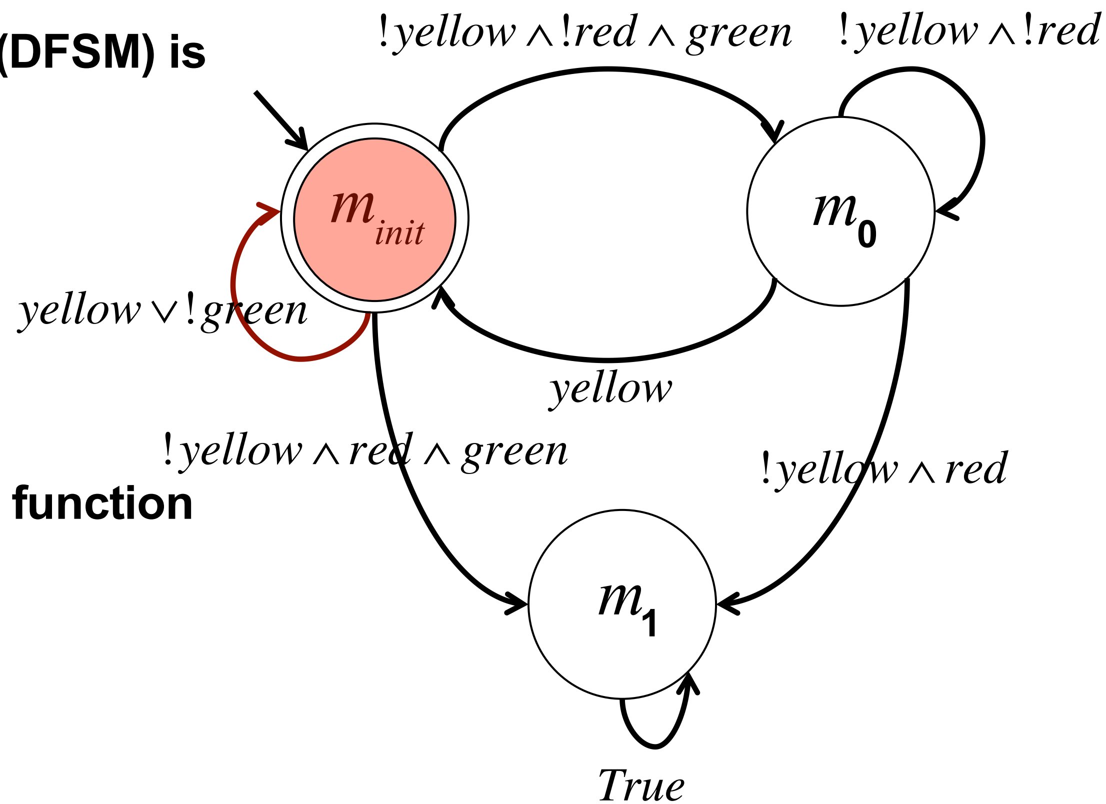

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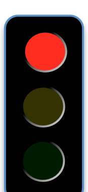

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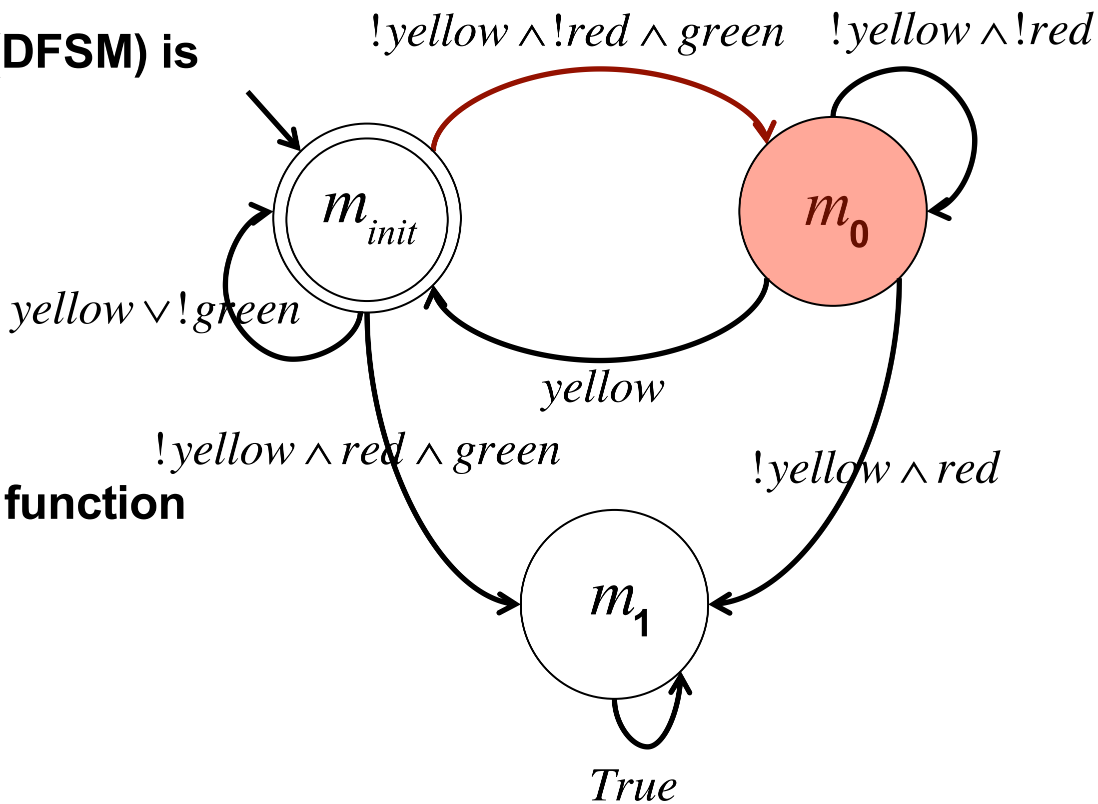

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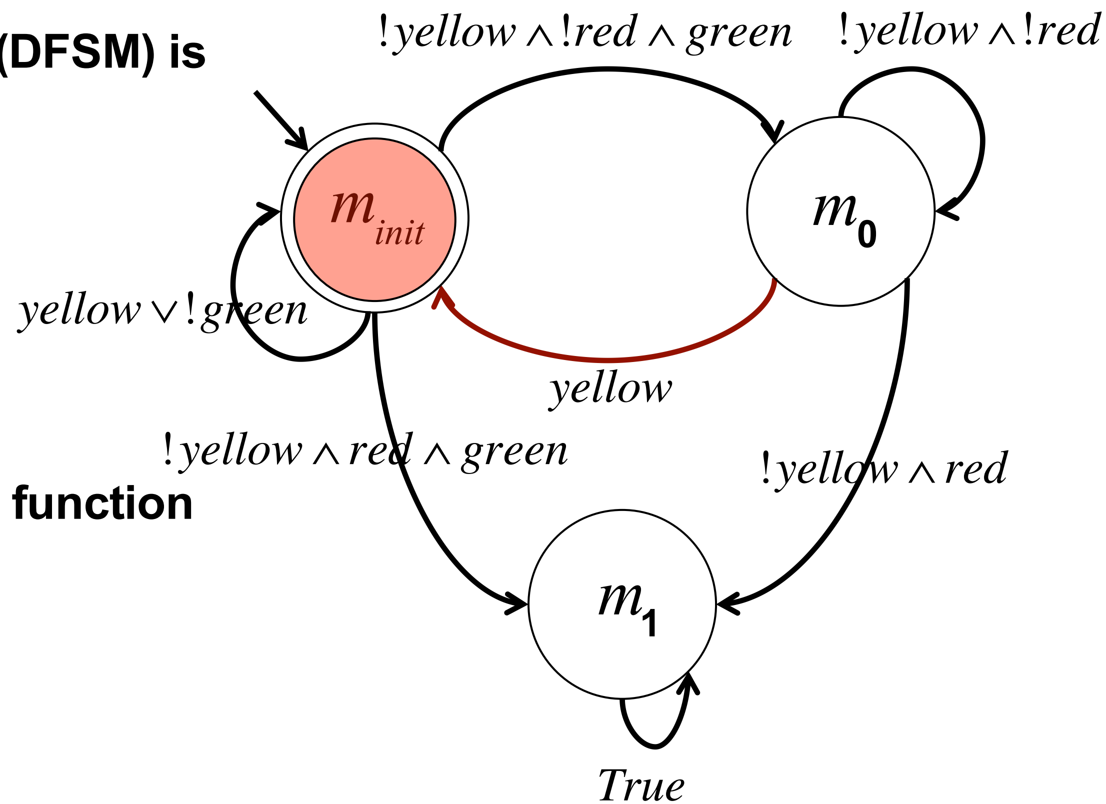

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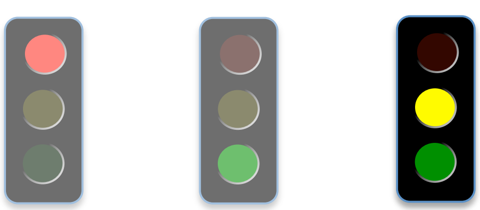

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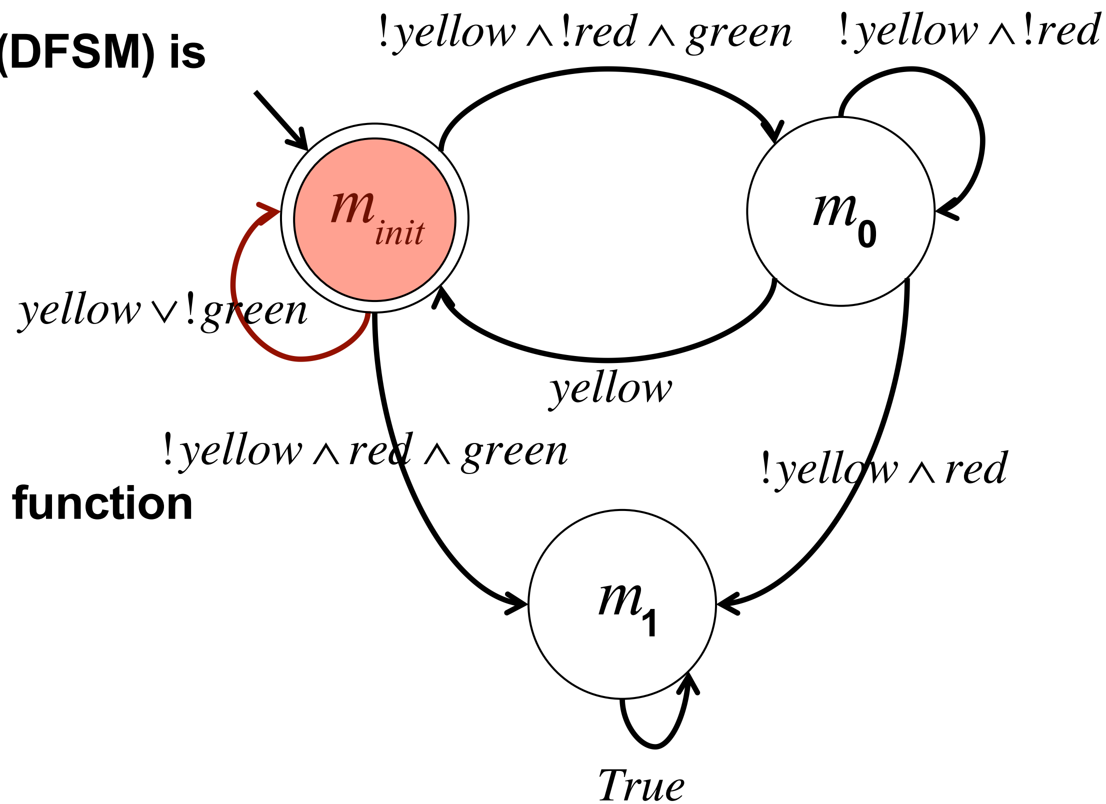

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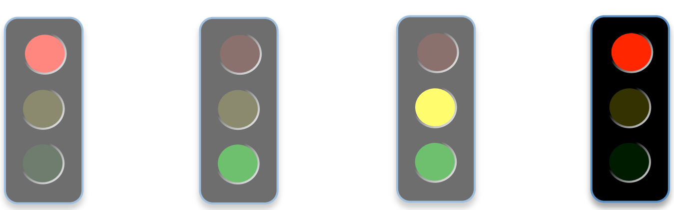

....

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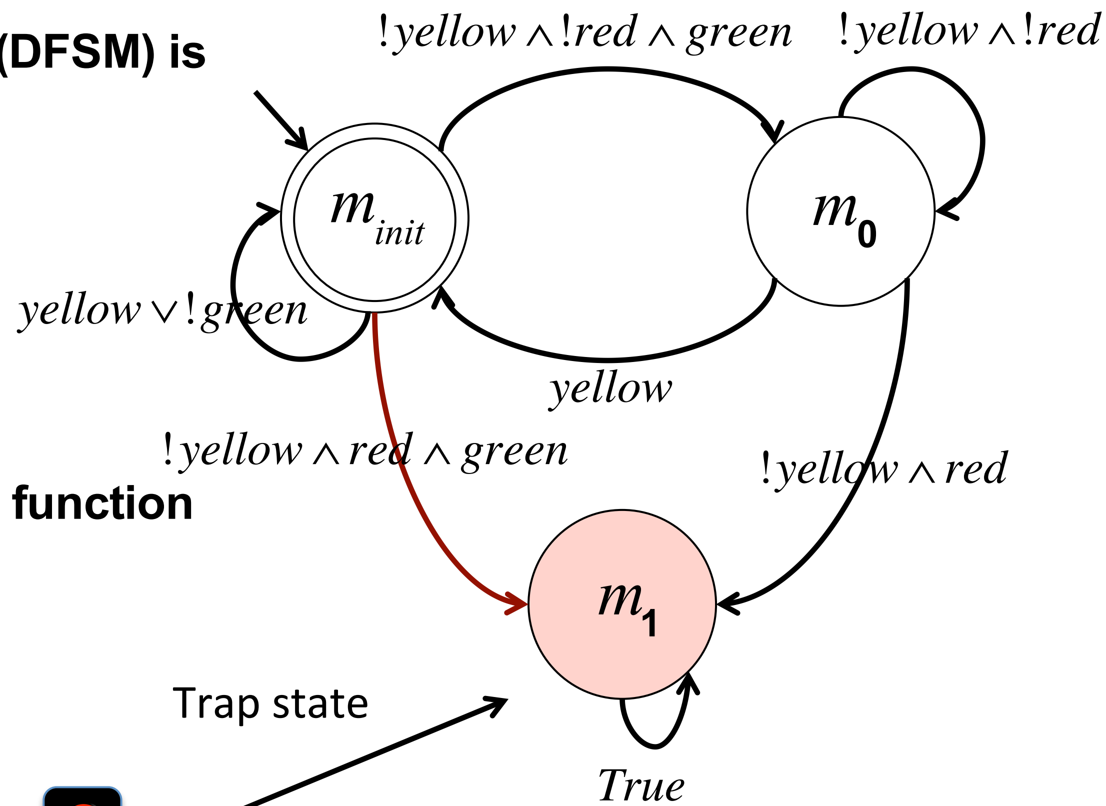
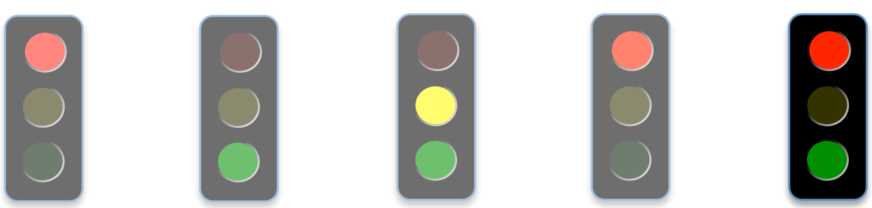
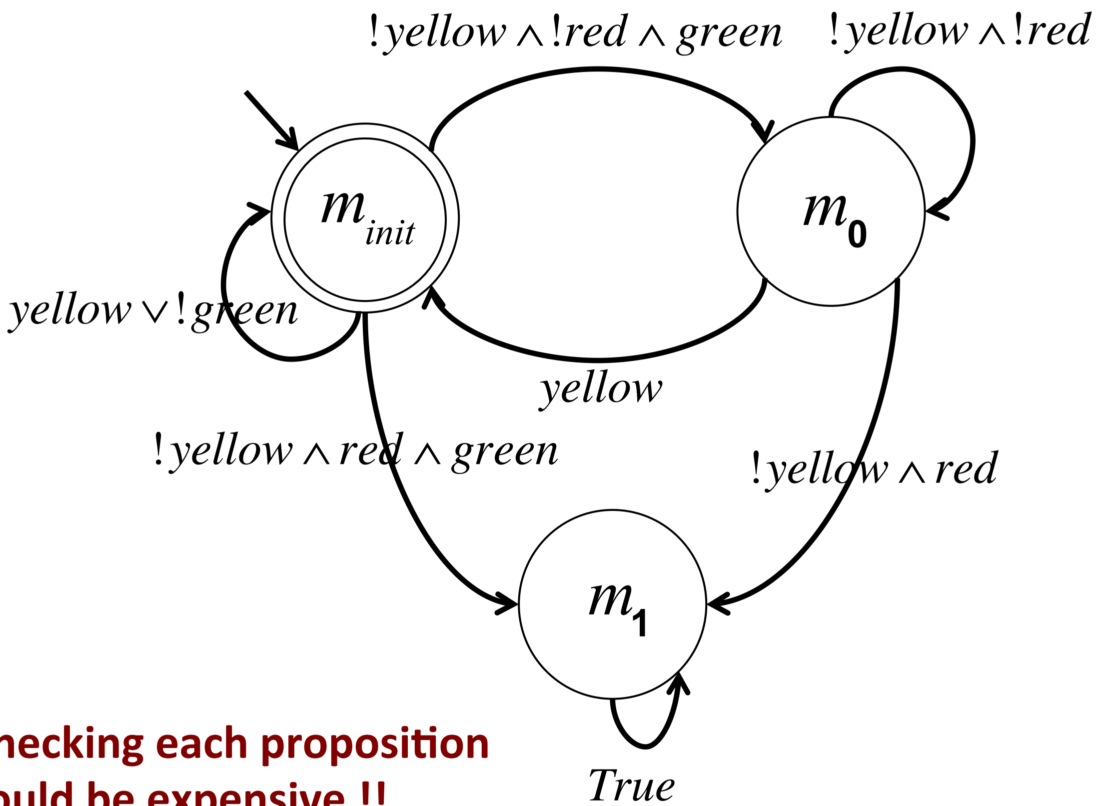


Unsafe Behavior:

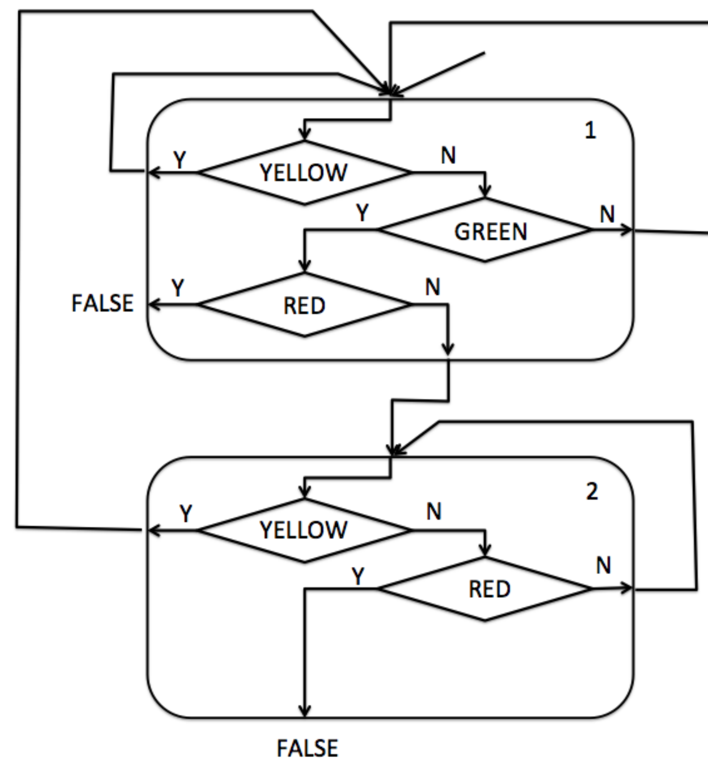


$$\varphi := \Box(green \rightarrow \neg red \ \mathrm{U}\ \mathrm{yellow})$$

Efficient DFSM: BTT-FSM



Binary Transition Tree Finite State Machine



Checking each proposition could be expensive !!

Literature

M. d'Amorim, G. Rosu: **Efficient Monitoring of omega-Languages**. CAV 2005: 364-378

LTL with Past

Syntax:

$$\varphi := T \mid p \mid \neg\varphi \mid \varphi_1 \vee \varphi_2 \mid \bigcirc\varphi \mid \varphi_1 \mathrm{U} \varphi_2 \mid \odot\varphi \mid \varphi_1 \mathrm{S} \varphi_2$$

next ($\bigcirc$), *until* (U), *previous* ($\odot$), *since* (S)

Semantics of the Past operators:

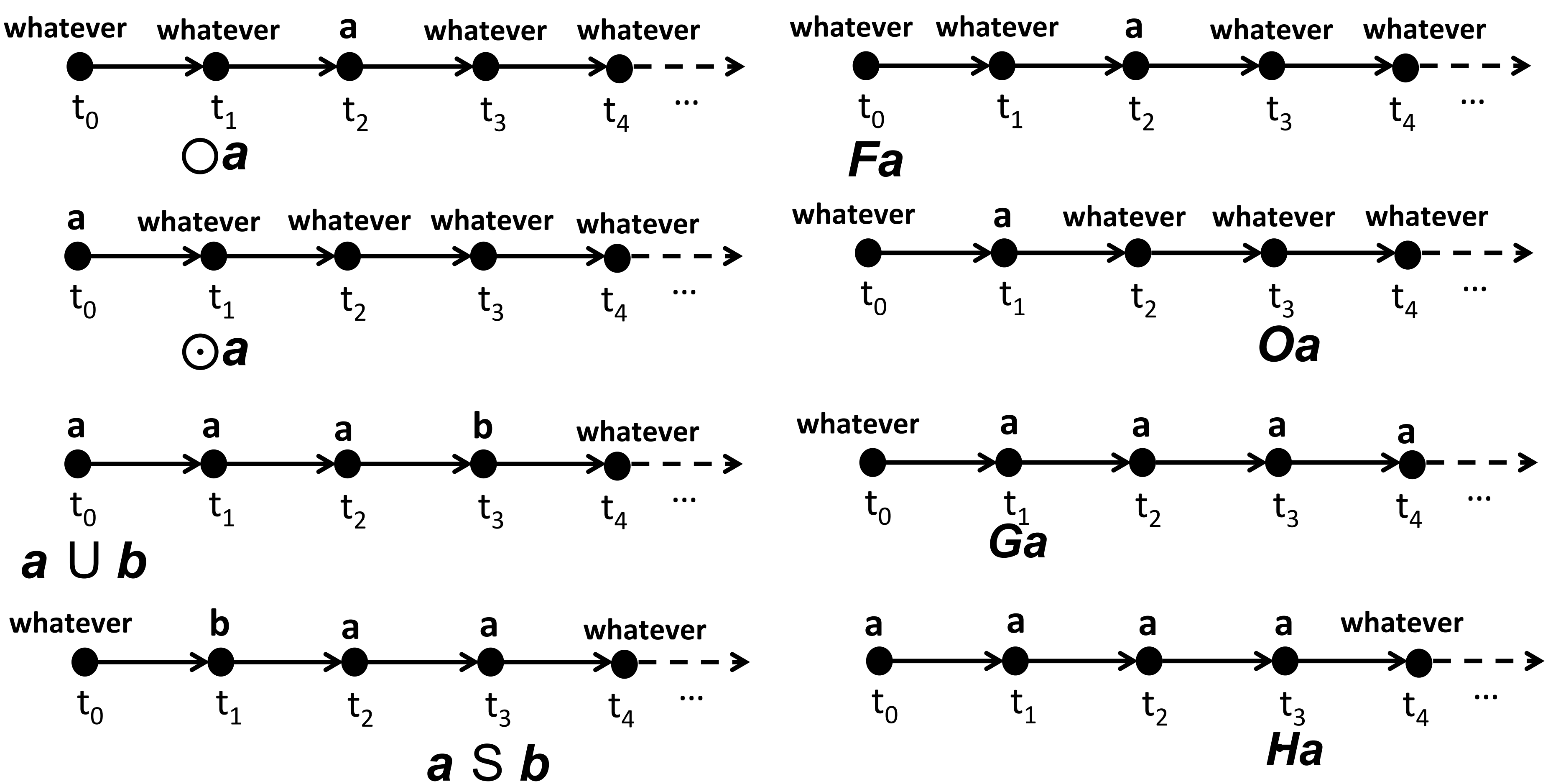
$$(\xi, t) \models \odot\varphi \quad \leftrightarrow \quad t > 0 \text{ and } (\xi, t-1) \models \varphi$$

$$(\xi, t) \models \varphi_1 \ \mathrm{S}\ \varphi_2 \quad \leftrightarrow \quad \exists t' :\ 0 \le t' < t, (\xi, t') \models \varphi_2 \text{ and } \forall t'' :\ 0 \le t'' < t, (\xi, t'') \models \varphi_1$$

Derived Temporal Operator:

$F\varphi = T\ \mathrm{U}\ \varphi$	$G\varphi = \neg F \neg \varphi$	$O\varphi = T\ \mathrm{S}\ \varphi$	$H\varphi = \neg O \neg \varphi$
Eventually	**Globally**	**Once**	**Historically**

LTL with Past



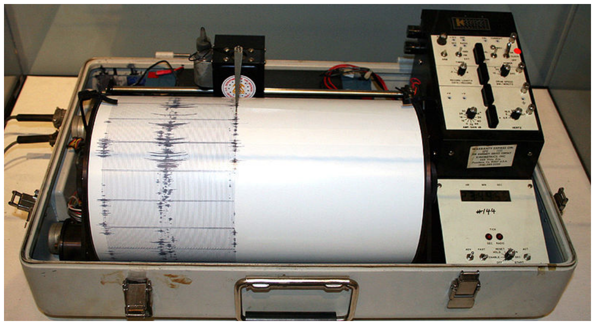
Beyond LTL

- **The use of LTL has been very successful in formal verification and synthesis of hardware digital circuits and software**

- **However, the expressivity of LTL is rather limited to discrete-time systems than to hybrid (discrete-continuous) systems**

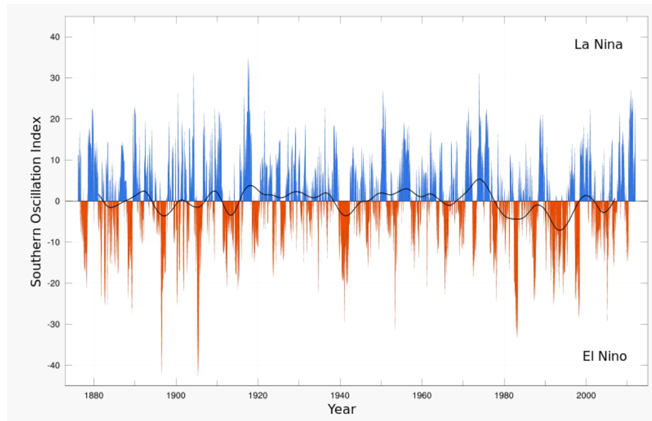
Monitoring Signals

From the Earth



Seismometer

From the Climate Changes

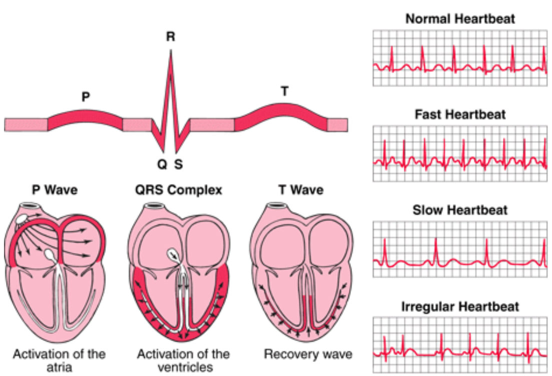


El Niño/La Niña-Southern Oscillation

From the Economy

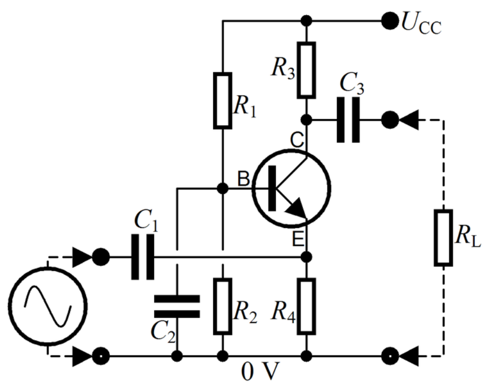
Stock Market

From the Heart



ECG

From Circuits



Amplifier

From Music

Music Sheet

From LTL to Signal Temporal Logic

- Extending LTL with real-time and real-valued constraints

- Example: request-grant property

Linear Temporal Logic
Boolean predicates, discrete-time

$$G\ (a \Rightarrow \mathsf{F}\ \mathsf{b})$$

Metric Temporal Logic
Boolean predicates, real-time

$$G\ (a \Rightarrow \mathsf{F}_{[0,0.5s]}\ \mathsf{b})$$

Signal Temporal Logic
Predicates over real values, real-time

$$G\ (x[t] > 0 \Rightarrow \mathsf{F}_{[0,0.5s]}\ y[t] > 0)$$

Signal Temporal Logic

MTL/STL Formulas

$$\varphi := \top \mid \mu \mid \neg\varphi \mid \varphi \wedge \psi \mid \varphi\ \mathbf{U}_{[a,b]}\ \psi$$

- $\bot = \neg\top$
- Eventually is $\mathsf{F}_{[a,b]}\ \varphi = \top\ \mathcal{U}_{[a,b]}\ \varphi$
- Always is $\mathsf{G}_{[a,b]}\varphi = \neg(\ \mathsf{F}_{[a,b]}\ \neg\varphi)$

STL Predicates

STL adds an analog layer to MTL. Assume signals $x_1[t],\ x_2[t],\ \ldots, x_n[t]$, then atomic predicates are of the form:

$$\mu = f(x_1[t], \ldots, x_n[t]) > 0$$

STL Semantics

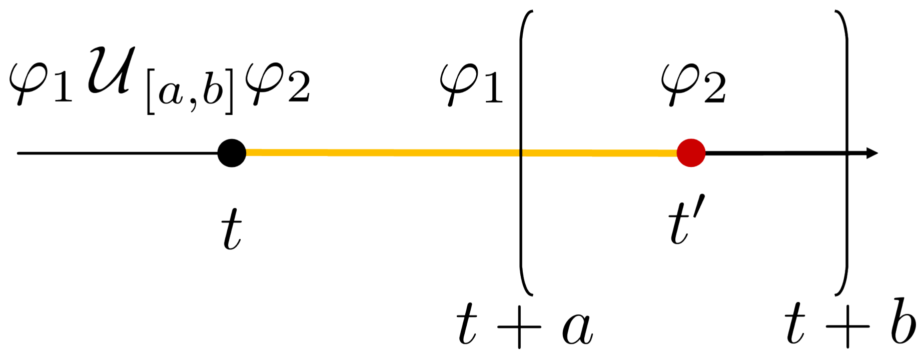
The validity of a formula φ w.r.t. a signal $x=(x_1,\ldots,x_n)$ at time t:

$$(\mathbf{x},t) \models \mu \quad \Leftrightarrow \quad f(x_1[t],\ldots,x_n[t]) > 0$$

$$(\mathbf{x},t) \models \varphi \wedge \psi \quad \Leftrightarrow \quad (x,t) \models \varphi \wedge (x,t) \models \psi$$

$$(\mathbf{x},t) \models \neg\varphi \quad \Leftrightarrow \quad \neg((x,t) \models \varphi)$$

$$(\mathbf{x},t) \models \varphi\, \mathcal{U}_{[a,b]}\, \psi \quad \Leftrightarrow \quad \exists t' \in [t+a, t+b] \text{ such that } (x,t') \models \psi \wedge \forall t'' \in [t,t'],\ (x,t'') \models \varphi\}$$



STL Semantics

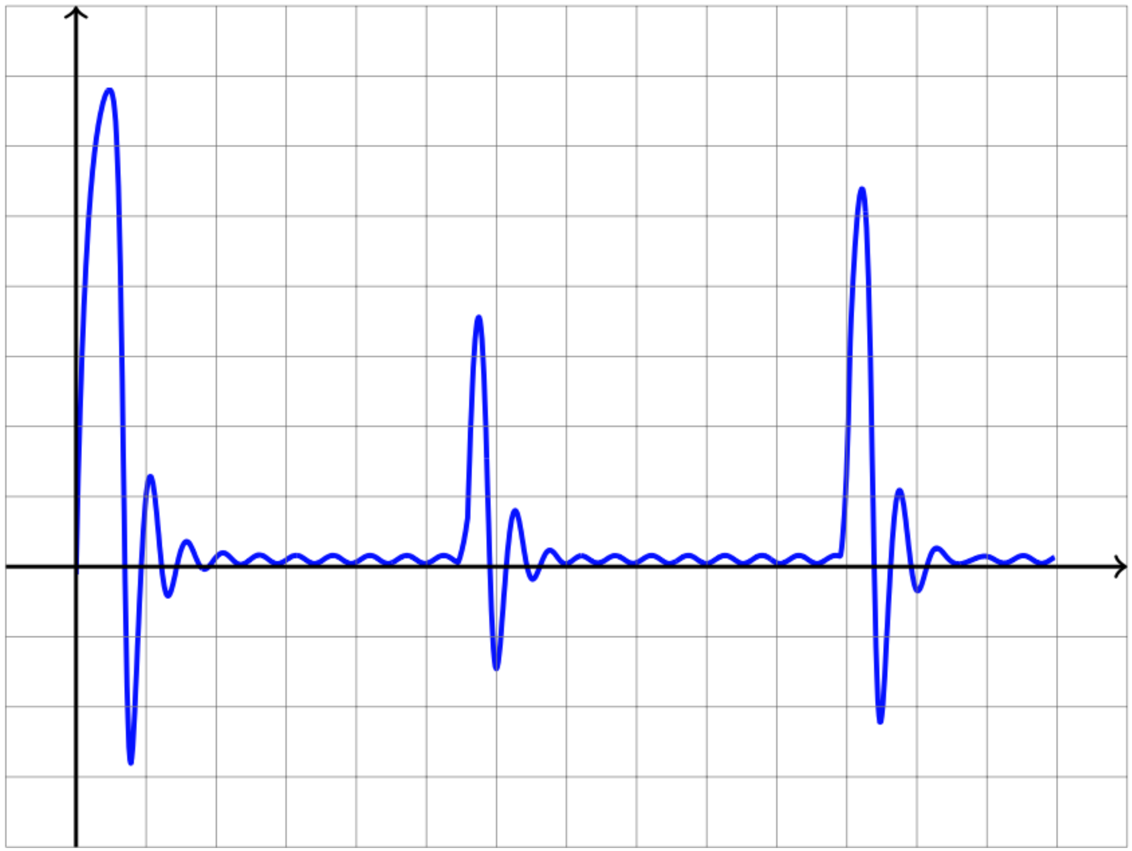
- Eventually is $\mathsf{F}_{[a,b]}\ \varphi = \top\ \mathcal{U}_{[a,b]}\ \varphi$

  $$(\mathbf{x}, t) \models\ \mathsf{F}_{[a,b]}\ \psi \Leftrightarrow \exists t' \in [t+a, t+b] \text{ such that } (x, t') \models \psi$$

- Always is $\mathsf{G}_{[a,b]}\varphi = \neg(\ \mathsf{F}_{[a,b]}\ \neg\varphi)$

  $$(\mathbf{x}, t) \models \mathsf{G}_{[a,b]}\psi \Leftrightarrow \forall t' \in [t+a, t+b] \text{ such that } (x, t') \models \psi$$

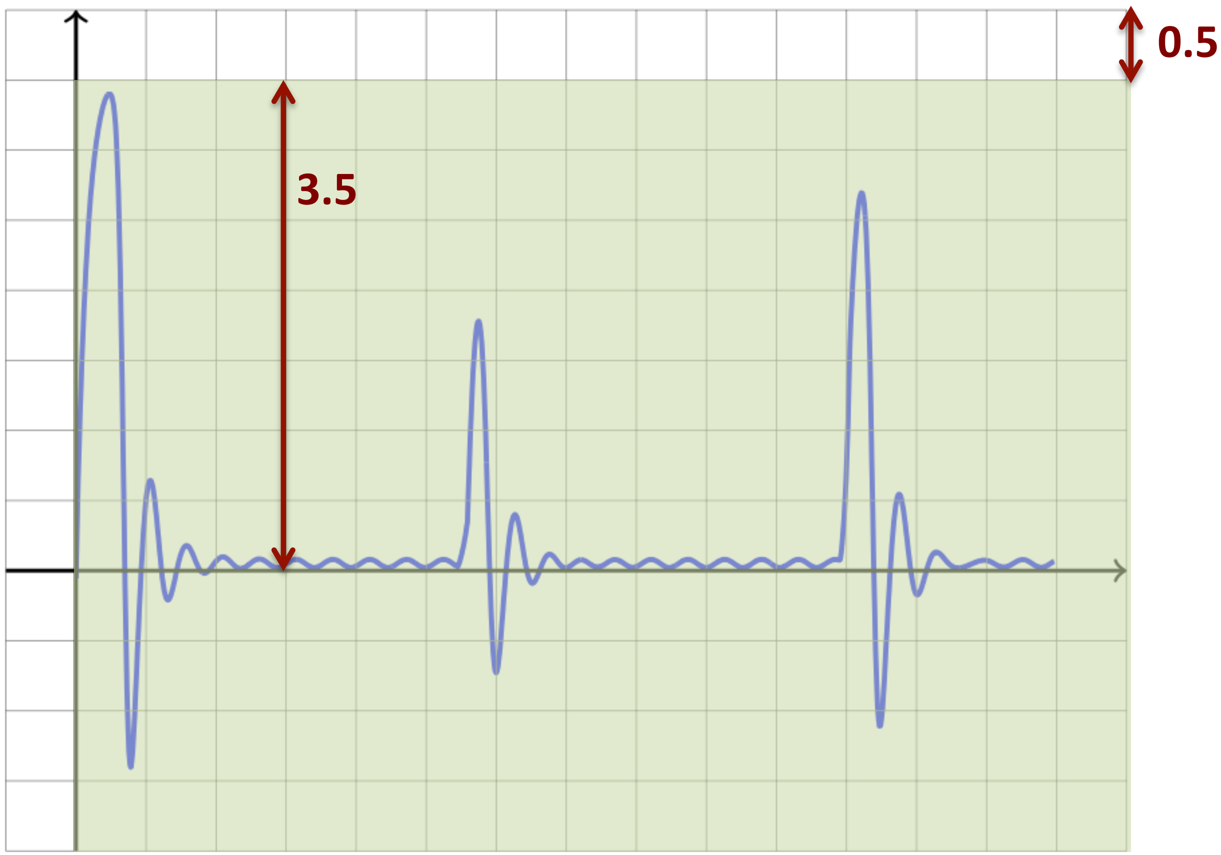
STL Examples



STL Examples

The signal is never above 3.5

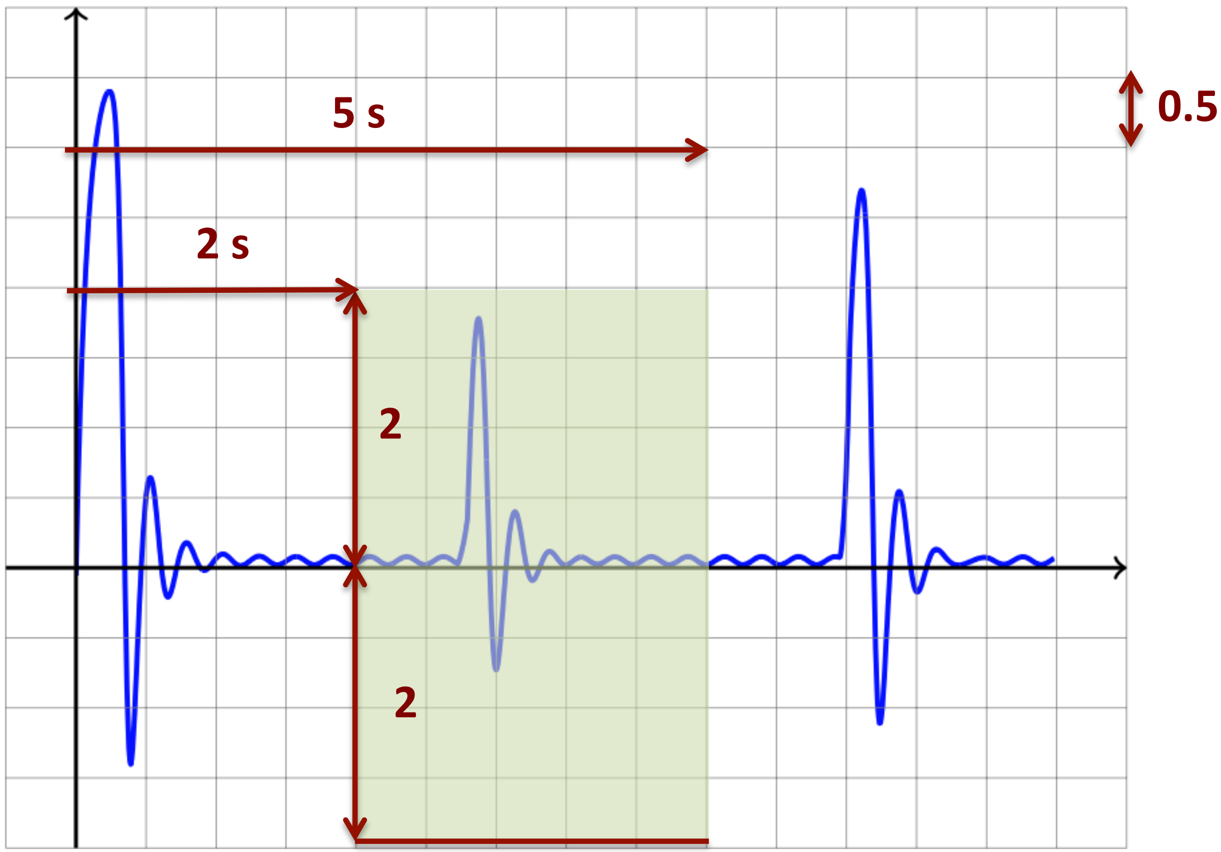
$$\varphi := \mathrm{G}(|x[t]| < 3)$$



STL Examples

Between 2s and 5s the signal is between -2 and 2

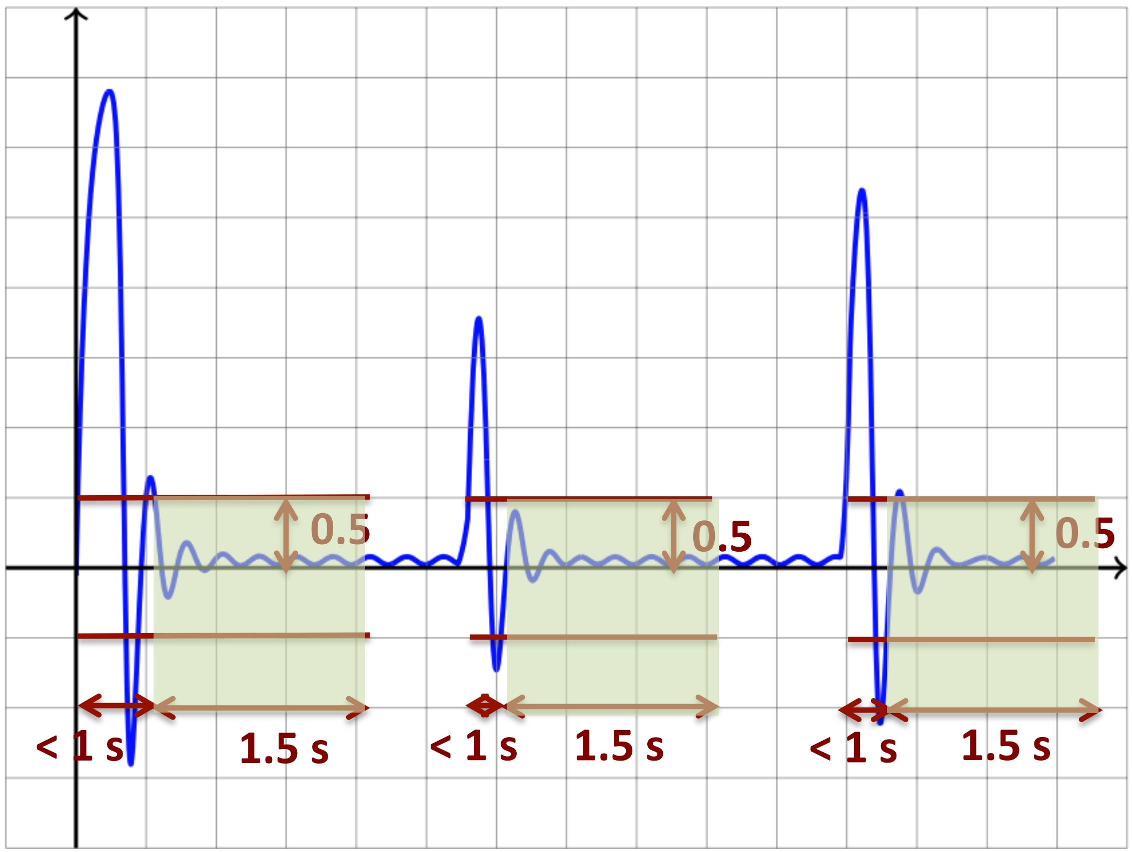
$$\varphi := G_{[2,5]}\left(\left|x[t]\right| < 2\right)$$



STL Examples

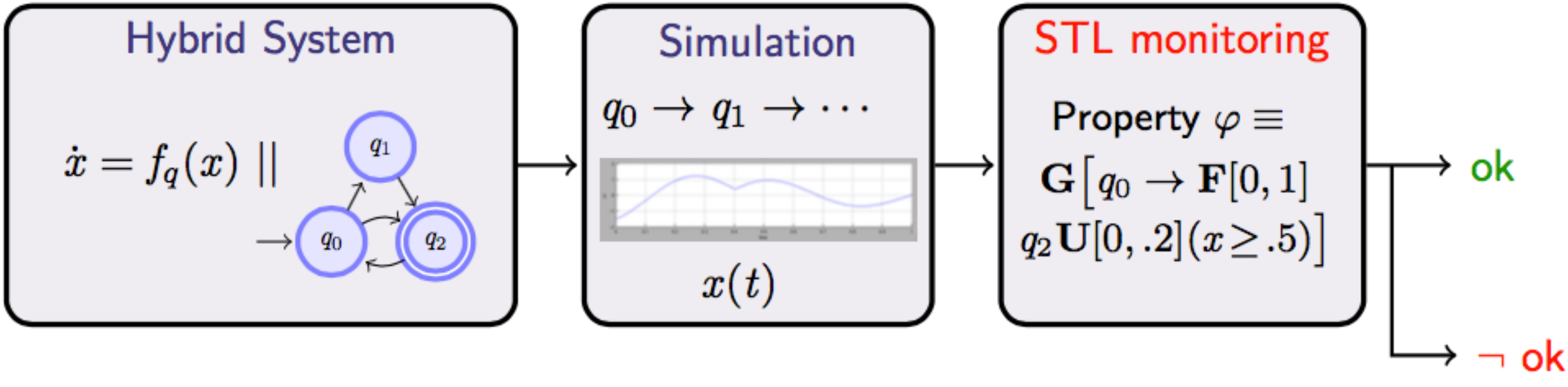
Always |x| > 0.5 ➔ within 1 s, |x| settles between 0.5 and 1.5 s

$$\varphi \coloneqq \mathrm{G}\left(\left(\|x[t]\| > 0.5\right) \rightarrow F_{[0,\,1]}\left(G_{[0,\,1.5]} x[t] < 0.5\right)\right)$$



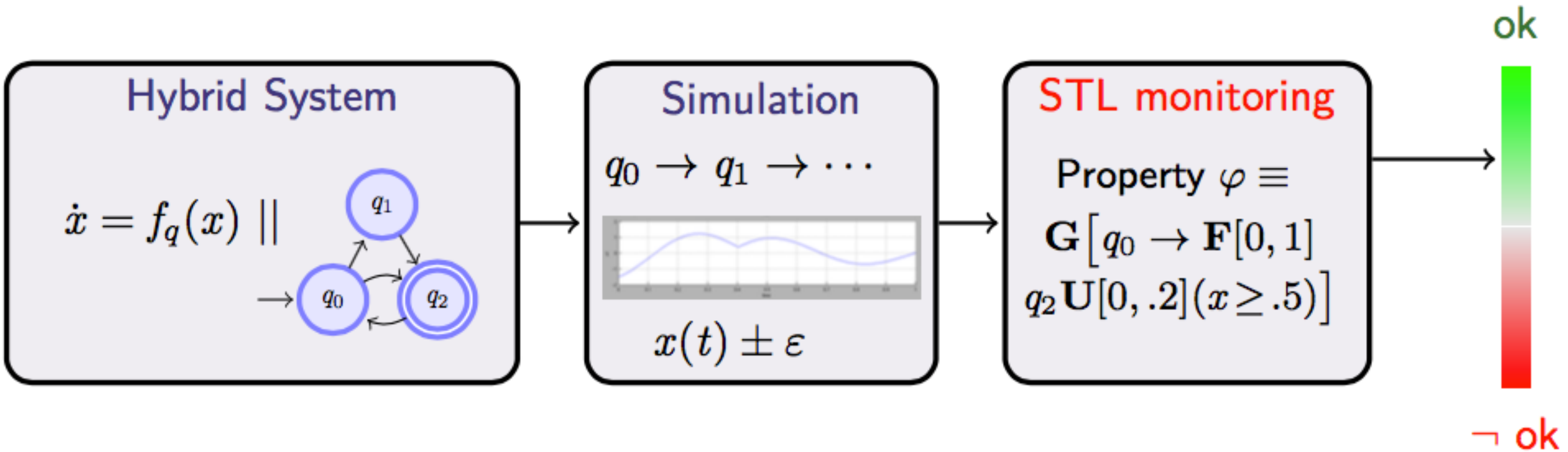
Model-Checking STL

- Models are generally hybrid systems producing hybrid traces
- Model-Checking is limited to restrictive cases
- *Monitoring simulated traces is more practical*



Model-Checking STL

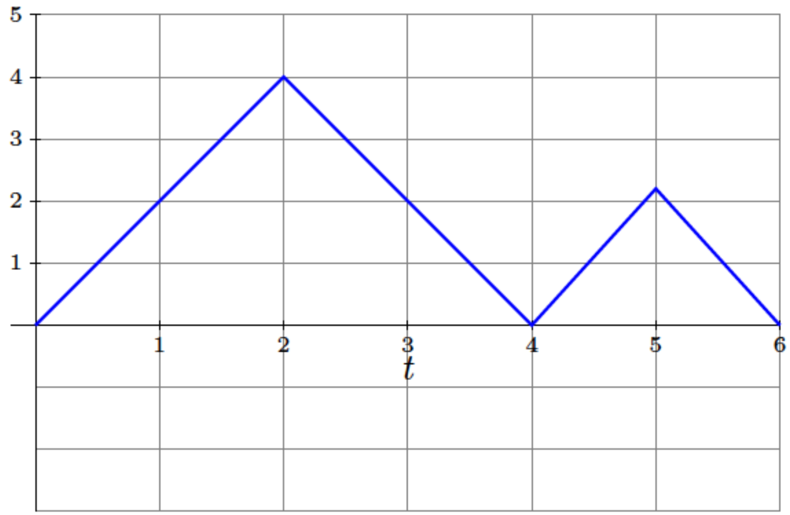
- Models are generally hybrid systems producing hybrid traces
- Model-Checking is limited to restrictive cases
- *Monitoring simulated traces is more practical*
- *Quantitative satisfaction of STL can address the problem of noise and approximation*



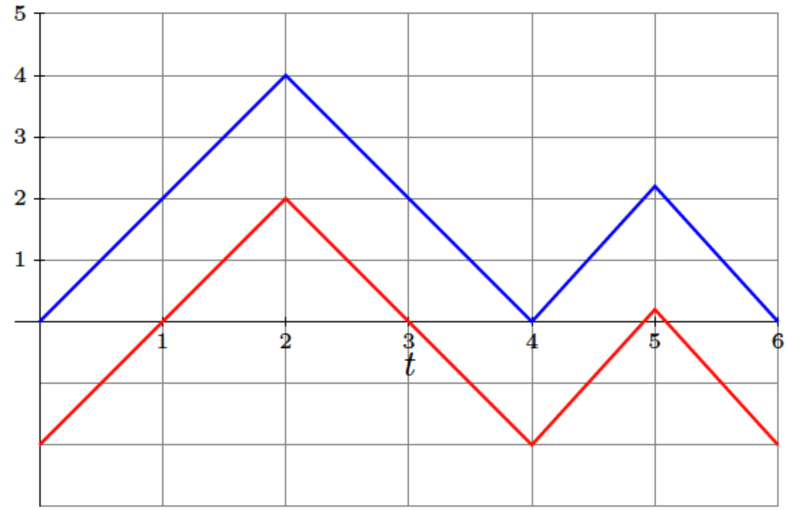
Robust Satisfaction Signal

$$
\begin{aligned}
\rho^{\mu}(x,t) &= f(x_1[t],\ldots,x_n[t]) \\
\rho^{\neg\varphi}(x,t) &= -\rho^{\varphi}(x,t) \\
\rho^{\varphi_1 \wedge \varphi_2}(x,t) &= \min(\rho^{\varphi_1}(x,t), \rho^{\varphi_2}(w,t)) \\
\rho^{\varphi_1 \ \mathcal{U}_{[a,b]} \ \varphi_2}(x,t) &= \sup_{\tau \in t+[a,b]} (\min(\rho^{\varphi_2}(x,\tau), \inf_{s \in [t,\tau]} \rho^{\varphi_1}(x,s))
\end{aligned}
$$

Robustness of STL

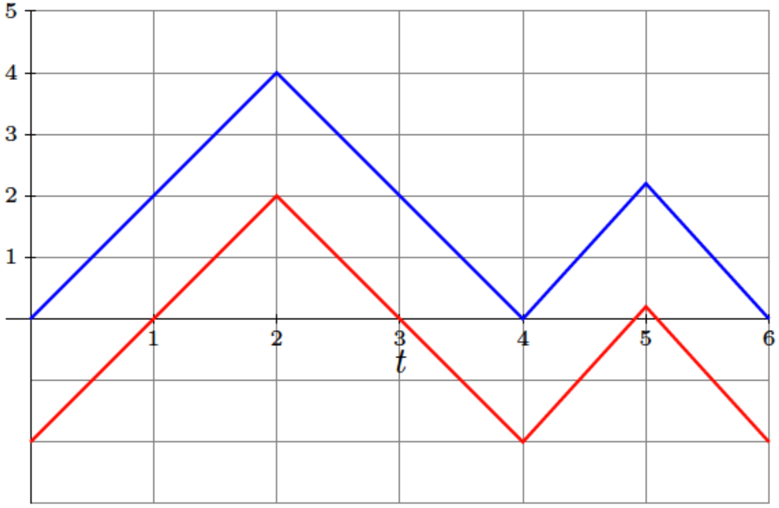


$y = f(t)$

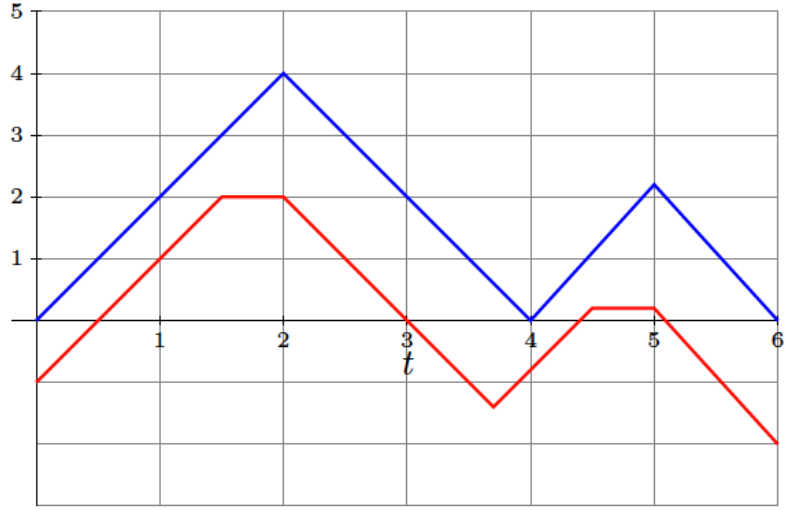


$\rho\left(y \geq 2, \omega, t\right)$

Robustness of STL



$\rho\left(y \geq 2, \omega, t\right)$



$\rho\left(\Diamond_{[0,0.5]} y \geq \mathbf{2}, w, t\right)$

Property of Robust Satisfaction Signal

- Sign indicates satisfaction status

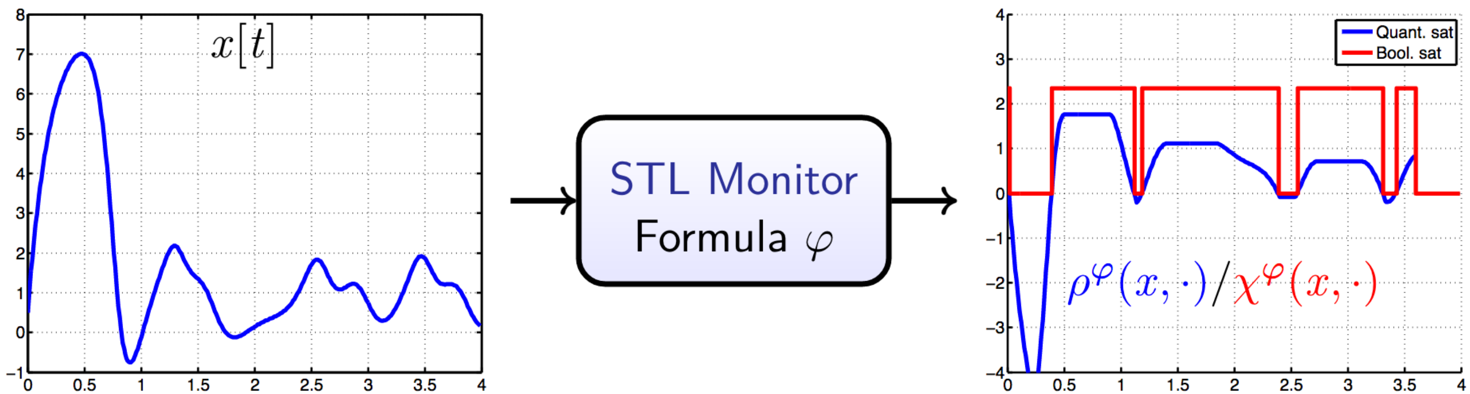
$$\rho^{\varphi}(x,t) > 0 \Rightarrow x,t \models \varphi$$
$$\rho^{\varphi}(x,t) < 0 \Rightarrow x,t \not\models \varphi$$

- Absolute value indicates tolerance

$$x,t \models \varphi \text{ and } \|x - x'\|_{\infty} \leq \rho^{\varphi}(x,t) \quad \Rightarrow \quad x',t \models \varphi$$
$$x,t \not\models \varphi \text{ and } \|x - x'\|_{\infty} \leq -\rho^{\varphi}(x,t) \quad \Rightarrow \quad x',t \not\models \varphi$$

Robust Monitoring

A robust STL monitor is a *transducer* that transform x into $\rho^{\varphi}(x, .)$



In practice

- Trace: time words over alphabet $\mathbb{R}$, linear interpolation

  Input: $x(\cdot) \triangleq (t_i, x(t_i))_{i \in \mathbb{N}}$ Output: $\rho^{\varphi}(x, \cdot) \triangleq (r_j, z(r_j))_{j \in \mathbb{N}}$
- Continuity, and piecewise affine property preserved

Temporal Frequency Logic

Donze, Maler, Bartocci, Nickovick, Grosu, Smolka, ATVA, 2012

They extend STL with frequency predicates

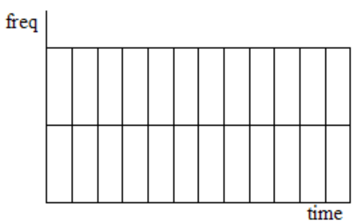
Continuous-time STFT:

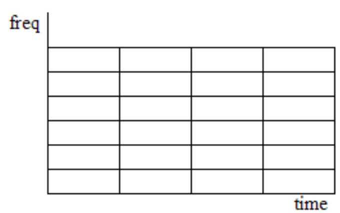
$$STFT\{x(t)\}(\tau,\omega) \equiv X(\tau,\omega) = \int_{-\infty}^{\infty} x(t)\omega(t-\tau)e^{-j\omega t}dt$$

Discrete-time STFT:

$$STFT\{x[n]\}(m,\omega) \equiv X(\tau,\omega) = \sum_{n=-\infty}^{\infty} x[n]\omega[n-m]e^{-j\omega n}$$

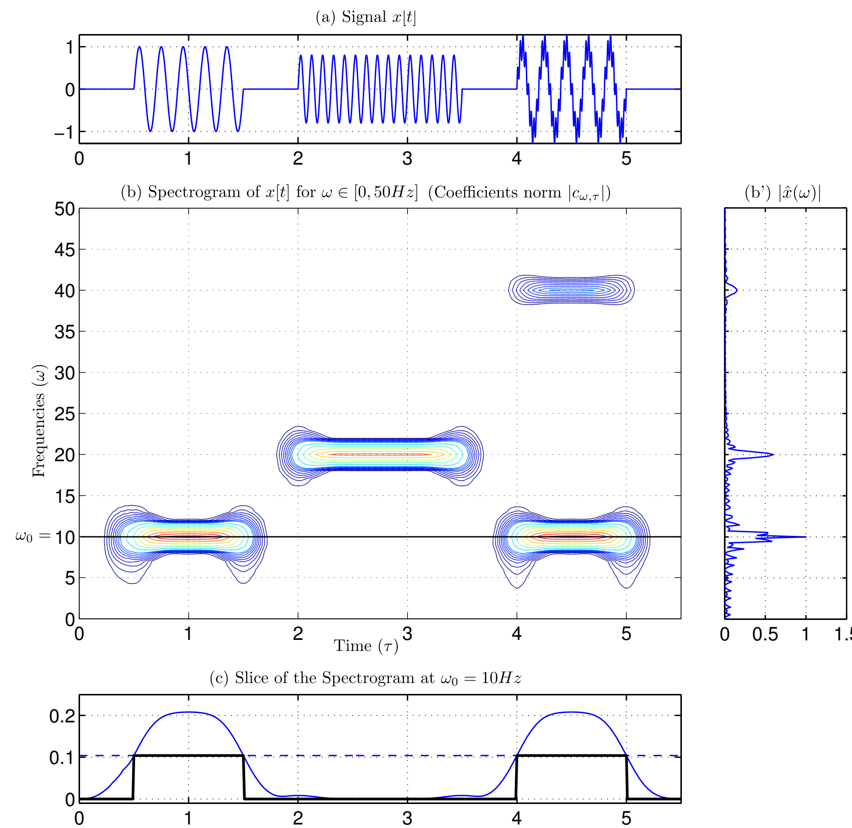
Fixed Resoultion: **It depends on the window size**



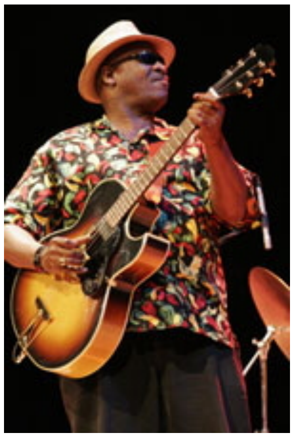


New predicate for the logic:

$$\mu = f(x,p) > \theta$$



Monitoring Music



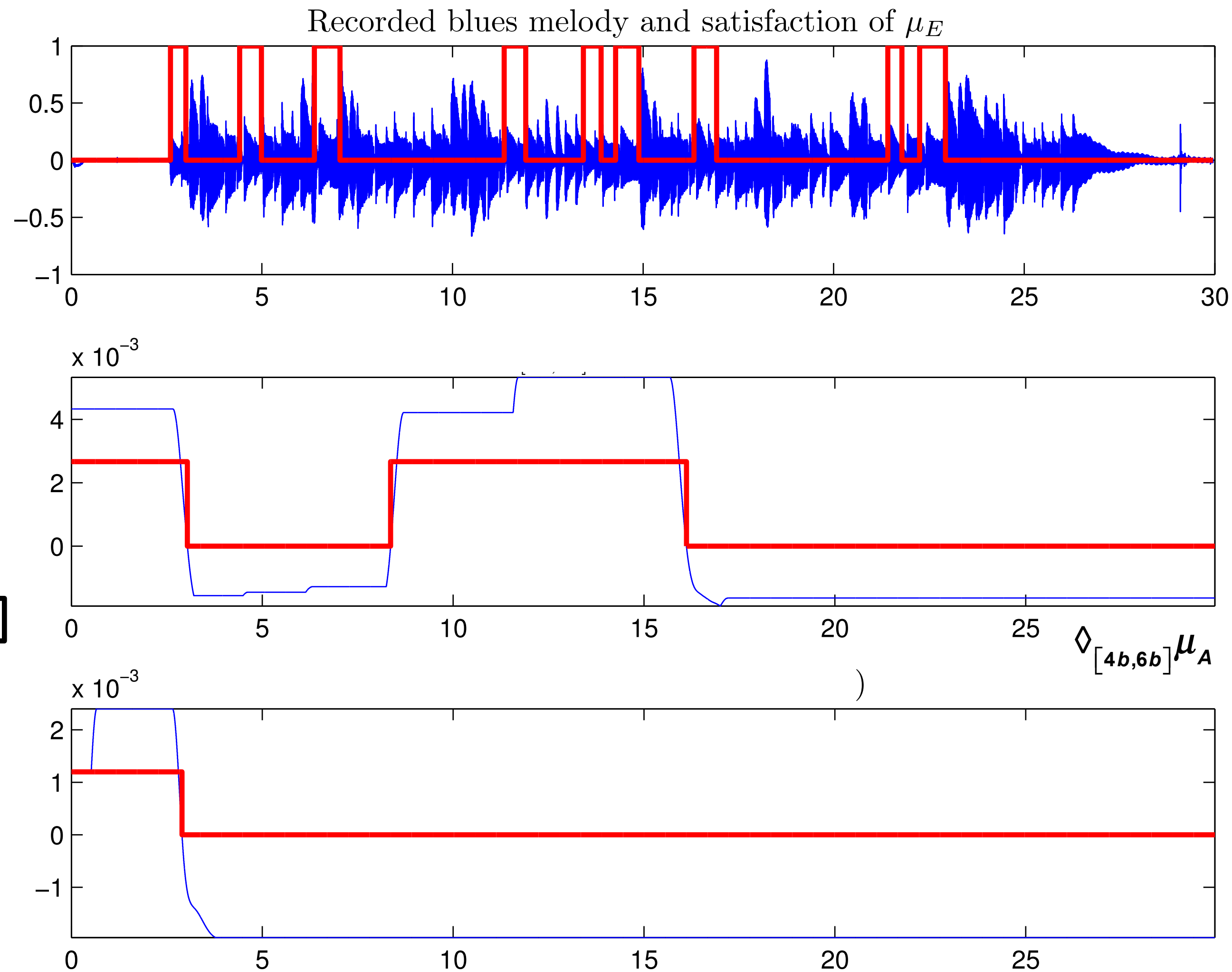
Blues pattern (12 bar):

E E E E | A A E E | B A E E

1 bar = 4 beats

Turn around

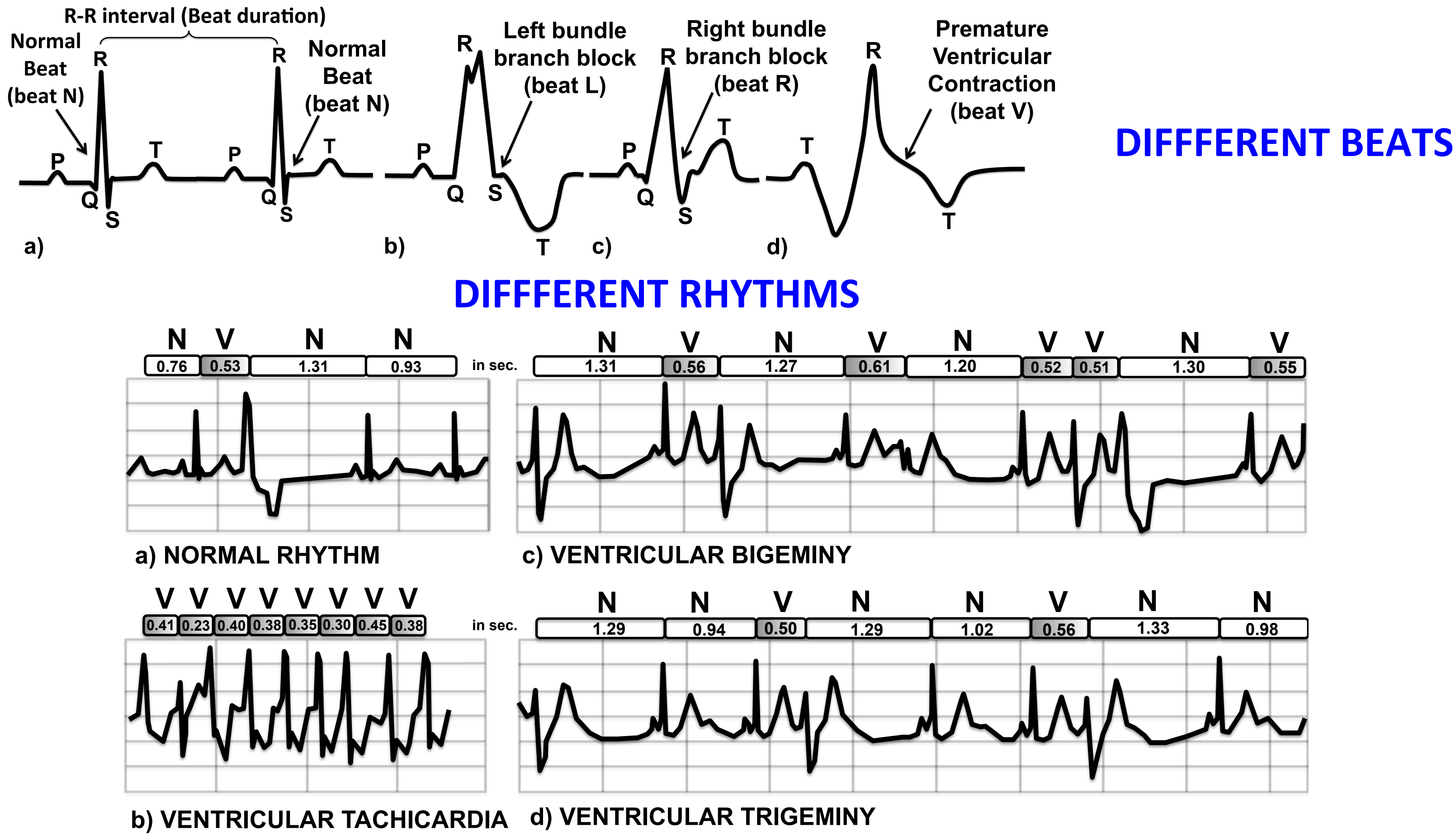
$$\mu_E \wedge \Diamond_{[5b,5b]}\mu_A \wedge \Diamond_{[8b,9b]}\left(\mu_B \wedge \Diamond_{[b,2b]}\mu_A \wedge \Diamond_{[2b,3b]}\mu_E\right)$$



$$\Diamond_{[8b,9b]}\left(\mu_B \wedge \Diamond_{[b,2b]}\mu_A \wedge \Diamond_{[2b,3b]}\mu_E\right)$$

Monitoring ECG

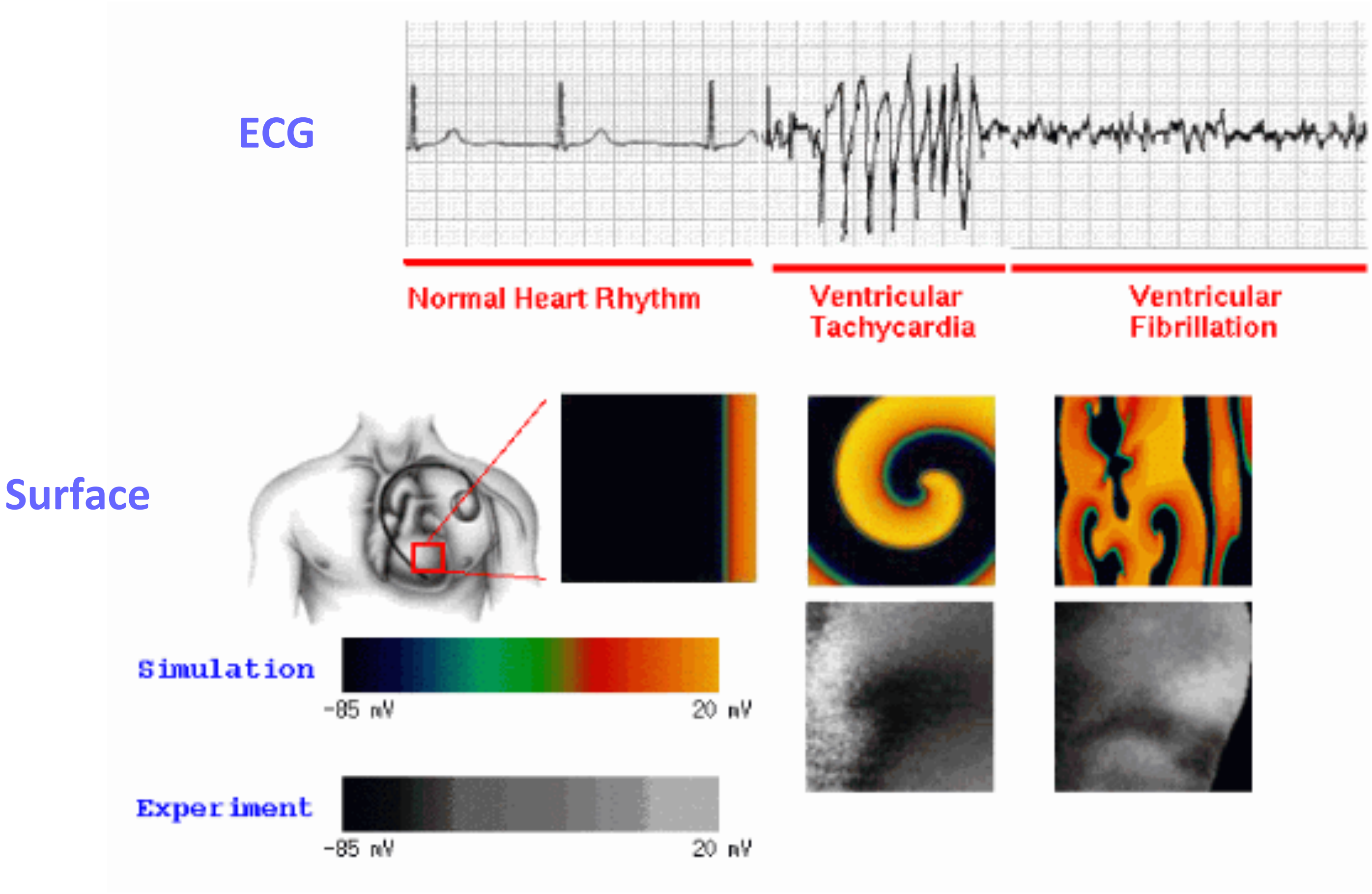
Bartocci, Bortolussi, Sanguinetti, FORMATS, 2014



Spatial Logics –
Linear Spatial Superposition Logic

Grosu, Smolka, Corradini, Wasilewska, Entcheva, Bartocci, CAMC 2009

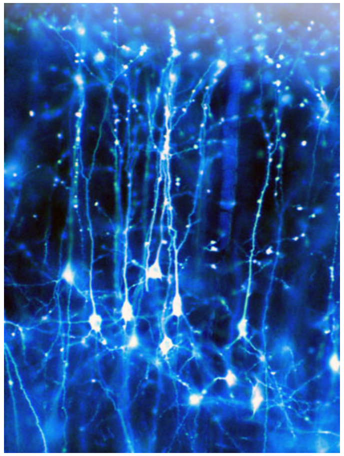
Emergent Behavior in Heart Cells



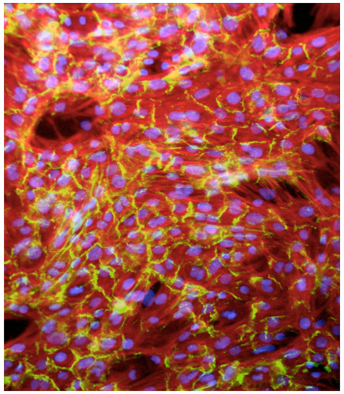
Arrhythmia afflicts more than 3 million Americans alone

Excitable Cells

- **Generate action potentials** (elec. pulses) in response to **electrical stimulation**
  - **Examples:** neurons, cardiac cells, etc.
- **Local regeneration** allows electric signal propagation **without damping**
- **Building block** for electrical signaling in **brain, heart, and muscles**



Neurons of a squirrel
University College London

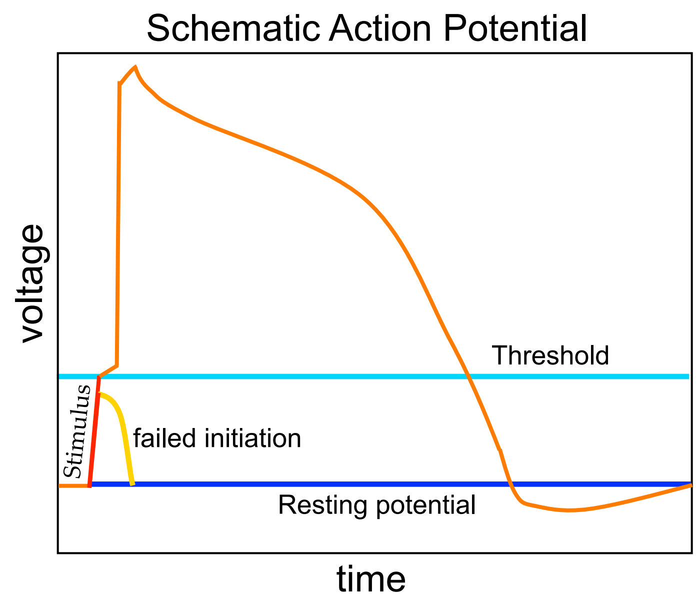


Artificial cardiac tissue
University of Washington

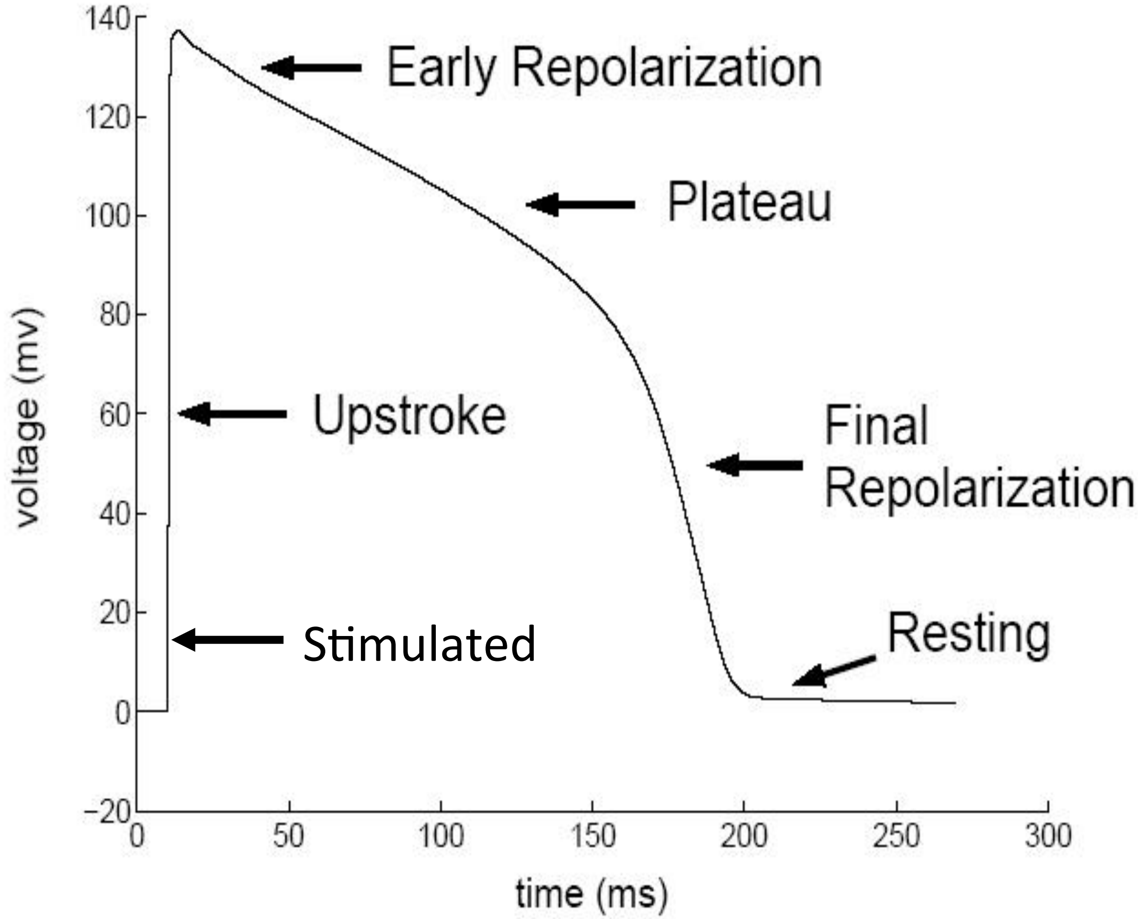
Action Potential (AP)

Membrane's AP depends on:

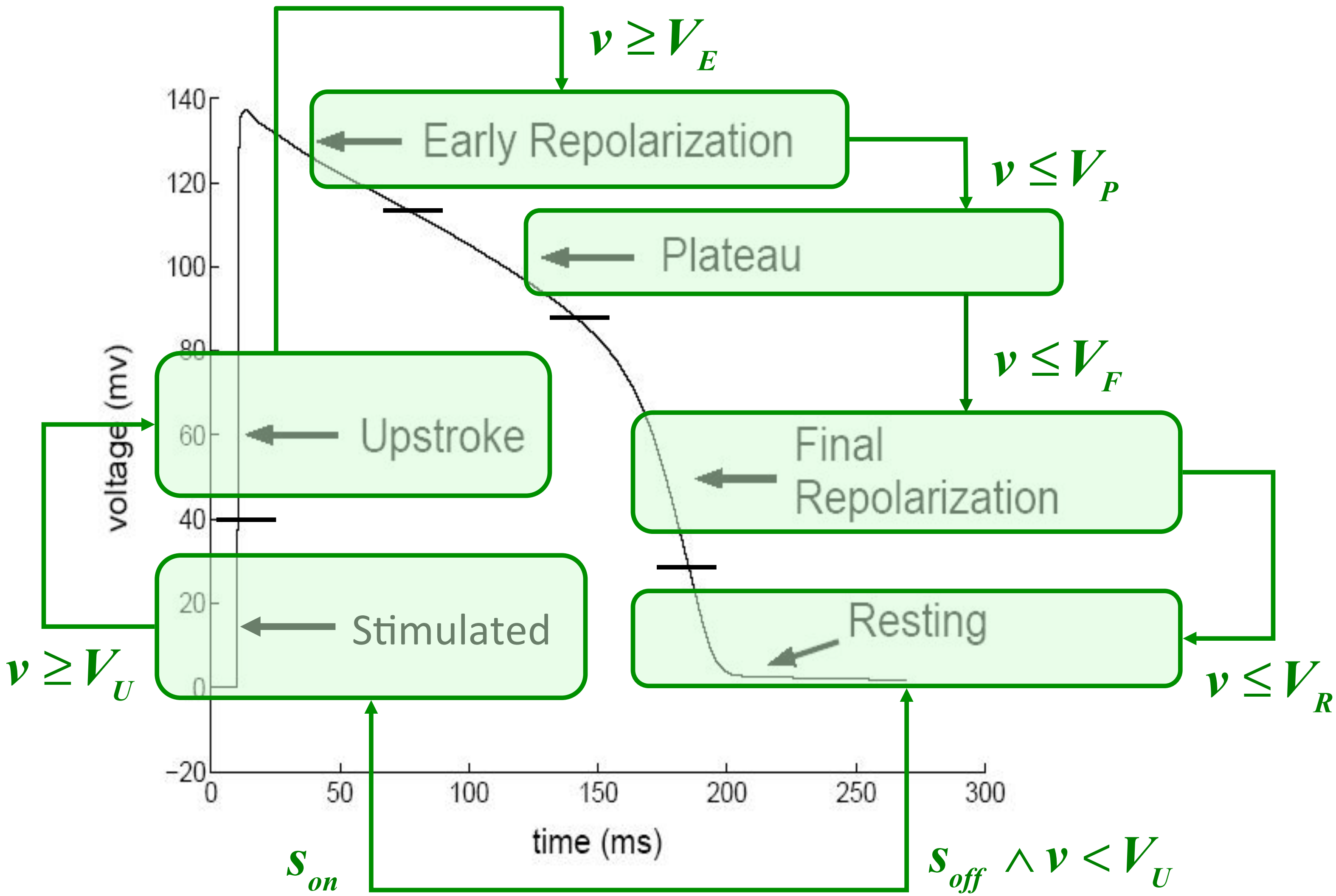
- **Stimulus (voltage or current):**
  - **External**
  - **Neighboring cells**
- **Cell's state**



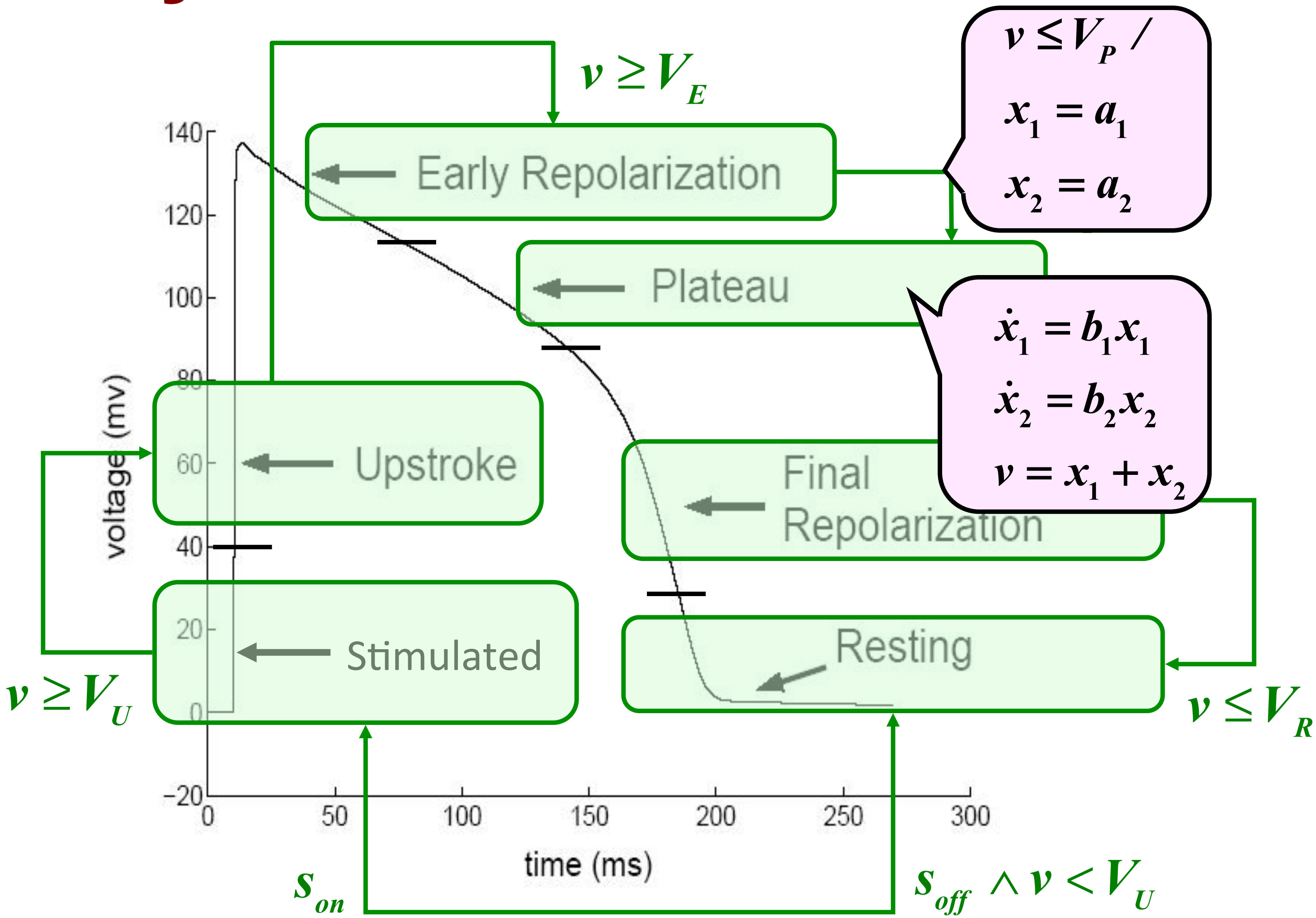
Hybrid Automaton Model



Hybrid Automaton Model

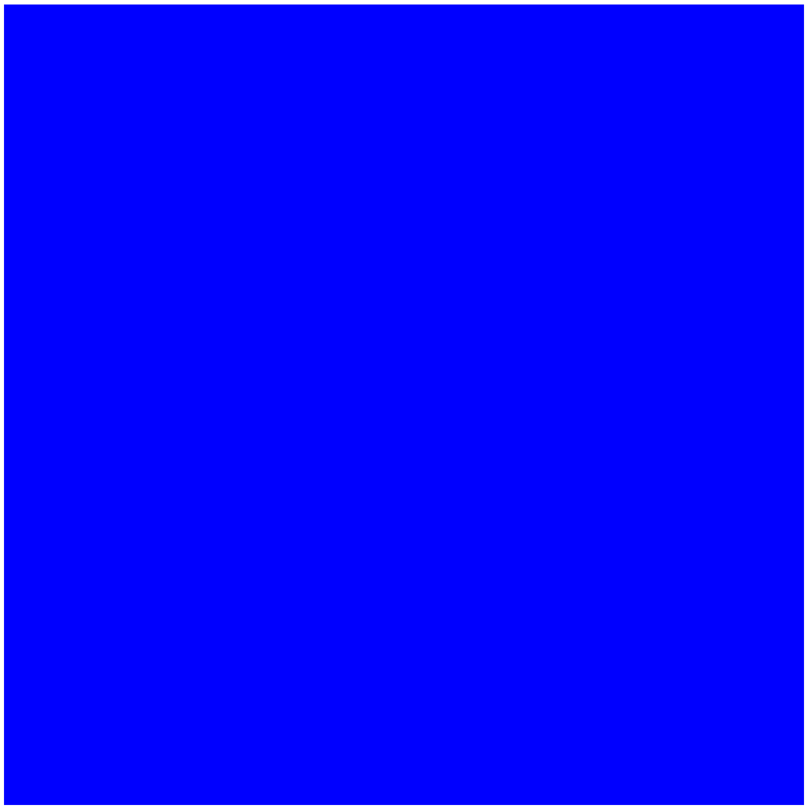


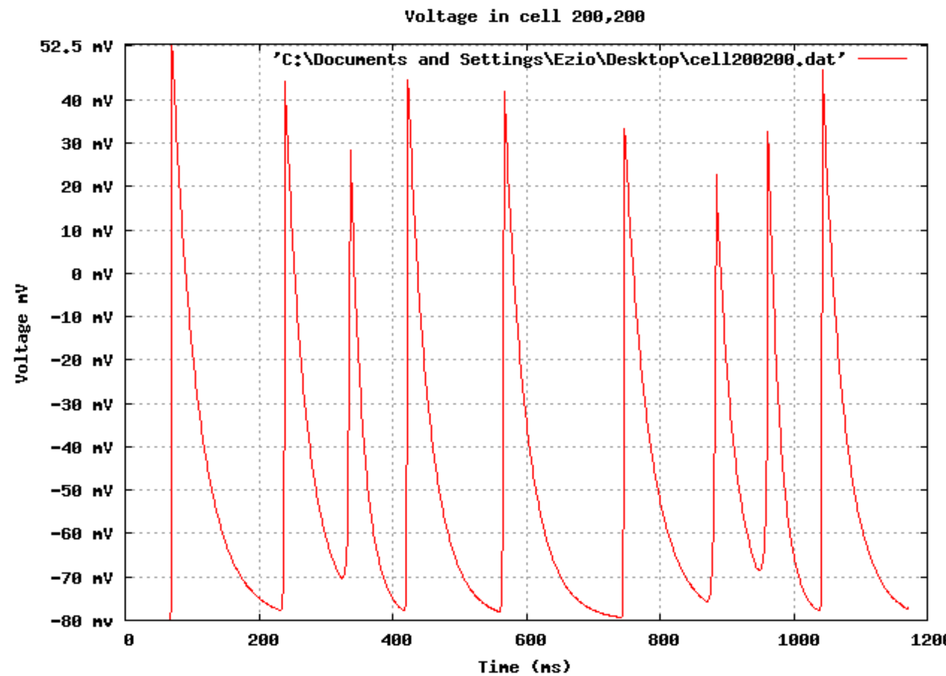
Hybrid Automaton Model

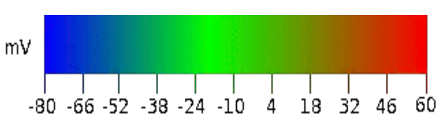


Fibrillation/Defibrillation

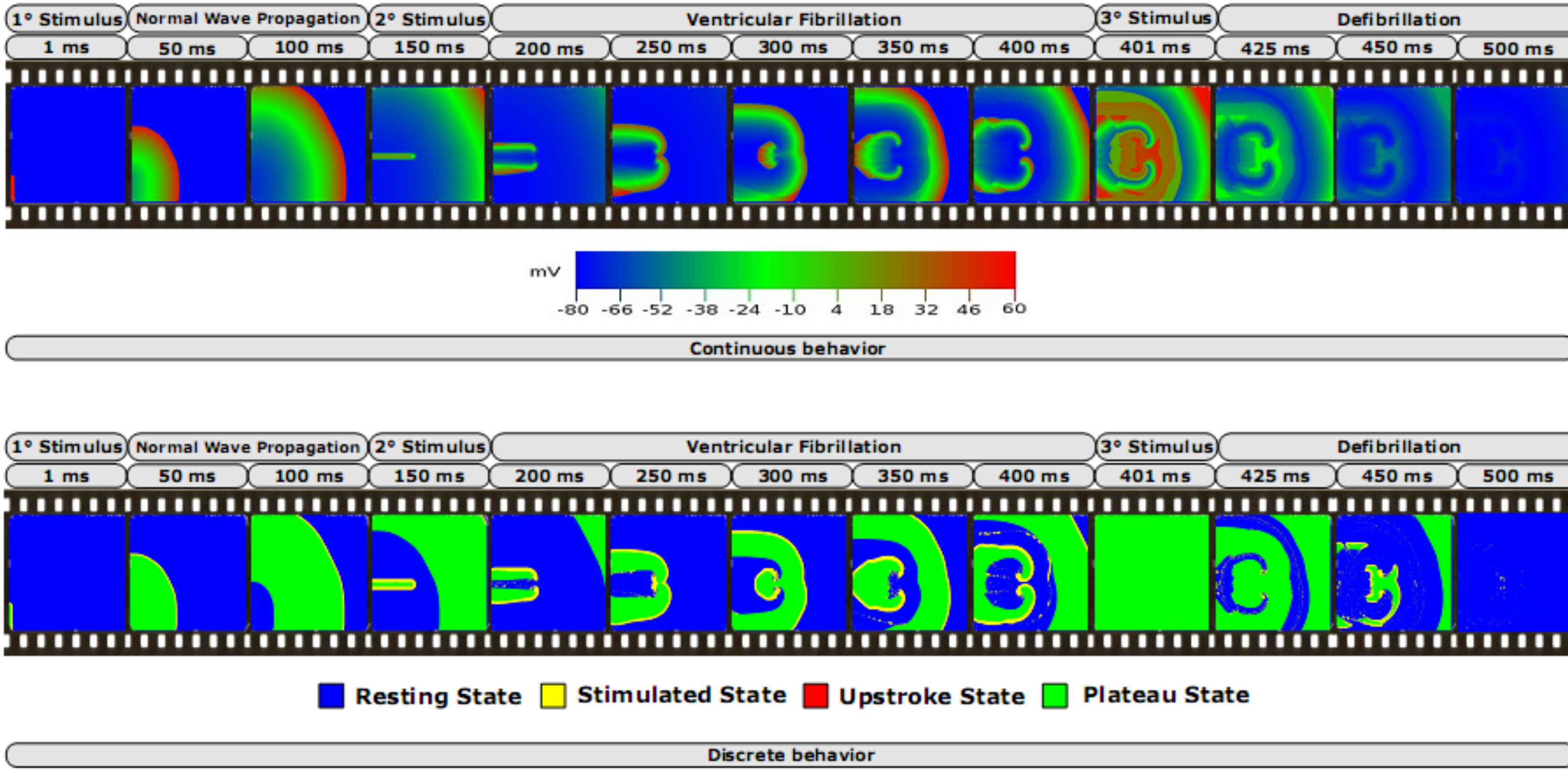
(400x400 neonatal-rat cells)







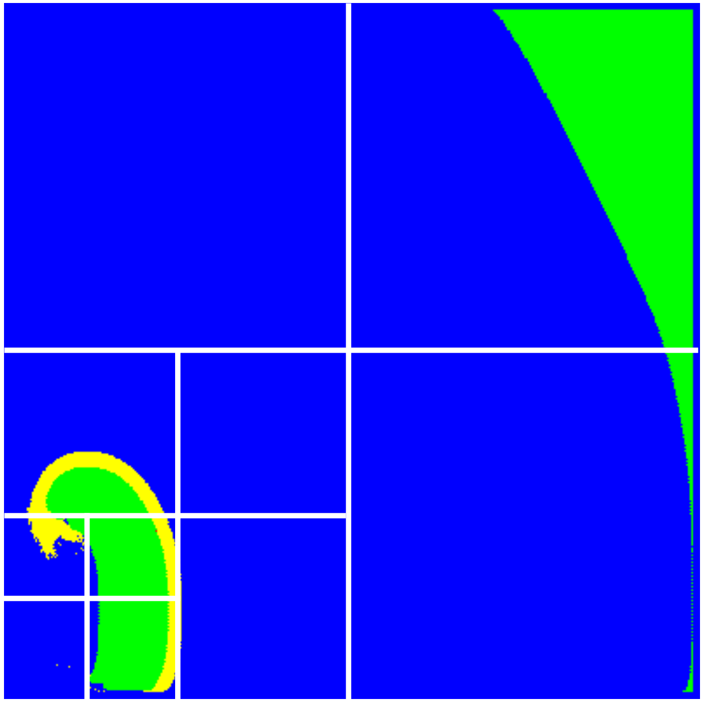
Finite Mode Abstraction



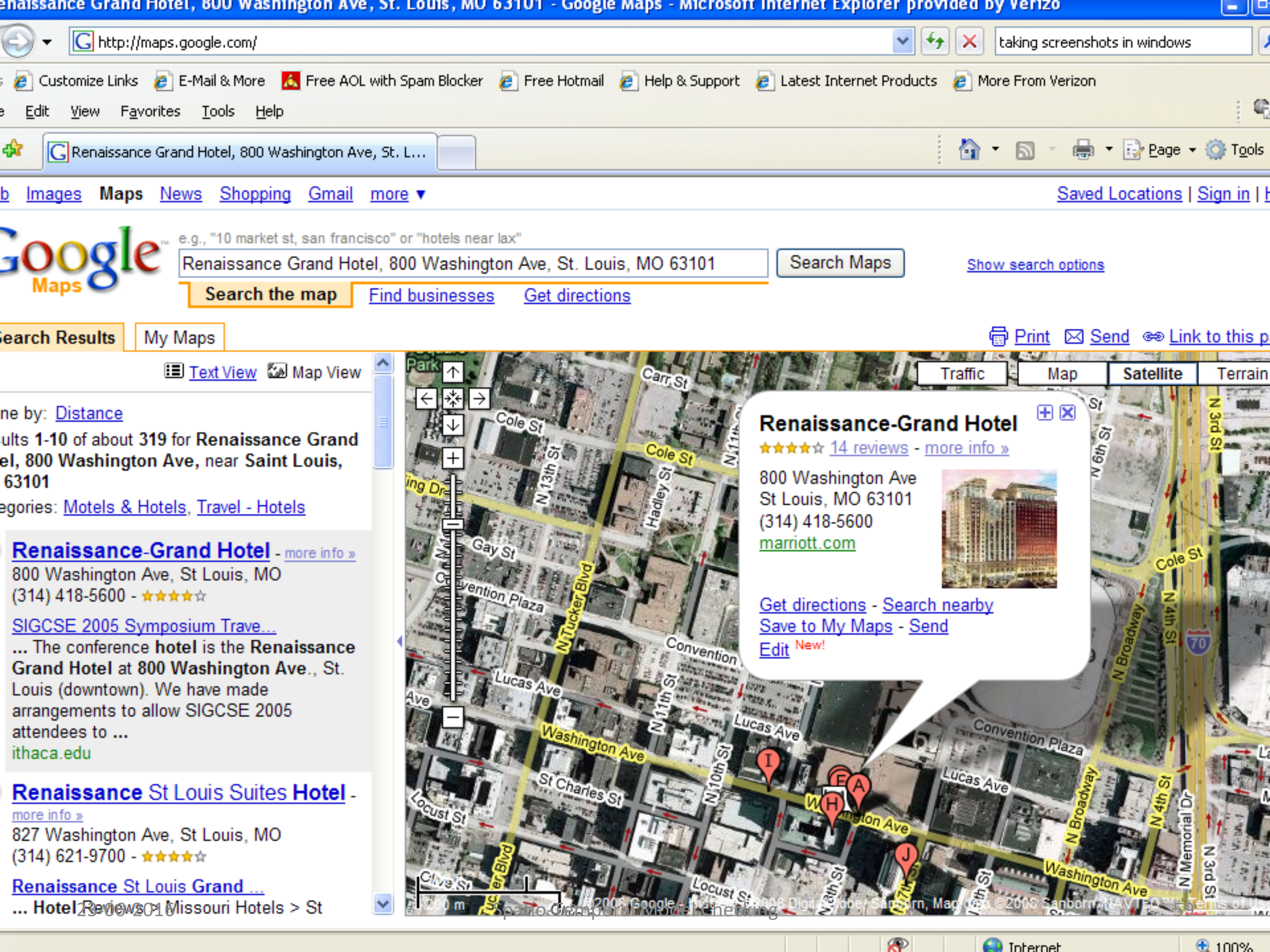
- **Preserves spatial properties ($4^{160,000}$ images)**

Problems to Solve

- **Detection problem:**
  - Does a simulated tissue contain a spiral ?
- **Specification problem:**
  - Encode above property as a logic formula?
  - Can we learn the formula?



How? Use Spatial Abstraction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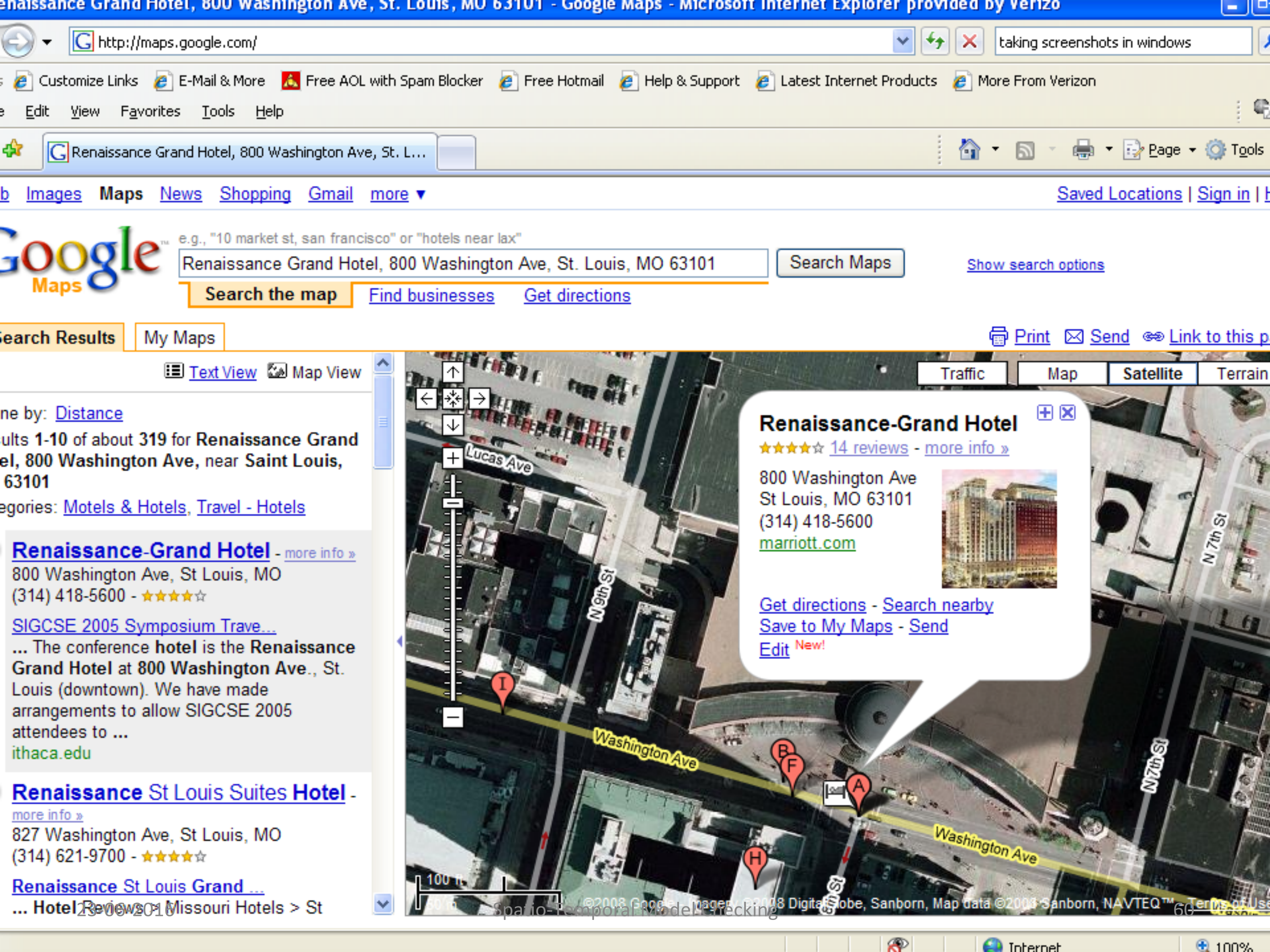
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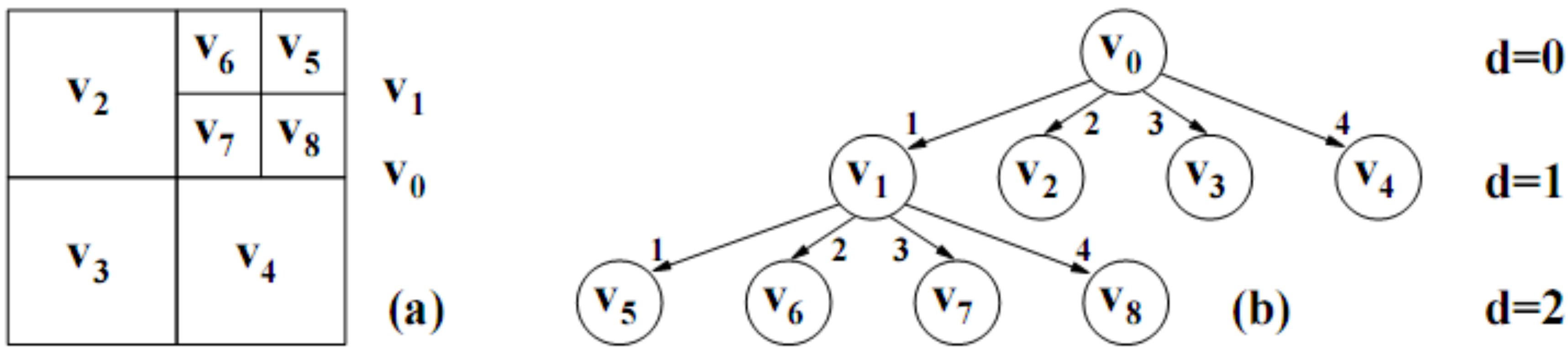
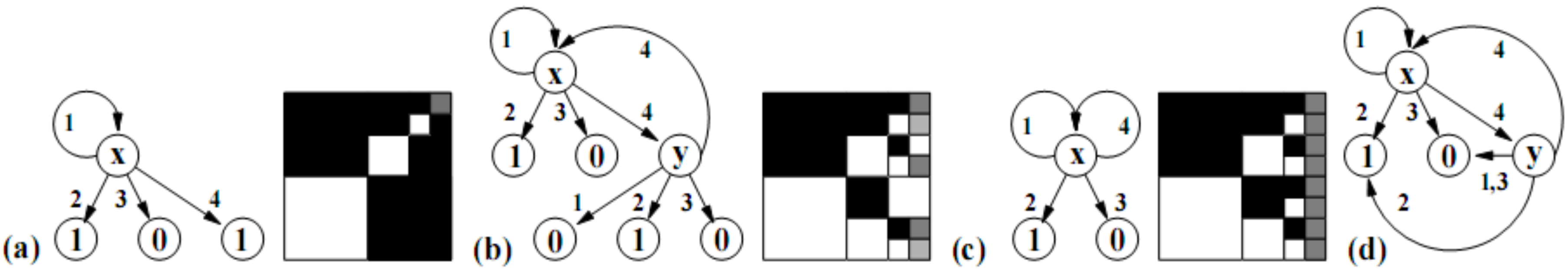
Superoposition Quadtrees (SQTs)



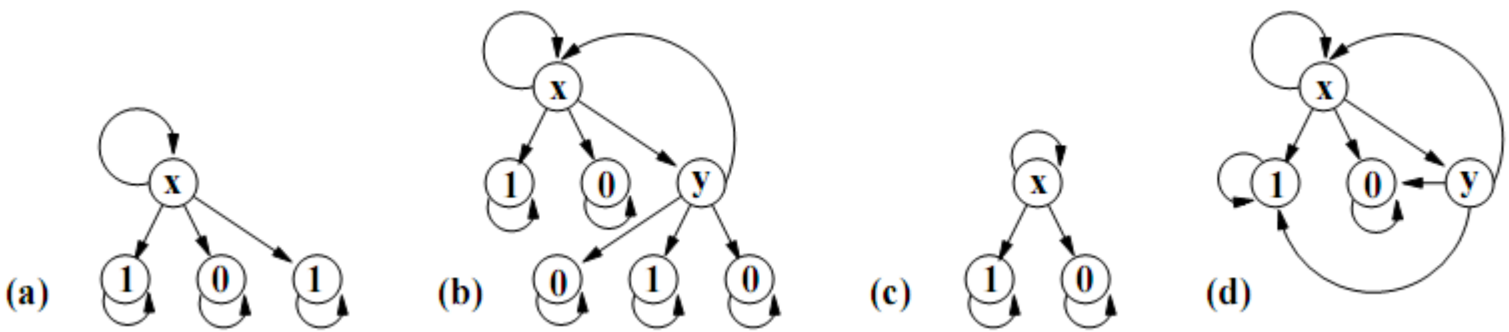
$$\exists! m \in \{s,u,p,r\}.\ p_l(m) = 1 \qquad p_i(m) = \frac{1}{4}\sum_{j=1}^{4} p_{ij}(m_j)$$

Abstract position and compute PMF p(m) ≡ P[D=m]

SQGs and Kripke Structures (KSs)

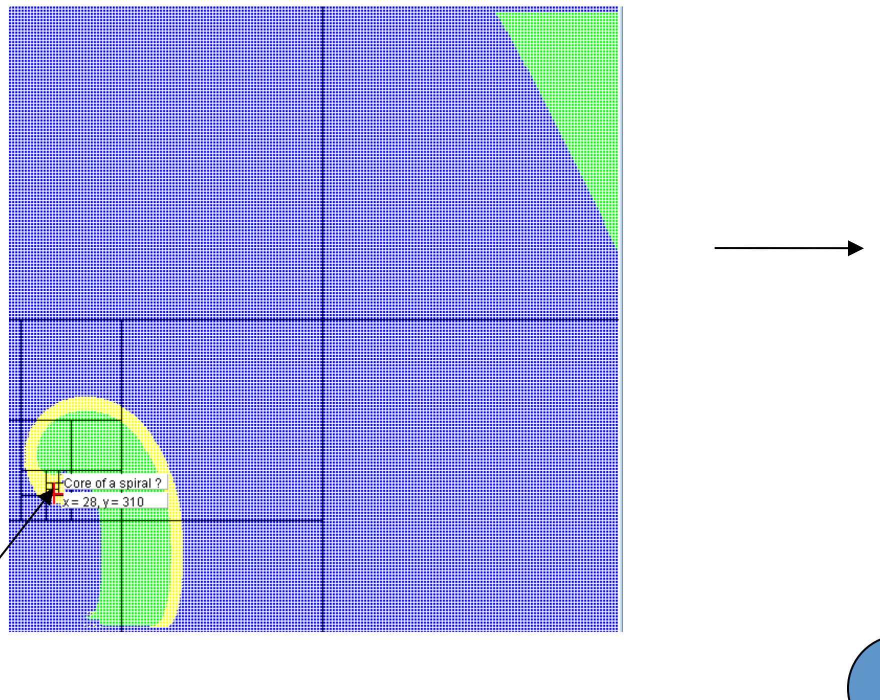


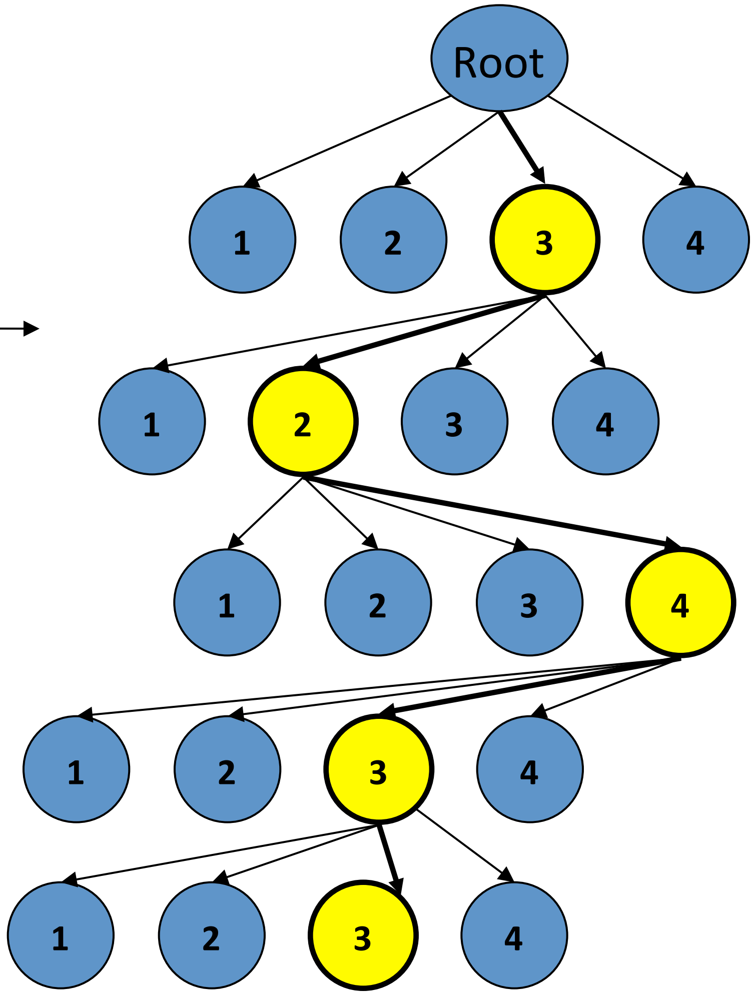
Superposition Quadgraphs (Fractals): modal SSL



Kripke Structure: linear / branching SSL

The Path to the Core of a Spiral





Click the core to determine the quadtree

Linear Spatial-Superposition Logic

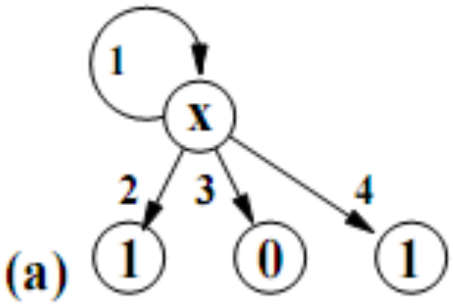
Syntax

$$\varphi \quad ::= \quad \top \mid \bot \mid P[D=m] \sim d \mid \neg\phi \mid \varphi \vee \psi \mid X\varphi \mid B\varphi \mid \varphi U \varphi \mid \varphi R \varphi$$
$$\sim \quad ::= \quad < \mid \leq \mid = \mid \geq \mid >$$

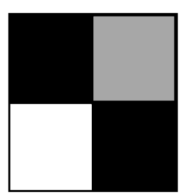
Semantics

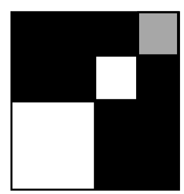
$$\pi \models^i_k \top \qquad and \quad \pi \not\models^i_k \bot$$
$$\pi \models^i_k p \quad \Leftrightarrow \quad p \in L(\pi[i])$$
$$\pi \models^i_k \neg\varphi \quad \Leftrightarrow \quad \pi \not\models^i_k \varphi$$
$$\pi \models^i_k \varphi \vee \psi \quad \Leftrightarrow \quad \pi \models^i_k \varphi \; or \; \pi \models^i_k \psi$$
$$\pi \models^i_k X\varphi \quad \Leftrightarrow \quad i < k \; and \; \pi \models^{i+1}_k \varphi$$
$$\pi \models^i_k B\varphi \quad \Leftrightarrow \quad 0 < i \leq k \; and \; \pi \models^{i-1}_k \varphi$$
$$\pi \models^i_k \varphi U \psi \quad \Leftrightarrow \quad \exists j.\; i \leq j \leq k.\; \pi \models^j_k \psi \; and \; \forall n.\; i \leq n < j.\; \pi \models^n_k \varphi$$
$$\pi \models^i_k \psi R \varphi \quad \Leftrightarrow \quad \forall j.\; i \leq j \leq k.\; \pi \models^j_k \varphi \quad or \quad \exists n.\; i \leq n < j.\; \pi \models^n_k \psi$$

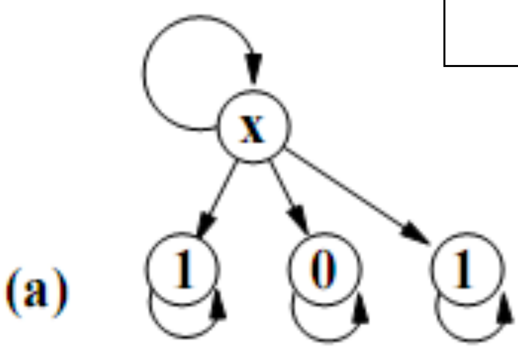
SQGs, KSs and LSL



$$x = \frac{x+2}{4} \Rightarrow x = \frac{2}{3}$$

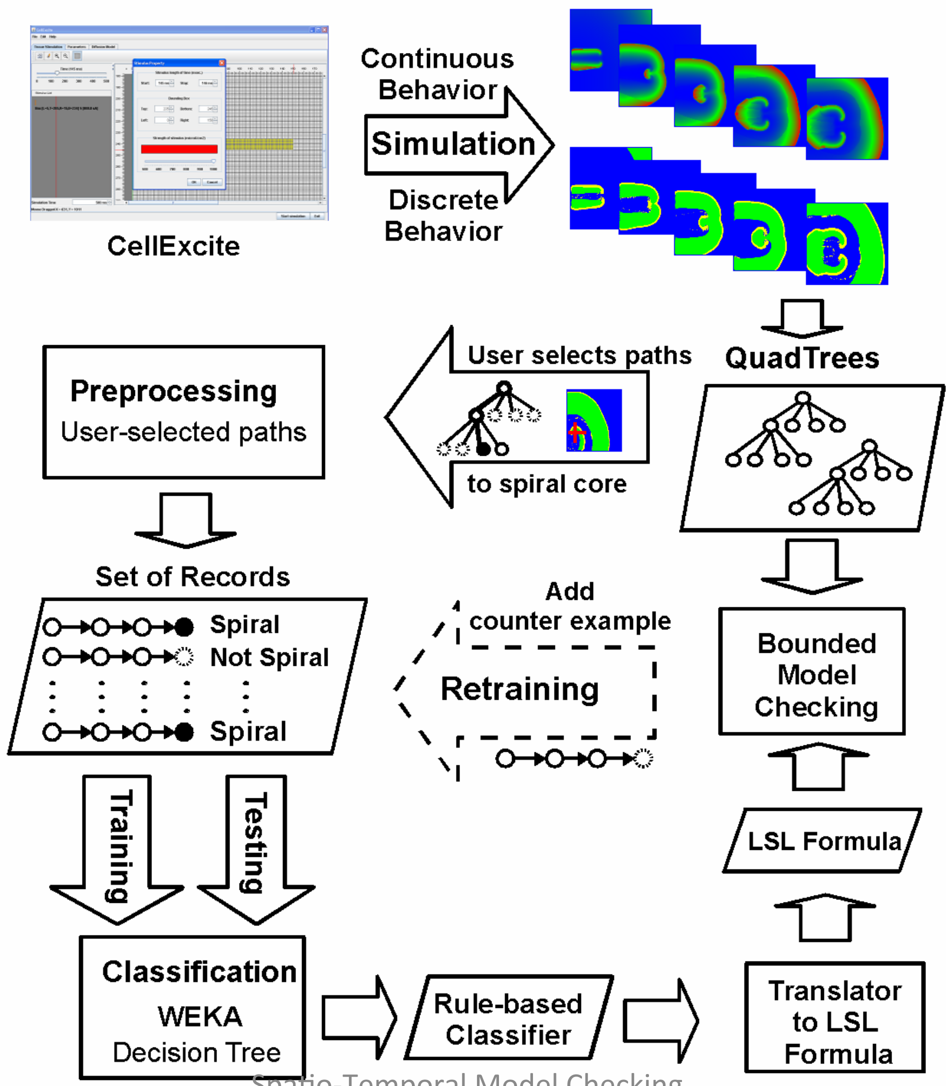






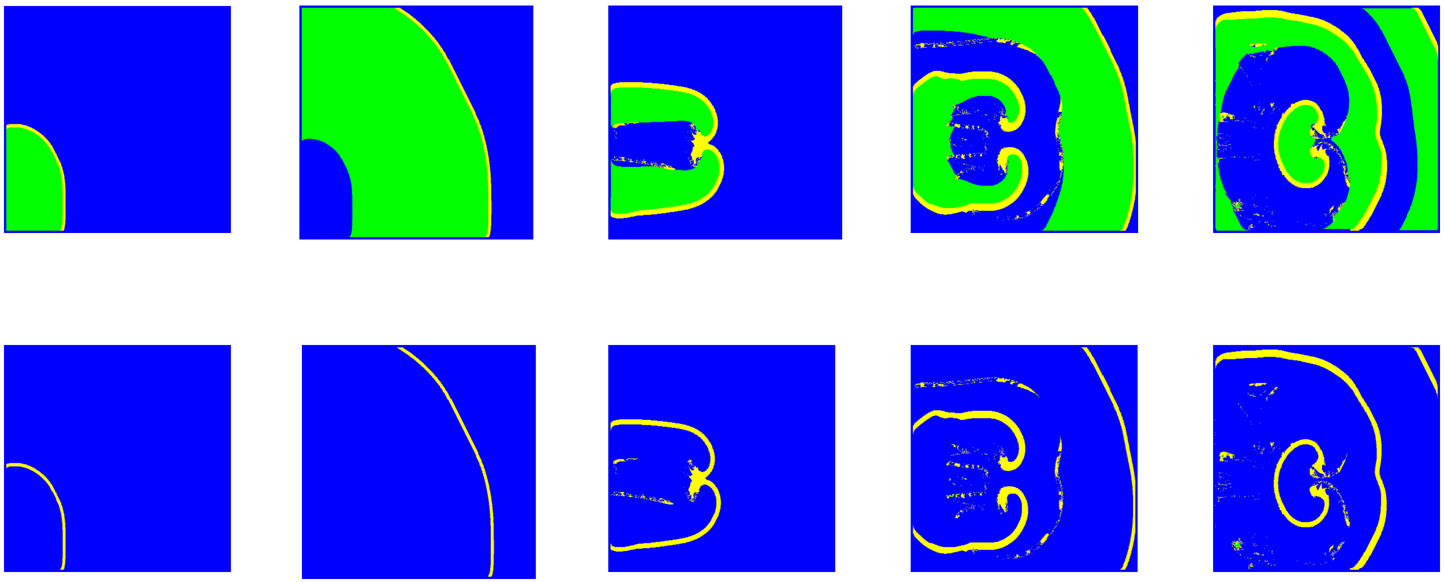
$$\neg G(P[D=b] = \frac{2}{3})$$

Overview of Our Approach



The Wave Front

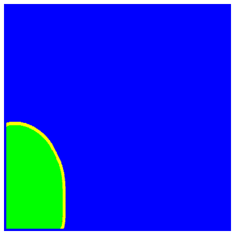
- **Measure density of mode stimulated (yellow)**



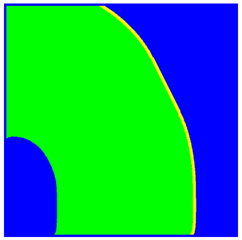
- **Yellow modes represent the wave front**

Learning Formula

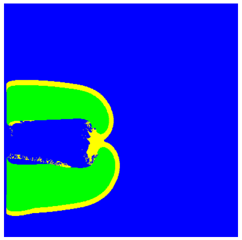
Input – Sequence of images (mode distribution)



1

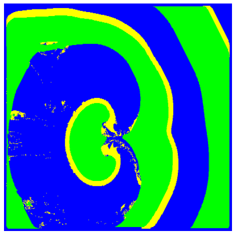


2



3

4



5

Output – Set of records with attributes (a table)

Record	a1	a2	a3	a4	…	Spiral
1	…	…	…	…	…	N
2	…	…	…	…	…	N
3	…	…	…	…	…	Y
4	…	…	…	…	…	Y
5	…	…	…	…	…	Y

Class Description Formula

Each record: corresponds to a discriminant rule

$$r_i \equiv (\wedge_{j \in I_i} a_{ij} = v_{ij} \Rightarrow c = v)$$

Table: corresponds to conjunction of rules

$$\wedge_{i=1}^{n} r_i = \wedge_{i=1}^{n} (\wedge_{j \in I_i} a_{ij} = v_{ij} \Rightarrow c = v)$$

$$= (\vee_{i=1}^{n} \wedge_{j \in I_i} a_{ij} = v_{ij}) \Rightarrow (c = v)$$

Class description formula (CDF): the antecedent

$$\vee_{i=1}^{n} \wedge_{j \in I_i} a_{ij} = v_{ij}$$

Creating/Checking an LSSL formula

Decision tree algorithm: simplifies the CDF

if $a_7 \leq 0.875$ then {if $a_2 > 0.049$ then c else ¬c}

else if $a_3 \leq 0.078$ then { if $a_0 > 0.025$ then c else ¬ c} else ¬c

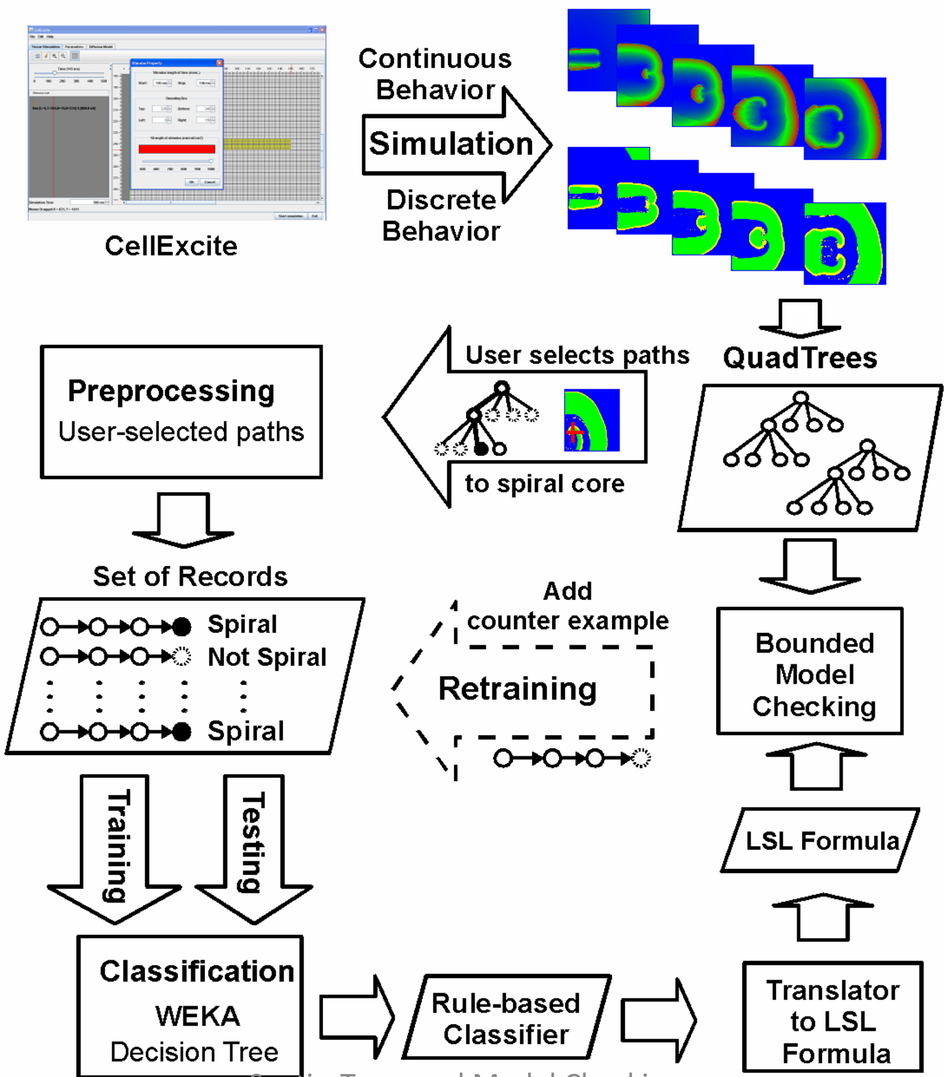
LSSL formula φ : gives meaning to attributes a_i

$X^7(P(D=s) \leq 0.875) \wedge X^2(P(D=s) > 0.049) \vee$

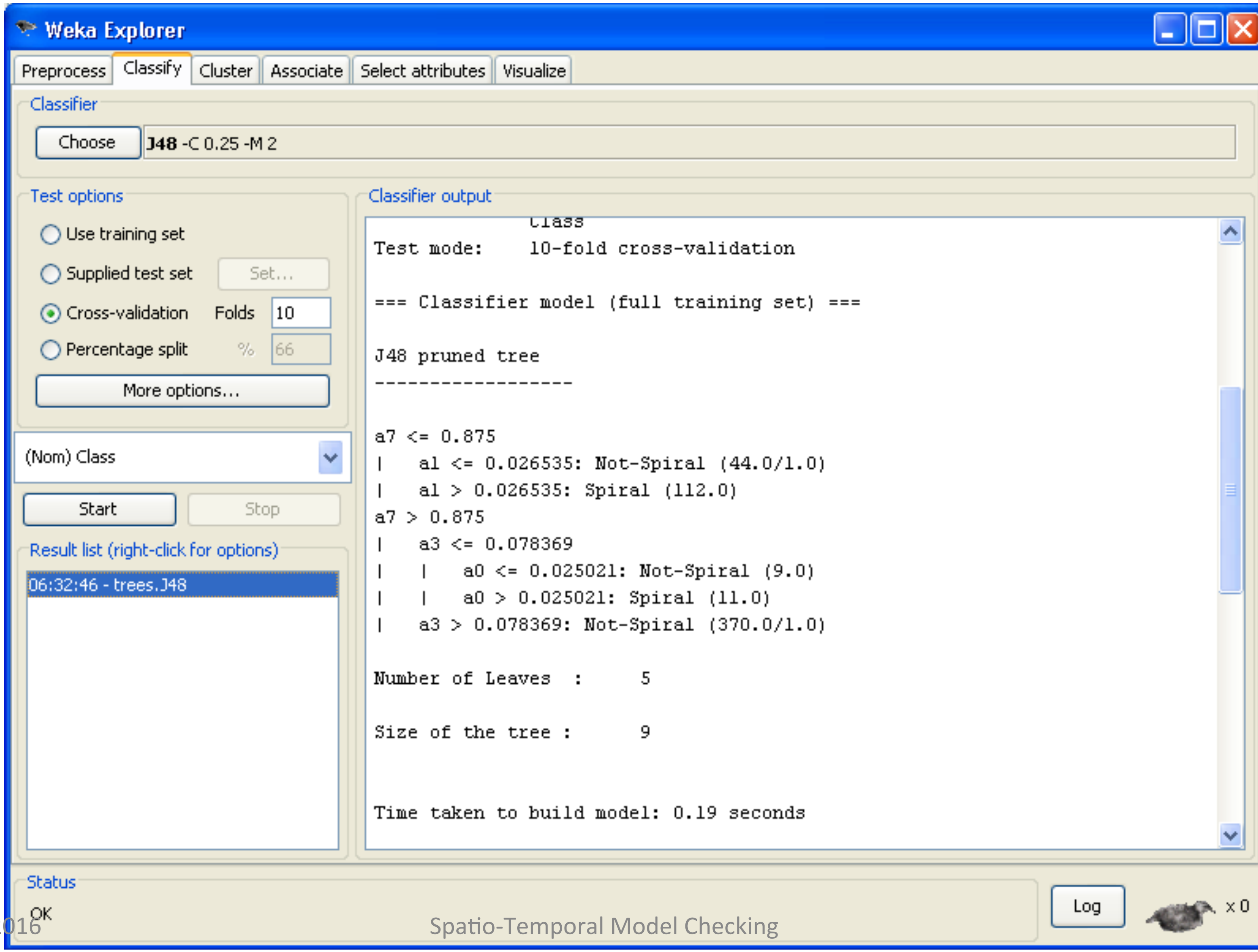
$X^7(P(D=s) > 0.875) \wedge X^3(P(D=s) \leq 0.078) \wedge (P(D=s) > 0.025)$

Spiral detection for SQT T: reduces to BMC of $T \models \varphi$

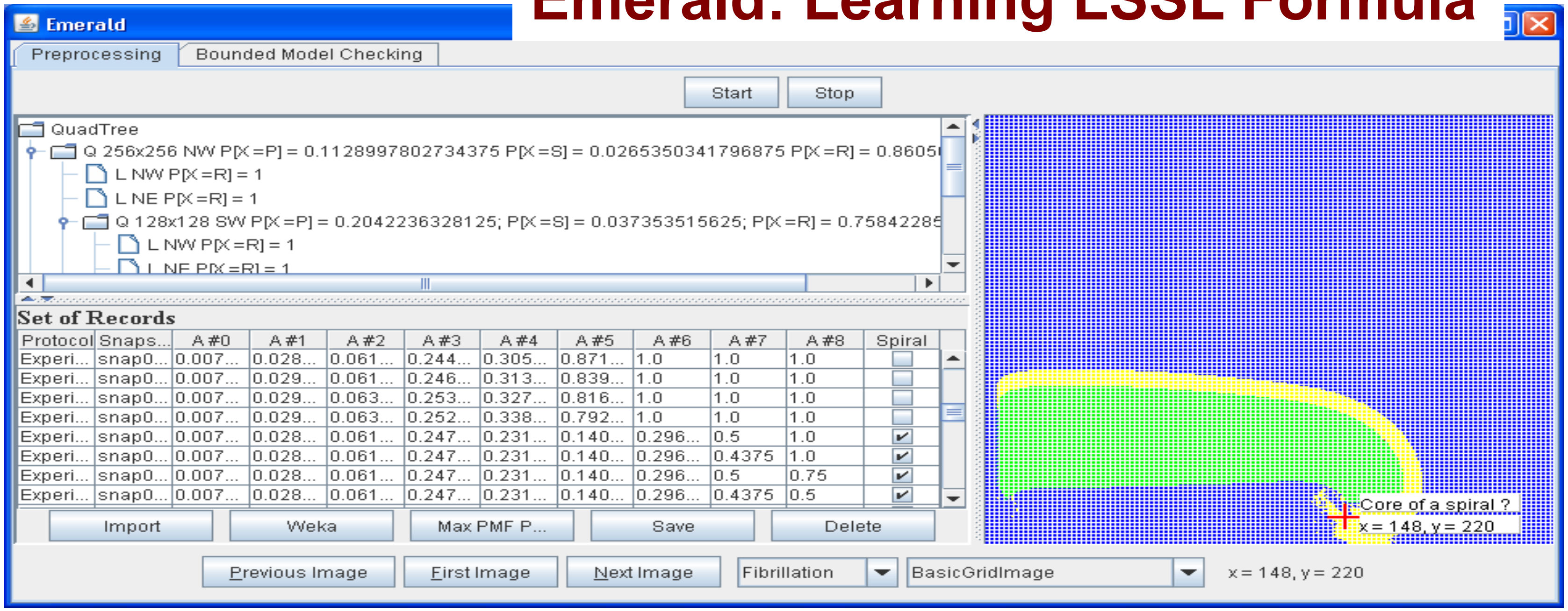
Overview of Our Approach



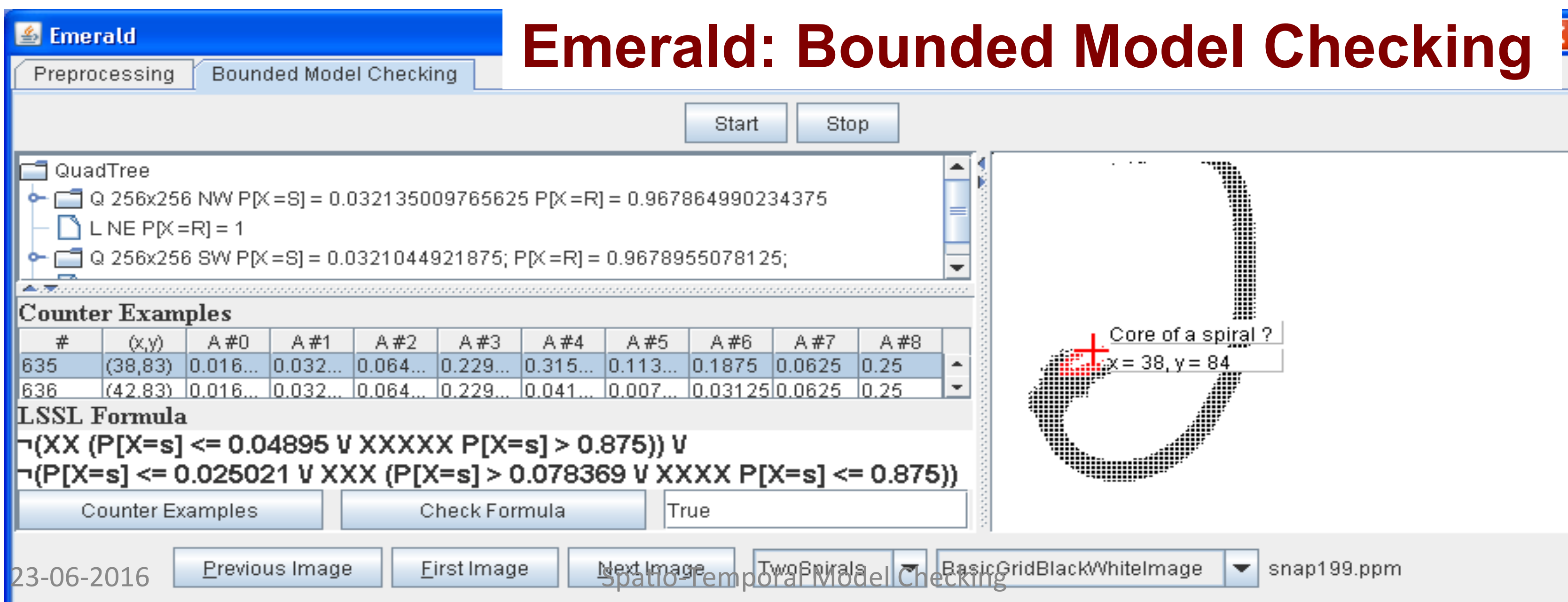
Using Weka


Weka Explorer
Preprocess
Classify
Cluster
Associate
Select attributes
Visualize
Classifier
Choose
J48 -C 0.25 -M 2
Test options
Use training set
Supplied test set
Set...
Cross-validation
Folds
10
Percentage split
%
66
More options...
(Nom) Class
Start
Stop
Result list (right-click for options)
06:32:46 - trees.J48
Classifier output
Test mode: 10-fold cross-validation
=== Classifier model (full training set) ===
J48 pruned tree
----
a7 <= 0.875
| a1 <= 0.026535: Not-Spiral (44.0/1.0)
| a1 > 0.026535: Spiral (112.0)
a7 > 0.875
| a3 <= 0.078369
| | a0 <= 0.025021: Not-Spiral (9.0)
| | a0 > 0.025021: Spiral (11.0)
| a3 > 0.078369: Not-Spiral (370.0/1.0)
Number of Leaves : 5
Size of the tree : 9
Time taken to build model: 0.19 seconds
Status
OK
Log
x 0

Emerald: Learning LSSL Formula



Emerald: Bounded Model Checking



Results

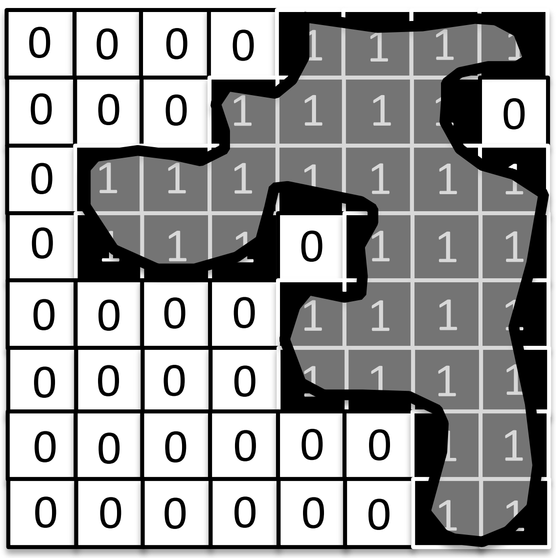
Path Classifier	Test Set 550	Test Set 600	Test Set 650
Trained (512 Paths)	87.00%	88.83%	88.23%
Retrained (512 Paths + 67 Counter-Examples)	97.10%	97.33%	93.07%

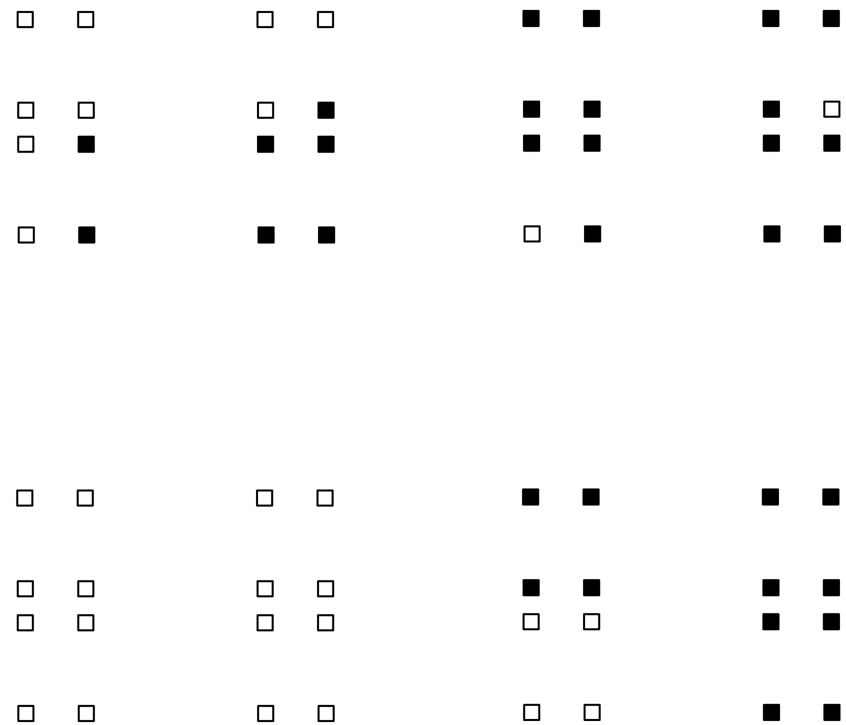
Prediction accuracy for spiral detection in Emerald

Spatial Logics
Tree Spatial Superposition Logic

Space Representation

Quadtree and Spatial Superposition



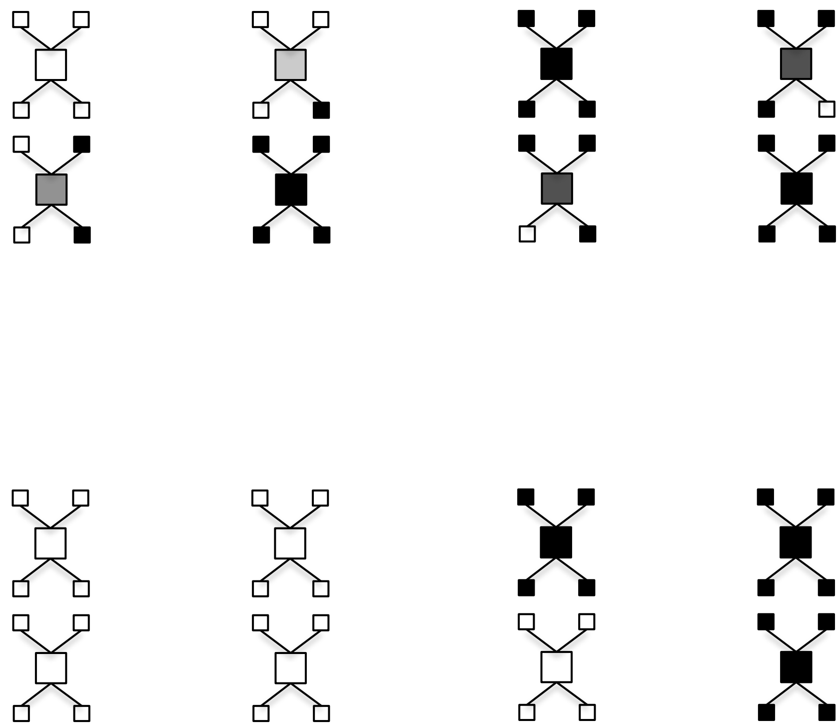


Leaves of the tree

Space Representation

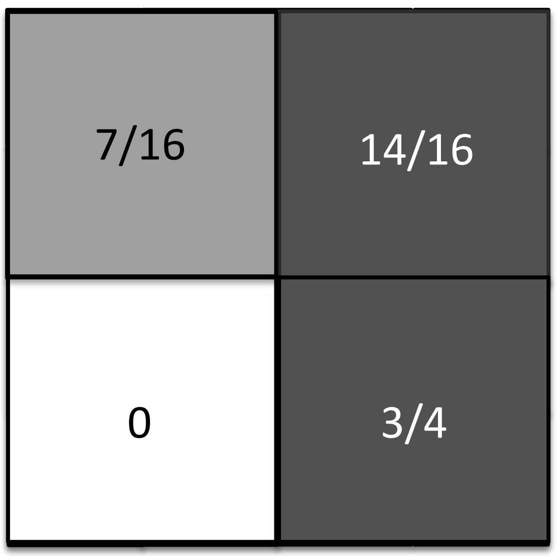
Quadtree and Spatial Superposition

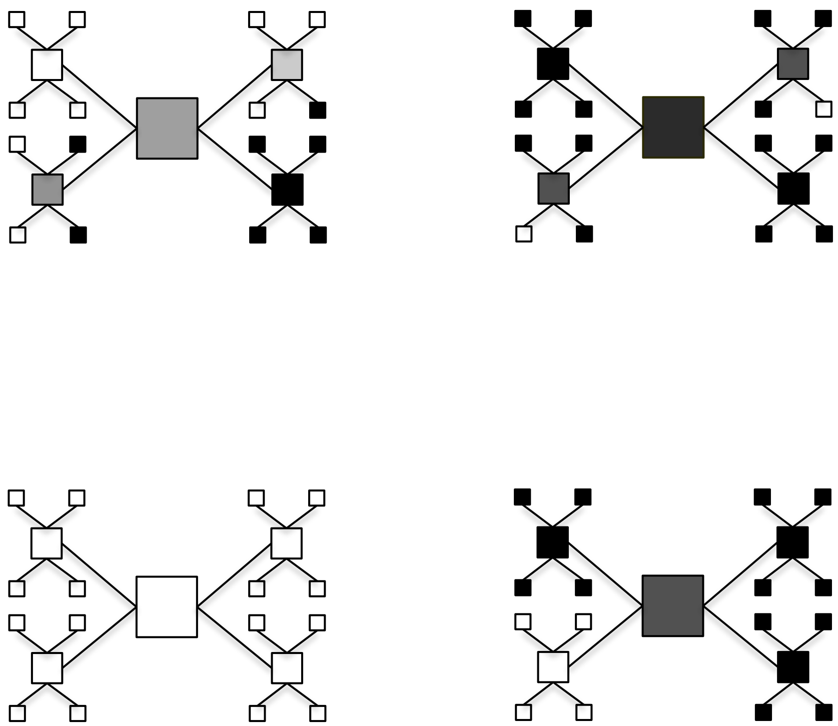
0	1/4	1	3/4
1/2	1	3/4	1
0	0	1	1
0	0	0	1



Space Representation

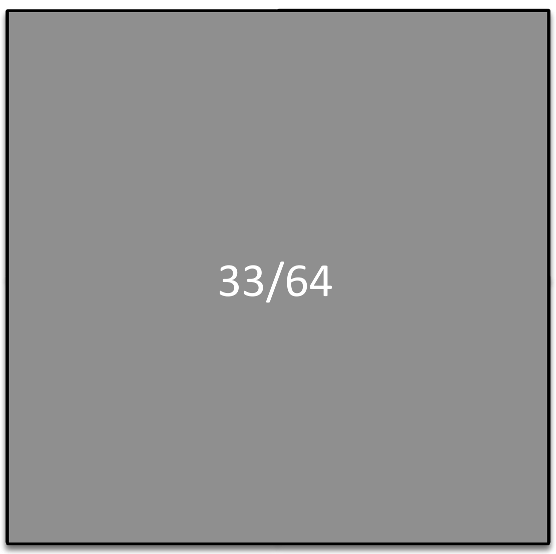
Quadtree and Spatial Superposition

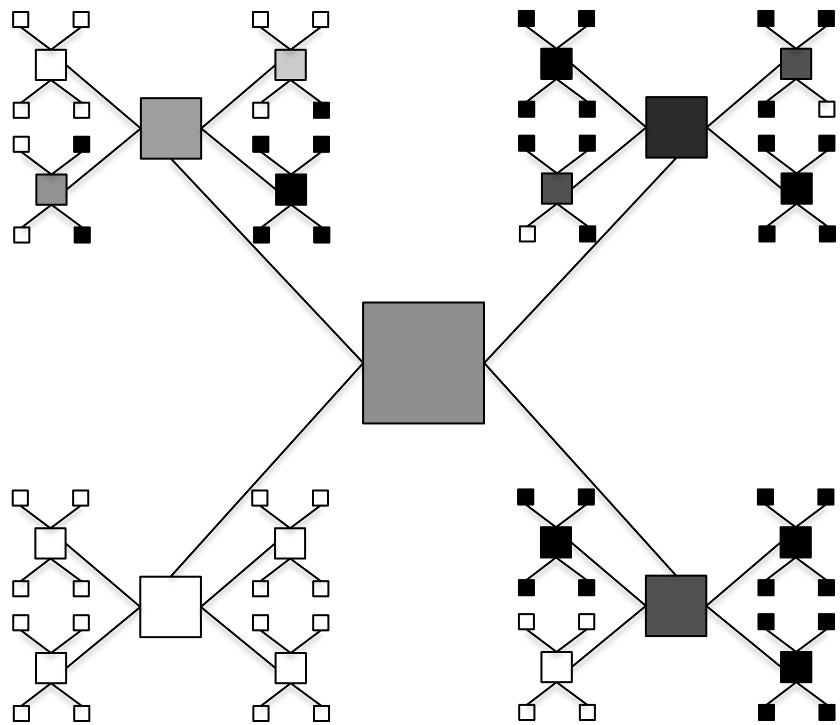




Space Representation

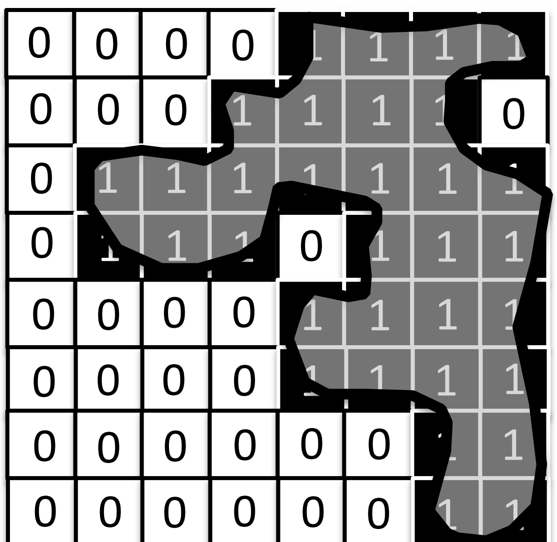
Quadtree and Spatial Superposition

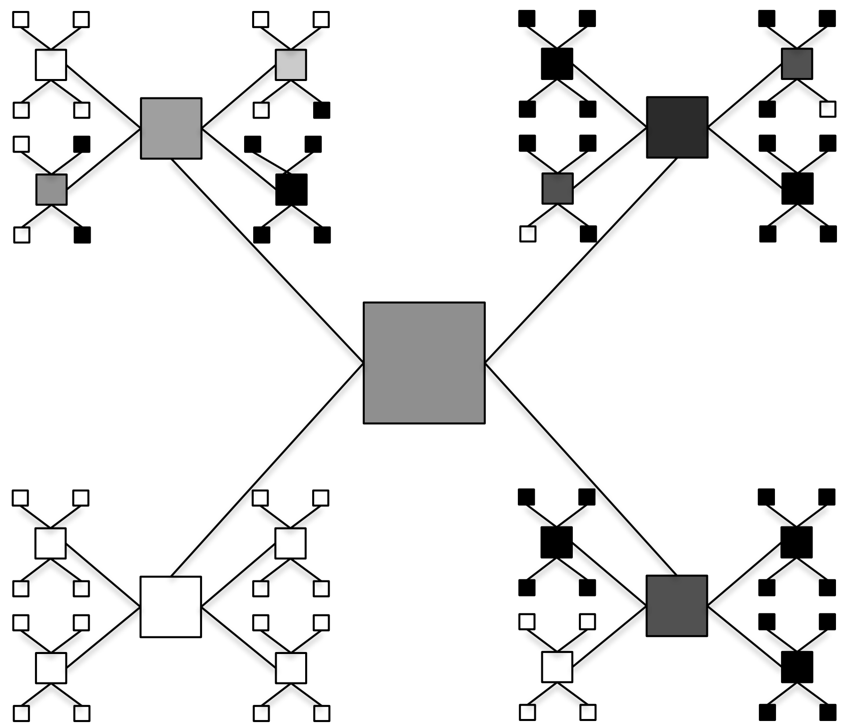




Space Representation

More compact representation



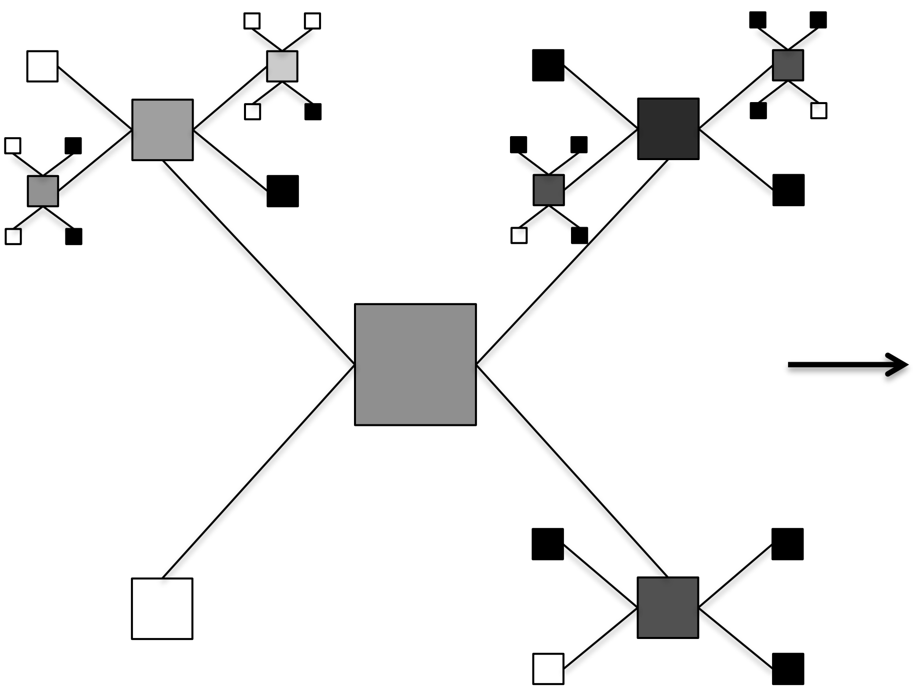


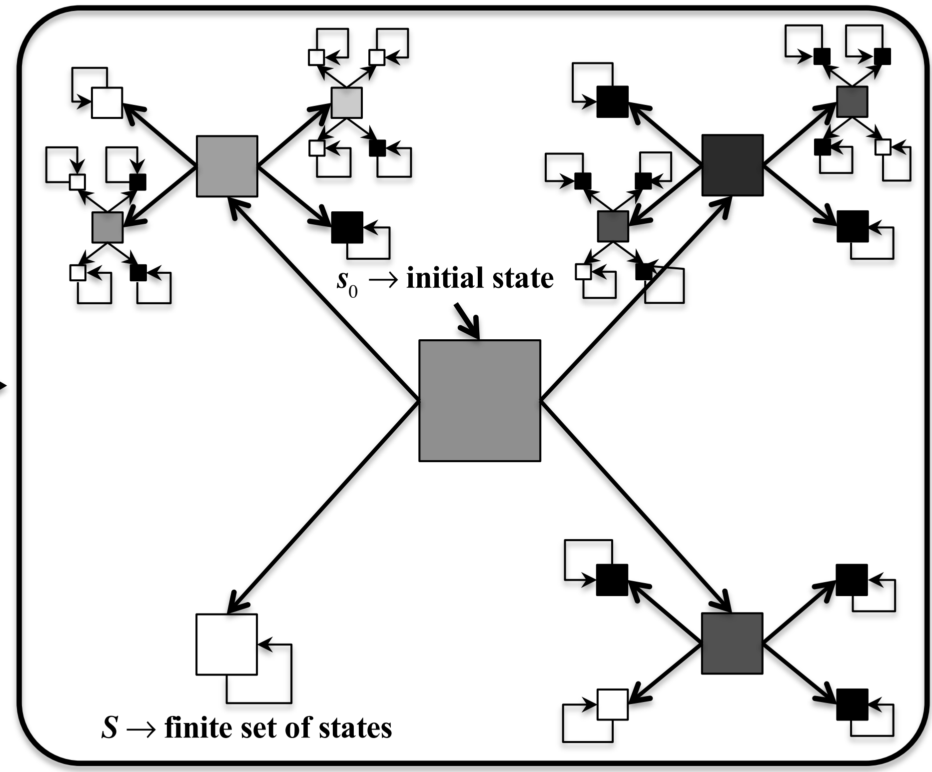
Pruning the tree

Reasoning over QuadTrees

From QuadTrees to Kripke Structures

$M = (S, s_0, R, L)$





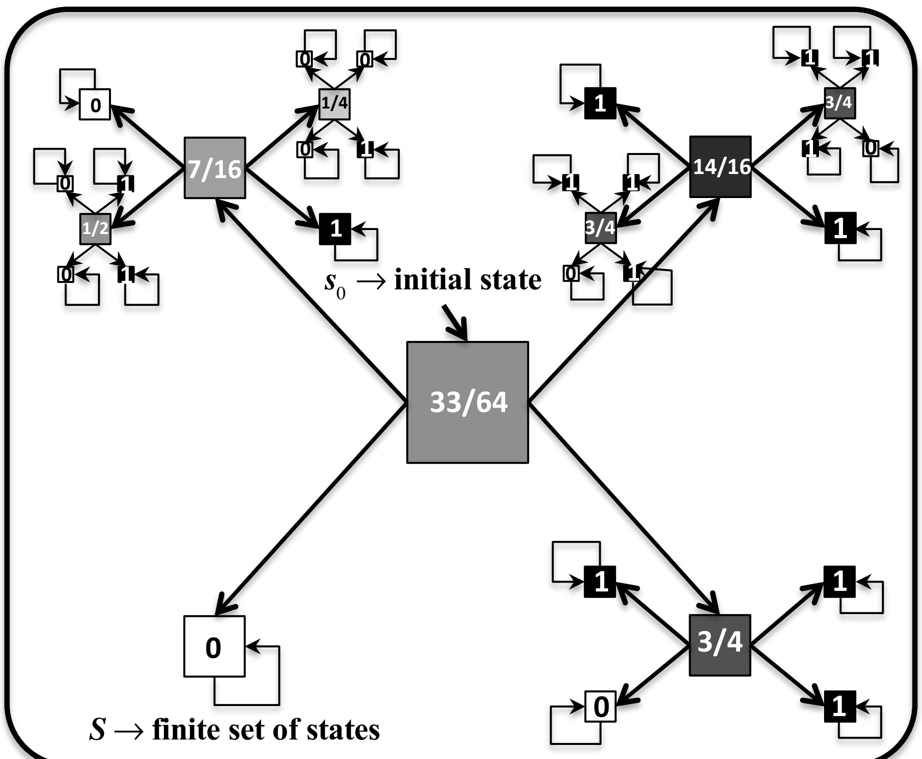
$R \in S \times S$ is a total transition relation
$\forall s \in S, \exists t \in S : (s,t) \in R$

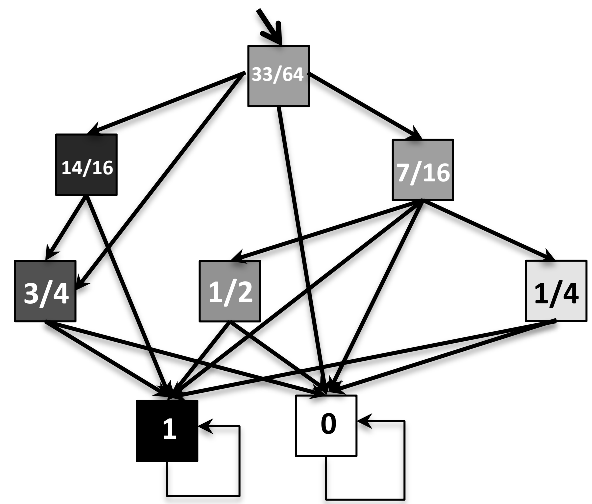
$L : S \rightarrow 2^{AP}$ is a labeling (or interpretation) function

Reasoning over QuadTrees

Compact Kripke Structures

$M = (S, s_0, R, L)$





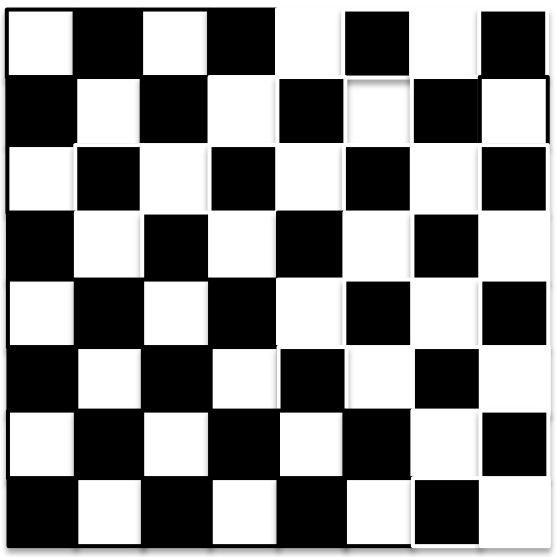
$R \in S \times S$ **is a total transition relation**
$\forall s \in S, \exists t \in S : (s,t) \in R$

$L : S \rightarrow 2^{AP}$ **is a labeling (or interpretation) function**

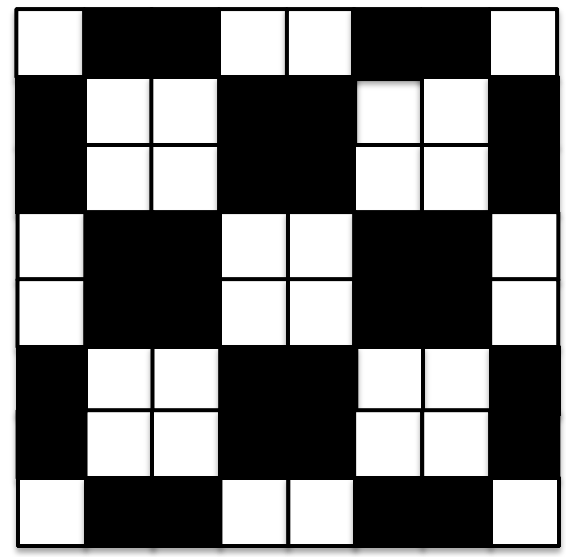
Tree Spatial Superposition Logic (TSSL)

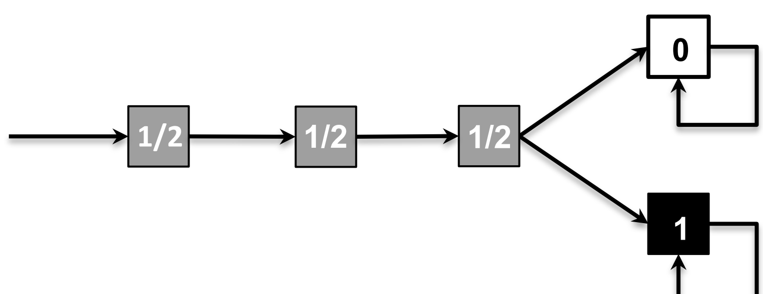
Aydin-Gol, Bartocci, Belta, CDC 2014

Problem: chessboard example



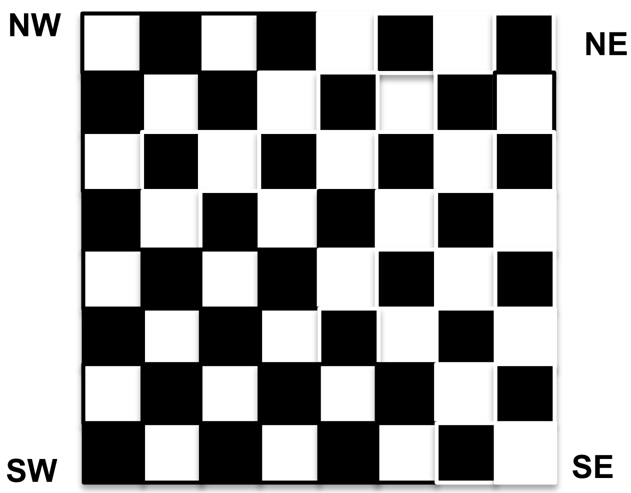
They both satisfies the same LSSL properties

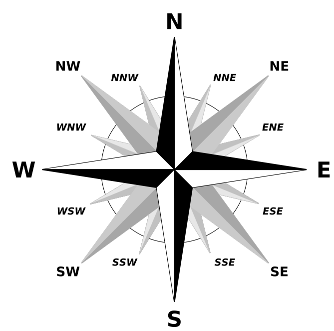


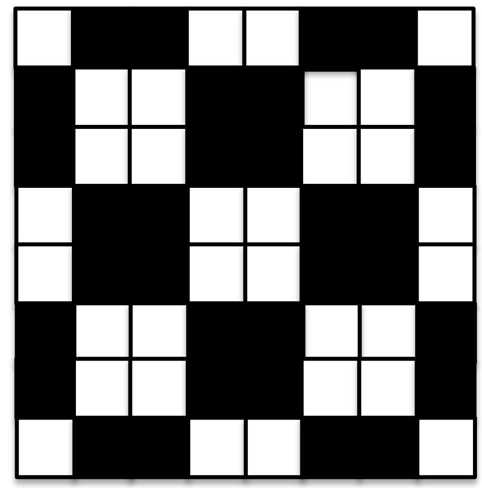


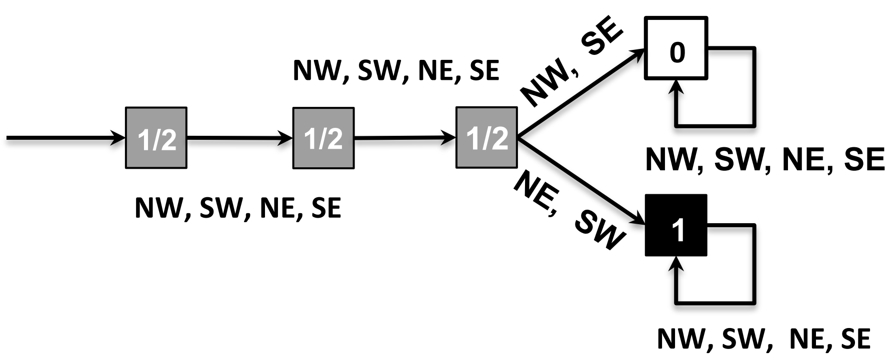
Tree Spatial Superposition Logic (TSSL)

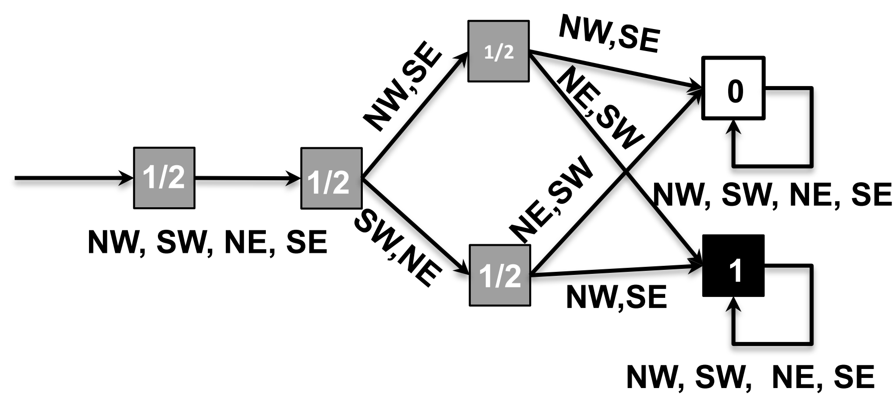
Adding directions (NW, NE, SW, SE) to transitions







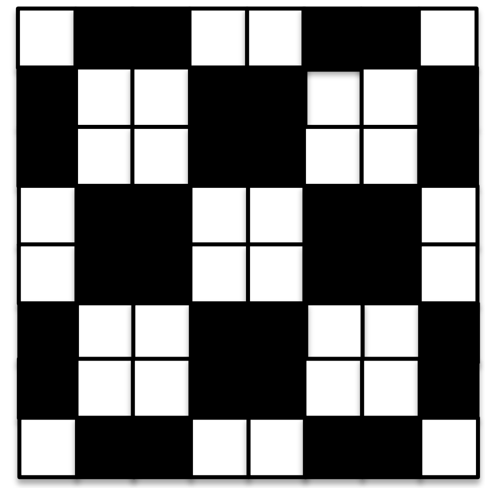




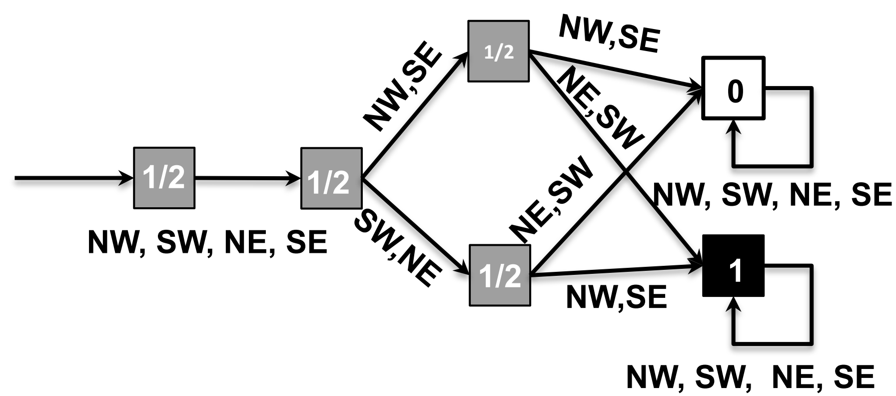
Tree Spatial Superposition Logic (TSSL)

Syntax $\varphi ::= \bot \mid m \sim d \mid \neg\varphi \mid \varphi \wedge \varphi \mid \exists_B \bigcirc \varphi \mid \forall_B \bigcirc \varphi \mid \exists_B \varphi U_k \varphi \mid \forall_B \varphi U_k \varphi$

$\sim \in \{\leq, \geq\}, d \in [0, b],$
$b \in \mathbb{R}_+, k \in \mathbb{N}_{>0}$
$B \subseteq D$
$B \neq \varnothing$



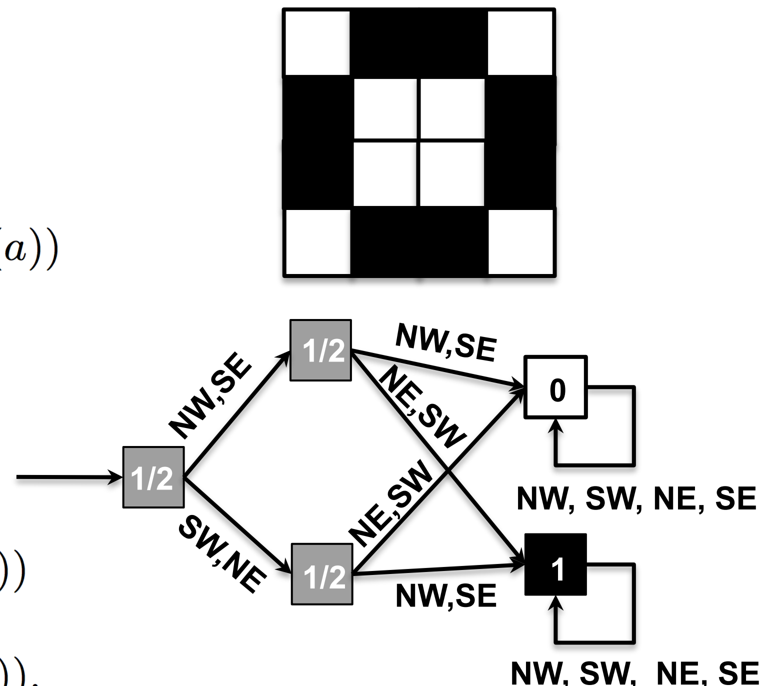
$$\forall_* \left(\boldsymbol{m} \geq \frac{1}{2} \right) \boldsymbol{U}_1 \left(\exists_{\{NW,SE\}} \boldsymbol{X}(\boldsymbol{\varphi}_1) \wedge \exists_{\{NE,SW\}} \boldsymbol{X}(\boldsymbol{\varphi}_2) \right)$$
$$\boldsymbol{\varphi}_1 = \exists_{\{NW,SE\}} \boldsymbol{X}(\boldsymbol{m} \leq 0) \wedge \exists_{\{NE,SW\}} \boldsymbol{X}(\boldsymbol{m} \geq 1)$$
$$\boldsymbol{\varphi}_2 = \exists_{\{NE,SW\}} \boldsymbol{X}(\boldsymbol{m} \leq 0) \wedge \exists_{\{NW,SE\}} \boldsymbol{X}(\boldsymbol{m} \geq 1)$$



Tree Spatial Superposition Logic (TSSL)

Quantitative Semantics

$$
\begin{aligned}
\rho_s(\top, a) &= b \\
\rho_s(m \sim d, a) &= (\sim \text{ is } \geq)?([m](a) - d) : (d - [m](a)) \\
\rho_s(\neg\varphi, a) &= -\rho_s(\varphi, a) \\
\rho_s(\varphi_1 \wedge \varphi_2, a) &= \min(\rho_s(\varphi_1, a), \rho_s(\varphi_2, a)) \\
\rho_s(\exists_B \bigcirc \varphi, a) &= 0.25 \max_{\pi^B \in LPath^B(a)} \rho_s(\pi_1^B) \\
\rho_s(\forall_B \bigcirc \varphi, a) &= 0.25 \min_{\pi^B \in LPath^B(a)} \rho_s(\pi_1^B) \\
\rho_s(\exists_B \varphi_1 U_k \varphi_2) &= \sup_{\pi^B \in LPath^B(a), i \in (0,k]} (\min(0.25 \\
&\quad \rho_s(\varphi_2, \pi_i^B), \inf_{j \in [0,i)} 0.25^j \rho_s(\varphi_1, \pi_j^B))) \\
\rho_s(\forall_B \varphi_1 U_k \varphi_2) &= \inf_{\pi^B \in LPath^B(a), i \in (0,k]} (\min(0.25 \\
&\quad \rho_s(\varphi_2, \pi_i^B), \inf_{j \in [0,i)} 0.25^j \rho_s(\varphi_1, \pi_j^B))).
\end{aligned}
$$



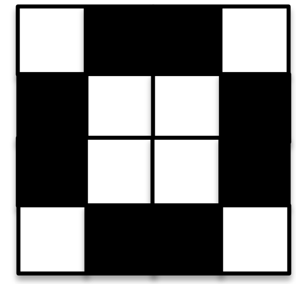
$$\rho_s\left(\exists_B X(m \geq 0.7), s\right) = \frac{0.5 - 0.7}{4} = -0.05$$

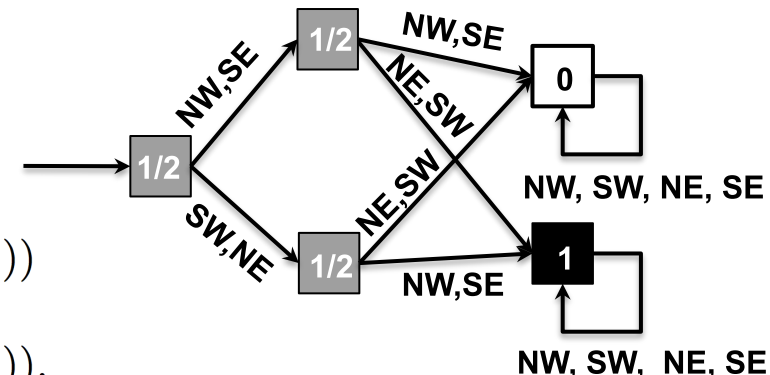
Tree Spatial Superposition Logic (TSSL)

Quantitative Semantics

$$
\begin{aligned}
\rho_s(\top, a) &= b \\
\rho_s(m \sim d, a) &= (\sim \text{ is } \geq)?([m](a) - d) : (d - [m](a)) \\
\rho_s(\neg\varphi, a) &= -\rho_s(\varphi, a) \\
\rho_s(\varphi_1 \wedge \varphi_2, a) &= \min(\rho_s(\varphi_1, a), \rho_s(\varphi_2, a)) \\
\rho_s(\exists_B \bigcirc \varphi, a) &= 0.25 \max_{\pi^B \in LPath^B(a)} \rho_s(\pi_1^B) \\
\rho_s(\forall_B \bigcirc \varphi, a) &= 0.25 \min_{\pi^B \in LPath^B(a)} \rho_s(\pi_1^B) \\
\rho_s(\exists_B \varphi_1 U_k \varphi_2) &= \sup_{\pi^B \in LPath^B(a), i \in (0,k]} (\min(0.25 \\
&\quad \rho_s(\varphi_2, \pi_i^B), \inf_{j \in [0,i)} 0.25^j \rho_s(\varphi_1, \pi_j^B))) \\
\rho_s(\forall_B \varphi_1 U_k \varphi_2) &= \inf_{\pi^B \in LPath^B(a), i \in (0,k]} (\min(0.25 \\
&\quad \rho_s(\varphi_2, \pi_i^B), \inf_{j \in [0,i)} 0.25^j \rho_s(\varphi_1, \pi_j^B))).
\end{aligned}
$$

$$
\begin{aligned}
\rho_s(\varphi, a_0) > 0 &\Rightarrow Q \models \varphi \\
\rho_s(\varphi, a_0) < 0 &\Rightarrow Q \not\models \varphi
\end{aligned}
$$





$$\rho_s(\exists_B X(m \geq 0.7), s) = \frac{0.5 - 0.7}{4} = -0.05$$

Spatio-Temporal Logics

SpaTel: Spatial-Temporal Logic

SpaTel: Spatial-Temporal Logic

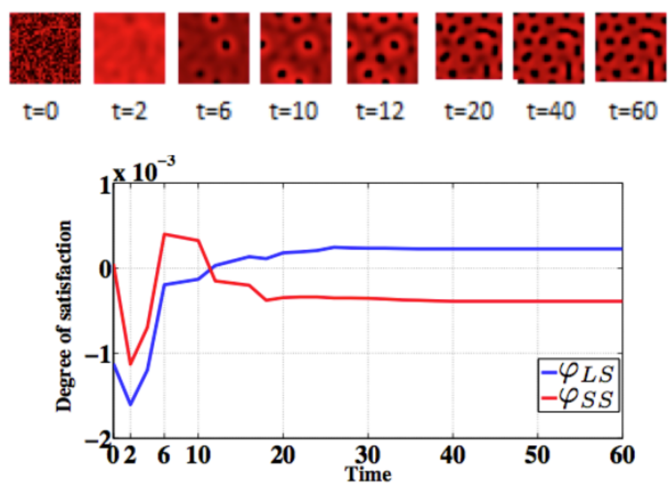
Spatial Temporal Logic = Temporal Logic + Spatial Logic

SpaTel | **STL** | **TSSL**

$\Phi_1 : F_{[0,30]} G_{[0,60]} \varphi_{SS}$

"The SS pattern appears within the first 30 sec and then persists for 60 sec."

$\Phi_2 : F_{[0,30]} G_{[0,60]} \varphi_{LS} \wedge G_{[0,60]} \neg \varphi_{SS}.$



"The LS pattern emerges within the first 30 seconds and remains for the next 60 seconds AND the SS pattern never occurs during the first 60 seconds, i.e. the large spots pattern is established unambiguously. "

(with Iman Haghighi, Austin Jones, Zhaodan Kong, Ezio Bartocci, and Radu Grosu)

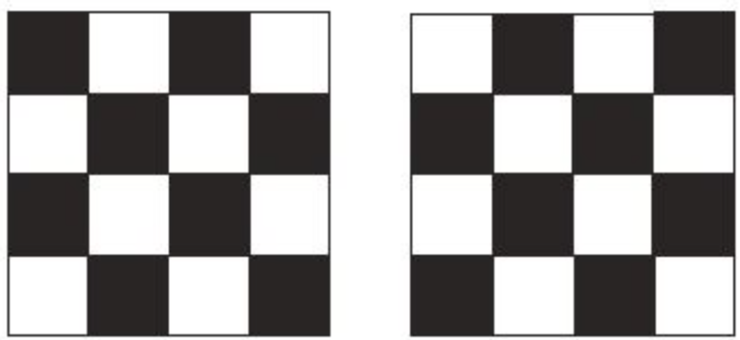
18

SpaTel: Spatial-Temporal Logic

Syntax

$\varphi ::= \bot \mid m \sim d \mid \neg\varphi \mid \varphi \wedge \varphi \mid \exists_B \bigcirc \varphi \mid \forall_B \bigcirc \varphi \mid \exists_B \varphi_1 \tilde{U}_k \varphi_2 \mid \forall_B \varphi_1 \tilde{U}_k \varphi_2$ ← **TSSL**

$\Phi ::= \varphi \mid \neg\Phi \mid \Phi_1 \wedge \Phi_2 \mid \Phi_1 U_I \; \Phi_2$ ← **Signal Temporal Logic**



$$\Phi = F_{[0,T]}(\varphi_{cboard} \rightarrow G_{[0,1]}\varphi_{flipcboard})$$

$$\varphi_{cboard} = \forall_{B^*} \tilde{F}_2 \left(\forall_{\{SW,NE\}} \bigcirc w \wedge \forall_{\{NW,SE\}} \bigcirc b \right)$$

$$\varphi_{flipcboard} = \forall_{B^*} \tilde{F}_2 \left(\forall_{\{NW,SE\}} \bigcirc w \wedge \forall_{\{SW,NE\}} \bigcirc b \right)$$

SpaTel: Spatial-Temporal Logic

Syntax

$\varphi ::= \bot \mid m \sim d \mid \neg\varphi \mid \varphi \wedge \varphi \mid \exists_B \bigcirc \varphi \mid \forall_B \bigcirc \varphi \mid \exists_B \varphi_1 \tilde{U}_k \varphi_2 \mid \forall_B \varphi_1 \tilde{U}_k \varphi_2$ ← **TSSL**

$\Phi ::= \varphi \mid \neg\Phi \mid \Phi_1 \wedge \Phi_2 \mid \Phi_1 U_I \; \Phi_2$ ← **Signal Temporal Logic**

Quantitative Semantics

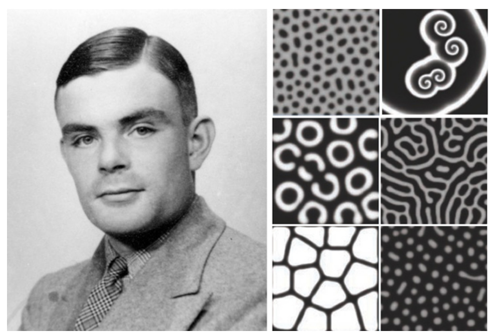
$$
\begin{aligned}
\rho_t(\neg\Phi, Q, t) &= -\rho_t(\Phi, Q, t) \\
\rho_t(\Phi_1 \wedge \Phi_2, Q, t) &= \min(\rho_t(\Phi_1, Q, t), \rho_t(\Phi_2, Q, t)) \\
\rho_t(\Phi_1 U_{[I_1, I_2]} \Phi_2, Q, t) &= \sup_{t' \in [t+I_1, t+I_2]} (\min(\rho_t(\Phi_2, Q, t'), \\
&\quad \inf_{t'' \in [t, t']} \rho_t(\Phi_1, Q, t''))) \\
\rho_t(\varphi, Q, t) &= \rho_s(\varphi, a_0(t))
\end{aligned}
$$

Pattern Synthesis Problem

Problem: **Find (the optimal)** $(p_1,\ldots,p_n) : M(p_1,\ldots,p_n) \models P$

Parameters — Model — Satisfies — Property

Example of models:



Parameters

$$\dot{u} = F(u,v) - d_u v + D_u \nabla u$$
$$\dot{v} = G(u,v) - d_v v + D_v \nabla v$$

REACTION — DEGRADATION — DIFFUSION

Turing Diffusion Model

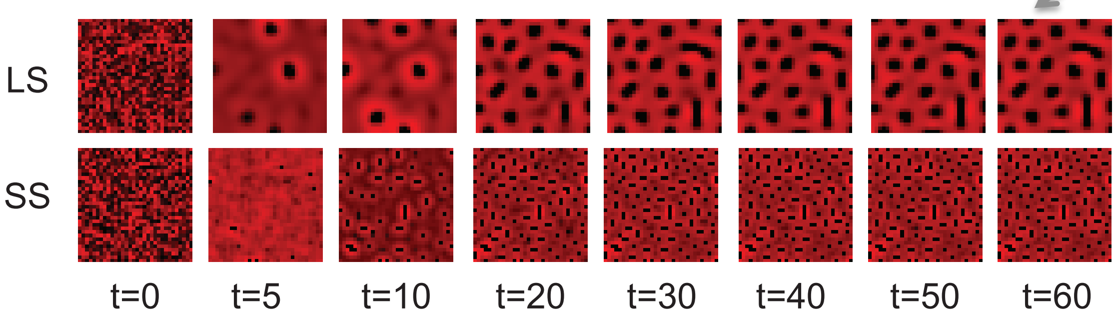
System Design and Parameter Synthesis

Problem: Find (the optimal) $(p_1,\ldots,p_n) : M(p_1,\ldots,p_n) \models P$

Parameters **Model** **Satisfies** **Property**

Example of properties:

Pattern 1



Pattern 2

Parameters

$$\dot{u} = F(u,v) - d_u v + D_u \nabla u$$
$$\dot{v} = G(u,v) - d_v v + D_v \nabla v$$

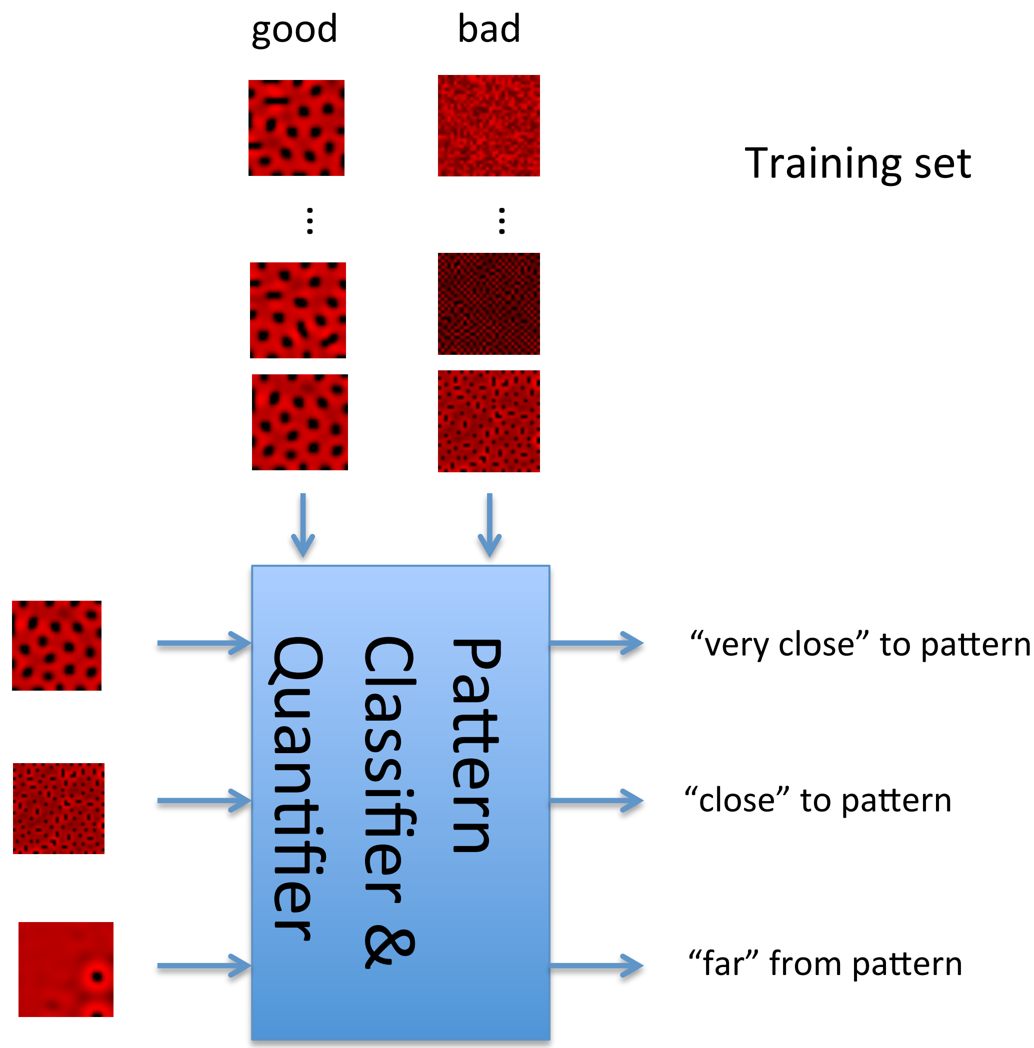
REACTION DEGRADATION DIFFUSION

Turing Diffusion Model

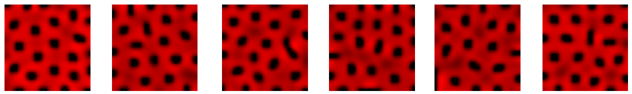
System Design and Parameter Synthesis

1. Learning (training)

2. Classification and Quantification



System Design and Parameter Synthesis

Desired pattern: 

Particle swarm optimization over the parameter space.

Fitness: The quantitative valuation $[\![\Phi_{LS}]\!]$ of the image from TSSL formula Φ_{LS}

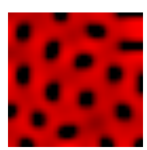
Turing diffusion system:

$$x_{k+1}(i,j) = x_k(i,j) + \Delta_t \left(D_x \left(\frac{1}{4} \sum_{m,n \in \{-1,1\}} x_k(i+m, j+n) - x_k(i,j) \right) + x_k(i,j) y_k(i,j) - x_k(i,j) - 12 \right)$$

$$y_{k+1}(i,j) = y_k(i,j) + \Delta_t \left(D_y \left(\frac{1}{4} \sum_{m,n \in \{-1,1\}} y_k(i+m, j+n) - y_k(i,j) \right) - x_k(i,j) y_k(i,j) + 16 \right)$$

Unknown parameters: D_x, D_y

$\Phi_{LS,3}$: learned from LS^{+} and LS^{-}_{3} (95.64 % on test)

PSO: $D_x = 6.25$ $D_y = 29.417$  $[\![\Phi_{LS,3}]\!](\mathcal{Q}) = 0.0011$

System Design and Parameter Synthesis

Algorithm 2 Parameter Synthesis

Input: SpaTeL formula Φ, system model S, parameter ranges $\mathcal{P}$, number of traces N, PSO parameters (W, r_p, r_g, m), termination constant k

Output: Parameter values Π^*

for $1 \leq j \leq m$ **do**

  $z_i \leftarrow$ initialize particle positions $v_i \leftarrow$ initialize particle velocities

end

while Π^* *has changed during the last* k *iterations* **do**

  for $1 \leq j \leq N$ **do**

    $Q_{u_j, z_i} \leftarrow$ draw a sample trace of the system $\rho_t(\Phi, Q_{u_j, z_i}) \leftarrow$ calculate quantitative valuation of Q_{u_j, z_i} with respect to Φ

  end

  $[z_i, v_i] \leftarrow$ update particles $\Pi^* \leftarrow$ the best position so far (z^{best})

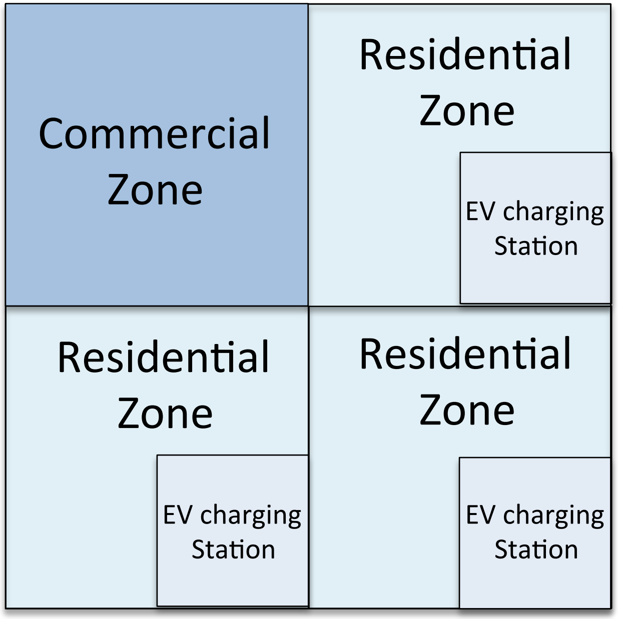
end

Update particles

$$v_i \leftarrow W v_i + \eta(0, r_p)(z_i^{best} - z_i) + \eta(0, r_g)(z^{best} - z_i)$$

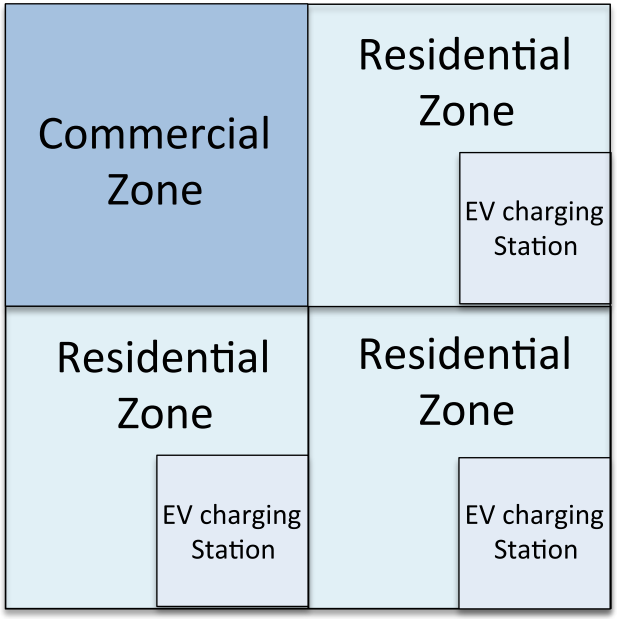
$$z_i \leftarrow z_i + v_i$$

Smart Neighborhood Model (DSM)



Inside each building, $n_i(t)$ appliances are consuming rate r_i KW.
The arrival distribution of appliances for building class *i* over the period [t,t+1] is a Poisson distributed with a rate $\lambda(U_i-p_j(t))/U_i$, where U_i is the utility of an appliance of class i and $p_j(t)$ is the broadcast price for neighborhood class j,j in {c,r} with residential building and EV station charged by the same price

Smart Neighborhood Model (DSM)



Specification

The total power consumption of the commercial buildings is always less than 150; the power consumption is below 150 in each EV station and below 25 in each of the residential neighborhoods in the first 12 hours; after 12 hours, the power consumption of each EV station is between 30 and 200; after 15 hours, the power consumption in all residential areas is above 5.

$$\Phi = \Phi_1 \wedge \Phi_2 \wedge \Phi_3 \wedge \Phi_4$$

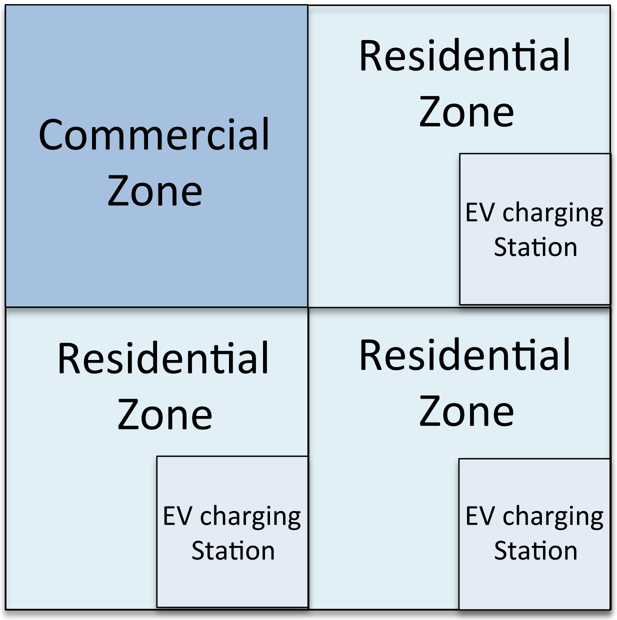
$$\Phi_1 = G_{[0,24]}\left(\forall_{NW} \bigcirc (m \leq 150)\right)$$

$$\Phi_2 = G_{[0,12]}\left(\forall_{\{NE,SE,SW\}} \bigcirc \left(\forall_{\{NW,SE,SW\}} \bigcirc (m \leq 25) \wedge \forall_{\{SE\}} \bigcirc (m \leq 150)\right)\right)$$

$$\Phi_3 = G_{[12,18]}\left(\forall_{\{NE,SE,SW\}} \bigcirc \left(\forall_{\{SE\}} \bigcirc (m \leq 200 \wedge m \geq 30)\right)\right)$$

$$\Phi_4 = G_{[15,18]}\left(\forall_{\{NE,SE,SW\}} \bigcirc \left(\forall_{\{NW,SE,SW\}} \bigcirc (m \geq 5)\right)\right)$$

Smart Neighborhood Model (DSM)



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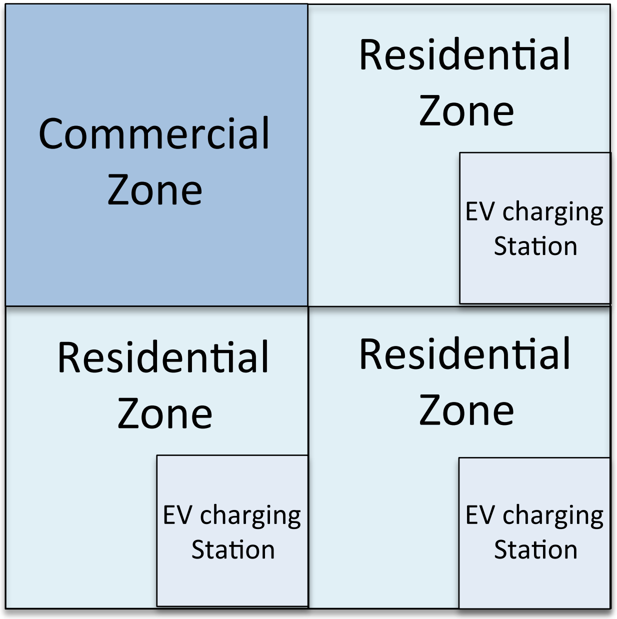
Smart Neighborhood Model (DSM)



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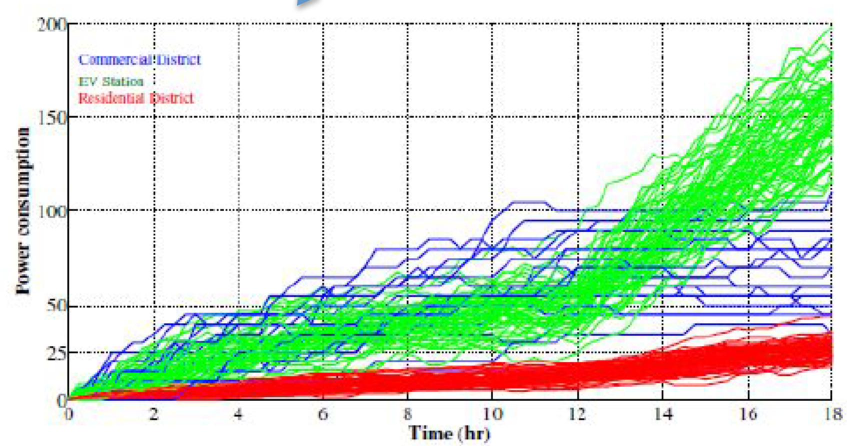
Parameter Synthesis

N	(RP_D^*, RP_N^*, CP_N^*)	$\hat{p}$
1	(4.69, 4.41, 19.70)	0.43
10	(4.42, 4.74, 19.70)	0.75
20	(4.40, 4.75, 19.70)	0.73
50	(4.15, 5.05, 19.70)	0.90



Spatio-Temporal Logics
Signal Spatio-Temporal Logic

Nenzi, Bortolussi, VALUETOOLS, 2014
Nenzi, Bortolussi, Ciancia, Loreti, Massink, RV, 2015

Signal Spatio-Temporal Logic (SSTL)

SSTL Syntax

$$\varphi := \mu \mid \neg\varphi \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1\, \mathcal{U}_{[t_1,t_2]}\, \varphi_2 \mid \diamonddiamond_{[w_1,w_2]}\varphi \mid \varphi_1\, \mathcal{S}_{[w_1,w_2]}\varphi_2$$

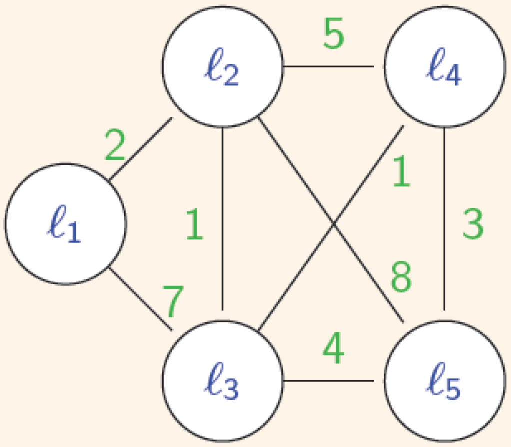
where $t_1, t_2, w_1, w_2 \in \mathbb{R}_{\geq 0}$.

In addition $\mathcal{F}_{[a,b]}\varphi := \top\, \mathcal{U}_{[a,b]}\varphi$, $\mathcal{G}_{[a,b]}\varphi := \neg\mathcal{F}_{[a,b]}\neg\varphi$, $\boxdot_{[w_1,w_2]}\varphi := \neg \diamonddiamond_{[w_1,w_2]} \neg\varphi$.

The space

A **weighted graph** is a tuple G=(L,E,w) where:

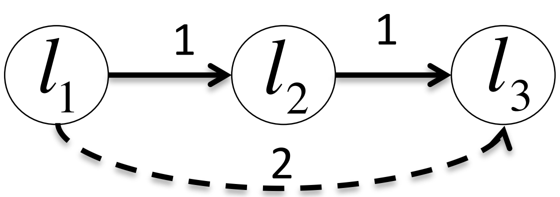
- $L = \{\ell_1, \ldots, \ell_n\}$
- E is the set of edges
- $w : E \to \mathbb{R}$ is the function that identy the weight associated with each edge.



Somewhere

$$(\vec{x}, t, \ell) \models \diamond_{[w_1, w_2]} \varphi \Leftrightarrow \exists \ell' \in L \text{ s.t.} \{(\ell', \ell) \in E^* \wedge w_1 \leqslant w(\ell', \ell) \leqslant w_2 \wedge (\vec{x}, t, \ell') \models \varphi\}$$

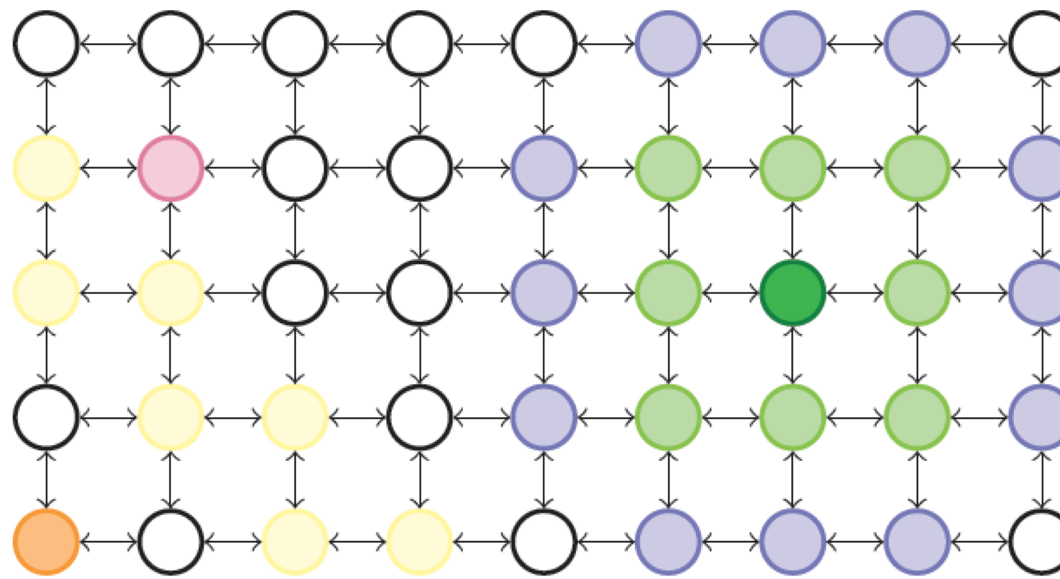
Shortest path



TRANSITIVE CLOSURE

$$E^* = \{(l_1, l_2), (l_2, l_3), (l_1, l_3)\}$$

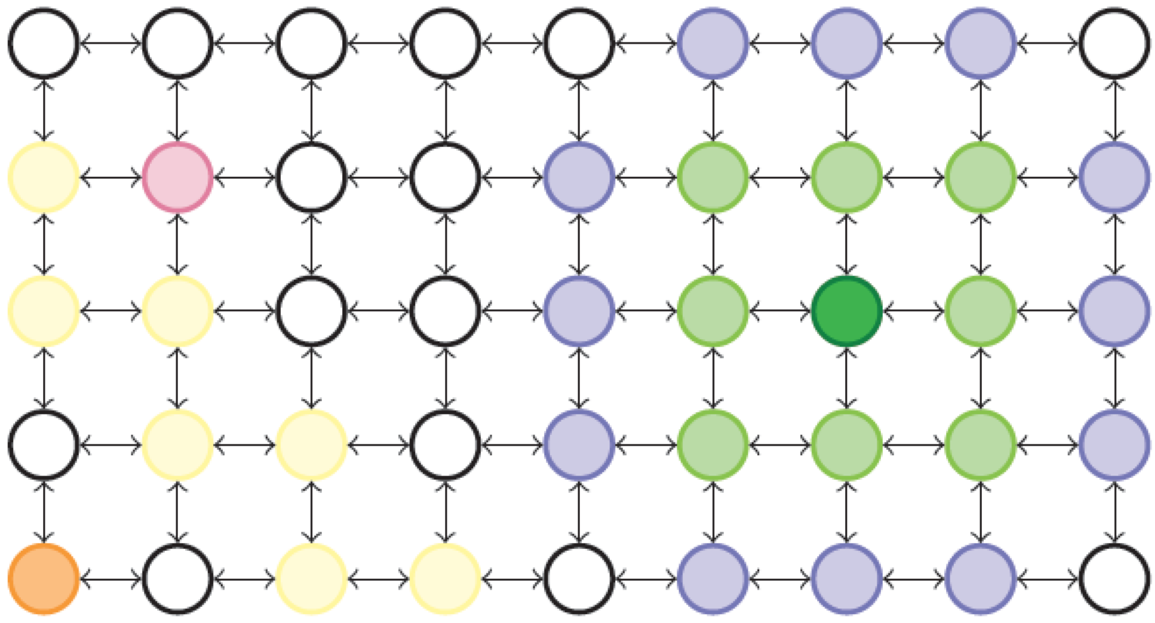
$$E = \{(l_1, l_2), (l_2, l_3)\}$$



The orange point satisfies $\diamond_{[3,5]}$ **purple**

Everywhere

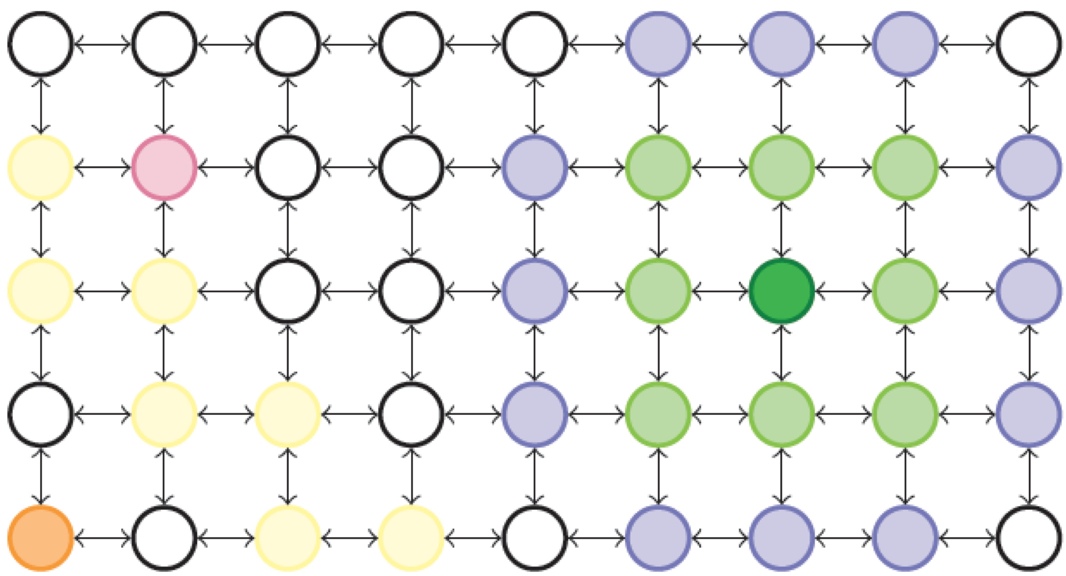
$$\boxdot_{[w_1,w_2]}\varphi := \neg \diamondsuit_{[w_1,w_2]} \neg\varphi$$



The orange point satisfies $\boxdot_{[2,3]}$ **yellow**

Surround

$$(\vec{x}, t, \ell) \models \varphi_1 \mathcal{S}_{[w_1,w_2]} \varphi_2 \Leftrightarrow \exists A \subseteq L^{\ell}_{[0,w_2]} : \ell \in A \wedge \forall \ell' \in A, (\vec{x}, t, \ell') \models \varphi_1 \wedge B^+(A) \subseteq L^{\ell}_{[w_1,w_2]} \wedge$$
$$\forall \ell'' \in B^+(A), (x, t, \ell'') \models \varphi_2.$$



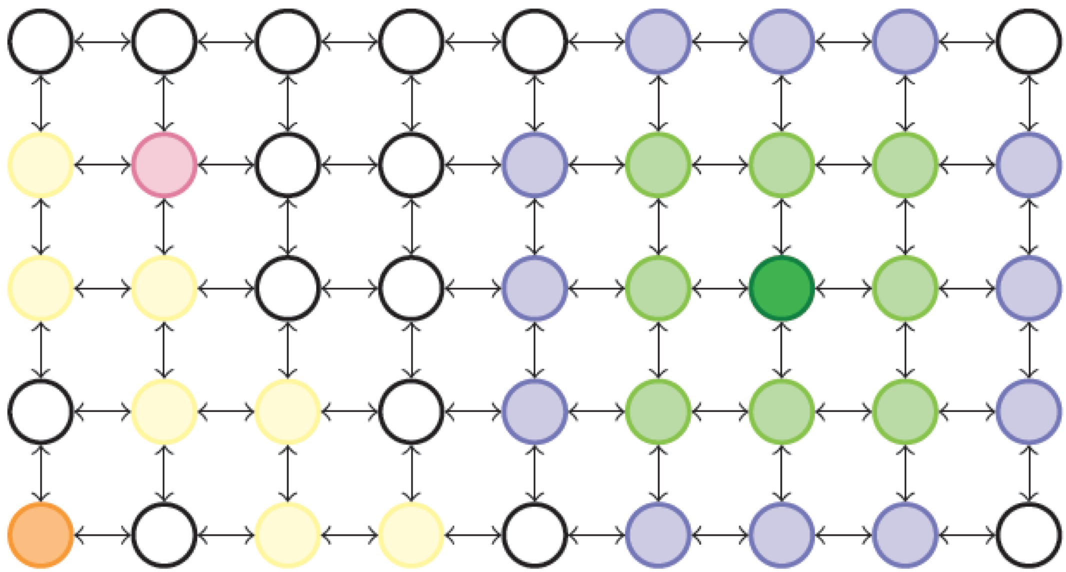
The dark green point satisfies **green $\mathcal{S}_{[2,3]}$ violet**

Green points satisfy **green $\mathcal{S}_{[0,100]}$ violet**

Spatio-temporal signals

$$(\vec{x}, t, \ell) \models \varphi_1 \mathcal{S}_{[w_1,w_2]} \varphi_2 \Leftrightarrow \exists A \subseteq L^{\ell}_{[0,w_2]} : \ell \in A \wedge \forall \ell' \in A, (\vec{x}, t, \ell') \models \varphi_1 \wedge B^+(A) \subseteq L^{\ell}_{[w_1,w_2]} \wedge$$

$$\forall \ell'' \in B^+(A), (x, t, \ell'') \models \varphi_2.$$



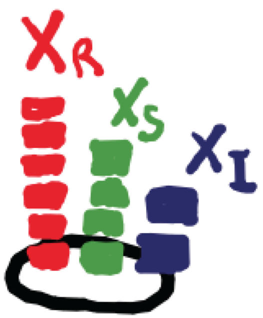
Green points satisfy **green** $\mathcal{S}_{[0,100]}$ **violet**

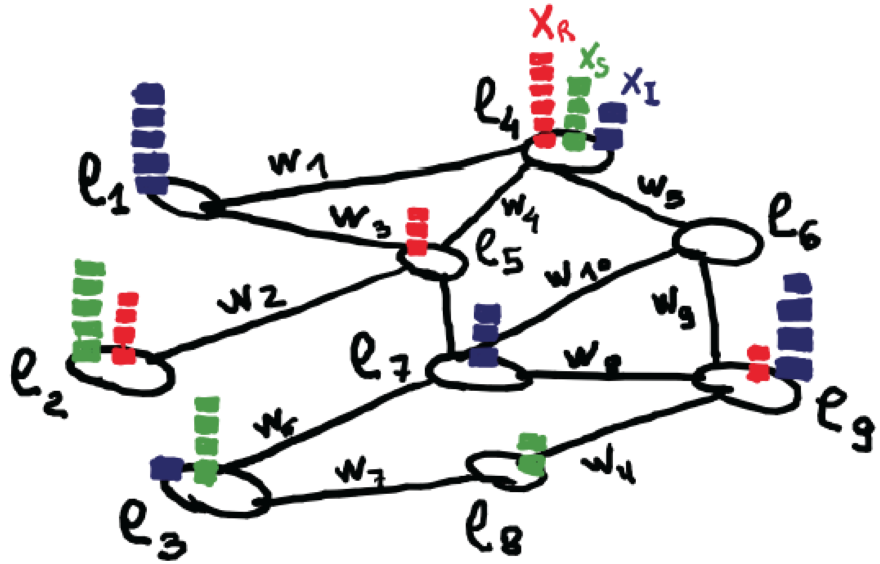
The dark green point satisfies **green** $\mathcal{S}_{[2,3]}$ **violet**

Spatio-temporal signals

Spatio-temporal trace

$\vec{x} : \mathbb{T} \times L \to \mathbb{R}^n, \qquad \vec{x}(t,\ell) = (x_1(t,\ell), \cdots, x_n(t,\ell))$



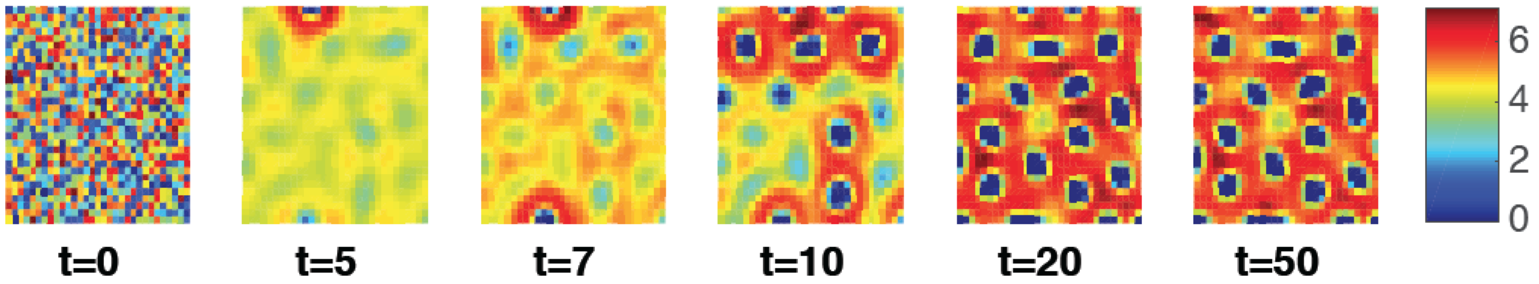


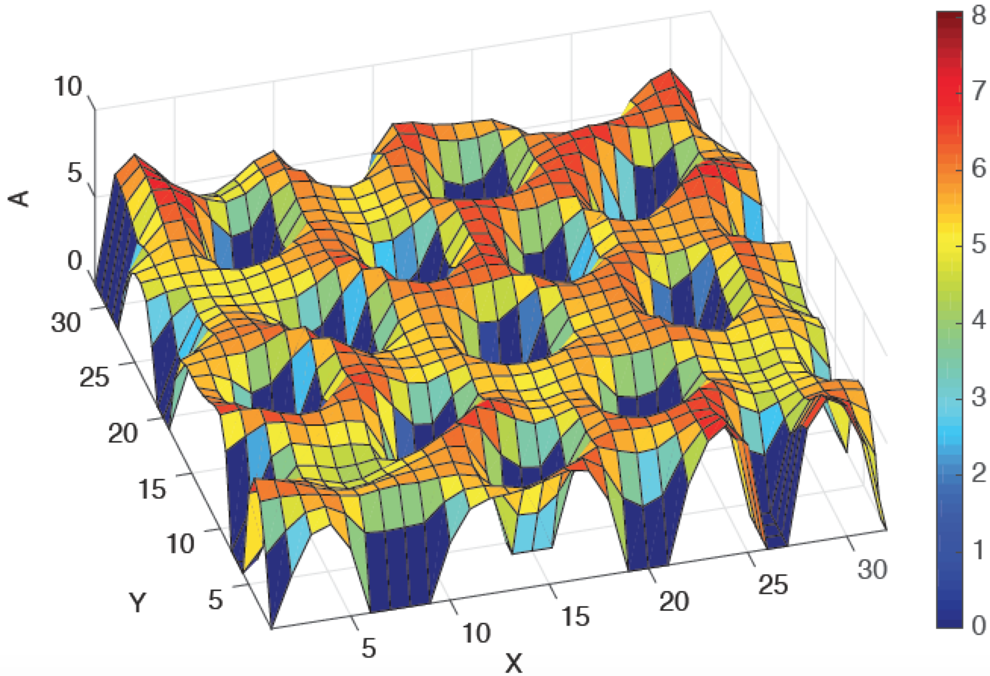
$\vec{x}(t) = (x_S(t), x_I(t), x_R(t))$

$\vec{x}(t,\ell) = (x_S(t,\ell), x_I(t,\ell), x_R(t,\ell))$

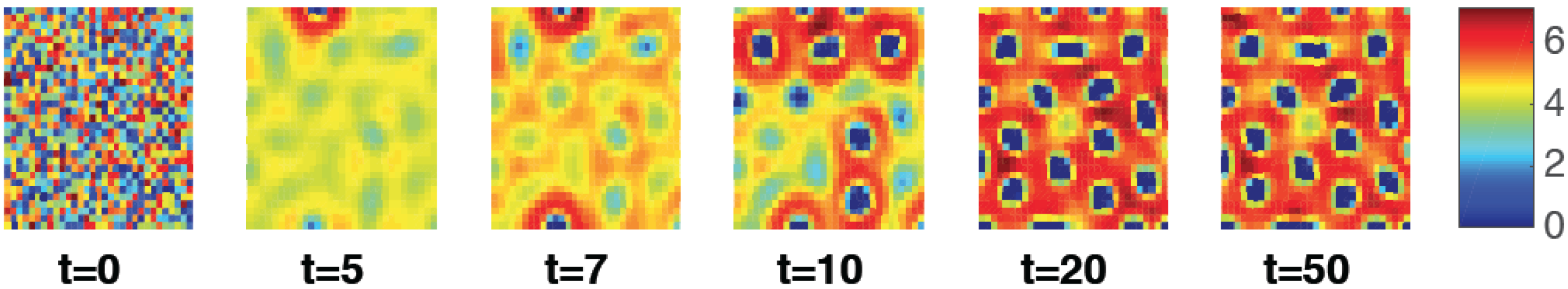
Pattern formation

The production of skin pigments that generate spots in animal furs:





Spots formation property



$\vec{x} = (x_A, x_B) : \mathbb{T} \times L \to \mathbb{R}^2$ is the trace (with $L = \{1,..,32\} \times \{1,..,32\}$)

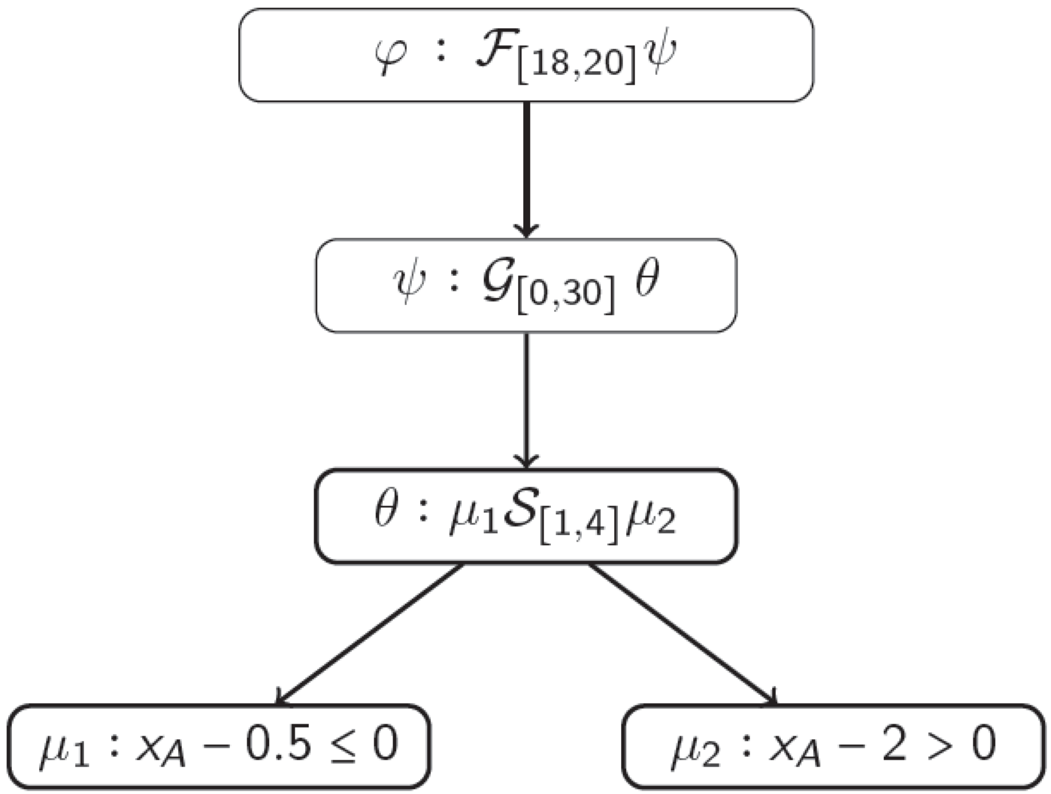
Spot formation property

$$\mathcal{F}_{[18,20]}\mathcal{G}_{[0,30]}((x_A \leq 0.5)\mathcal{S}[1,4](x_A > 2))$$

Monitoring SSTL

$$\varphi := \mathcal{F}_{[18,20]}\mathcal{G}_{[0,30]}((x_A \leq 0.5)\mathcal{S}[1,4](x_A > 2))$$

The parse tree of the formula:



Monitoring SSTL

$\mu_1 : x_A - 0.5 \leq 0$

$s_{\mu_1}(t,\ell),\ \rho_{\mu_1}(t,\ell)$

$x_A(t,\ell) - 0.5$

$x_A(t,\ell), x_B(t,\ell)$

$\mu_2 : x_A - 2 > 0$

$s_{\mu_2}(t,\ell),\ \rho_{\mu_2}(t,\ell)$

$x_A(t,\ell) - 2$

$x_A(t,\ell), x_B(t,\ell)$

Spatial Boolean signal

Spatial Quantitative signals

Secondary signals

Primary signals

Spatial Boolean Signal

$s_\varphi : [0, T] \times L \to \{0,1\}$ such that $s_\varphi(t,\ell) = 1 \Leftrightarrow (\vec{x}, t, \ell) \models \varphi$

Spatial Quantitative Signal

$\rho_\varphi : [0, T] \times L \to \mathbb{R} \cup \pm\infty$ such that $\rho_\varphi(t,\ell) = \rho(\varphi, \vec{x}, t, \ell)$

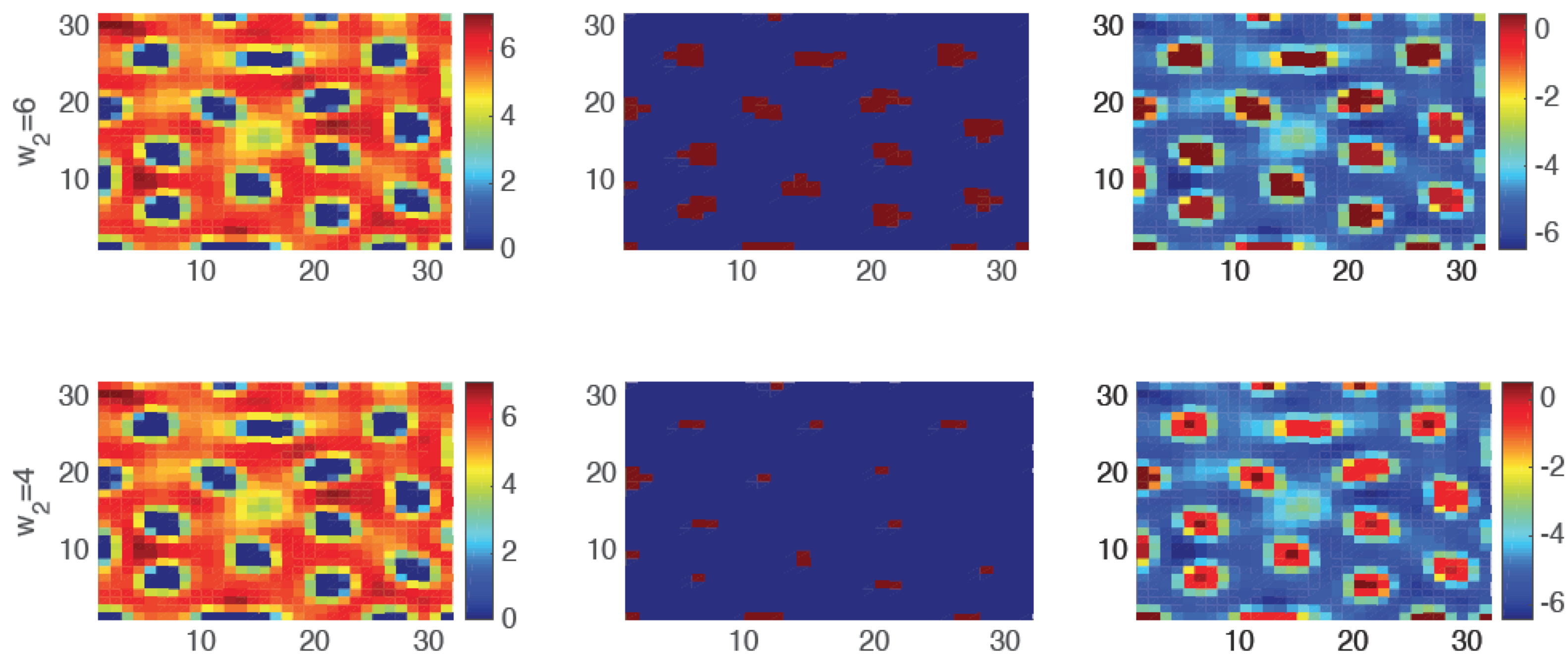
Quantitative Semantics

$$\begin{aligned}
\rho(\mu, \mathbf{x}, t, \ell) &= f(\mathbf{x}(t,\ell)) \quad \textit{where } \mu \equiv (f \geq 0) \\
\rho(\neg\varphi, \mathbf{x}, t, \ell) &= -\rho(\varphi, \mathbf{x}, t, \ell) \\
\rho(\varphi_1 \wedge \varphi_2, \mathbf{x}, t, \ell) &= \min(\rho(\varphi_1, \mathbf{x}, t, \ell), \rho(\varphi_2, \mathbf{x}, t, \ell)) \\
\rho(\varphi_1 \,\mathcal{U}_{[t_1,t_2]} \varphi_2, \mathbf{x}, t, \ell) &= \sup_{t' \in t+[t_1,t_2]} (\min\{\rho(\varphi_2, \mathbf{x}, t', \ell), \inf_{t'' \in [t,t']} (\rho(\varphi_1, \mathbf{x}, t'', \ell))\} \\
\rho(\diamondsuit_{[w_1,w_2]} \varphi, \mathbf{x}, t, \ell) &= \max\{\rho(\varphi, \mathbf{x}, t, \ell') \mid \ell' \in L, (\ell', \ell) \in E^* \\
&\quad \textit{and } w_1 \leqslant w(\ell', \ell) \leqslant w_2\} \\
\rho(\varphi_1 \,\mathcal{S}_{[w_1,w_2]} \varphi_2, \mathbf{x}, t, \ell) &= \max_{A \subseteq L^{\ell}_{[0,w_2]}, \ell \in A, B^+(A) \subseteq L^{\ell}_{[w_1,w_2]}} (\min(\min_{\ell' \in A} \rho(\varphi_1, \mathbf{x}, t, \ell'), \\
&\quad \min_{\ell'' \in B^+(A)} \rho(\varphi_2, \mathbf{x}, t, \ell''))).
\end{aligned}$$

Example

Spot formation property

$$\phi_{spot_{form}} := \mathcal{F}_{[T_{pattern}, T_{pattern}+\delta]}\mathcal{G}_{[0,T_{end}]}((x_A \leq h)\mathcal{S}[w_1, w_2](x_A > h))$$



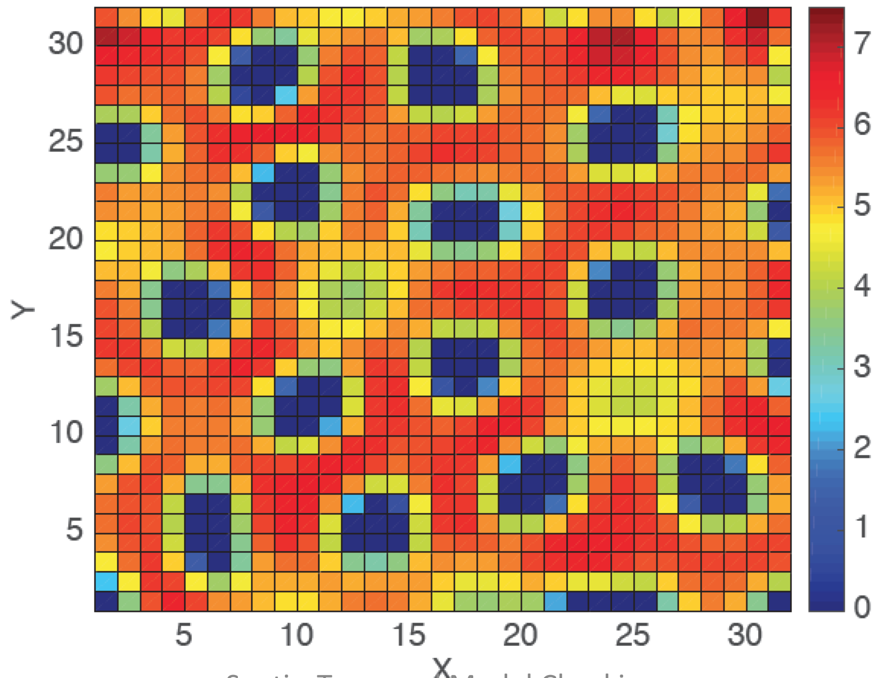
$x_A(50,\ell)$ **Boolean sat.** **Quantitative sat.**

Example

Pattern property

$$\phi_{pattern} := \boxdot[0, w] \diamondsuit [0, w'] \phi_{spot_{form}},$$

- **w** is the distance to cover all space
- **w′** measures the distance between spots



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