Fractional Sobolev spaces related to an ultraparabolic operator

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We propose a functional framework of fractional Sobolev spaces for a class of ultraparabolic Kolmogorov-type operators satisfying the weak Hörmander condition. We characterize these spaces as real interpolation of natural-order intrinsic Sobolev spaces recently introduced in [1] and prove continuous embeddings into L^p and intrinsic Hölder spaces from [2]. These embeddings naturally extend the standard Euclidean ones, coherently with the homogeneous structure of the associated Kolmogorov group. Our approach to interpolation is based on approximation of intrinsically regular functions, the latter heavily relying on integral estimates of the intrinsic Taylor remainder. The embeddings exploit the aforementioned interpolation property and the corresponding embeddings of natural-order intrinsic spaces. The talk is based on paper [3].

[1] Pascucci P., Pesce A.: Sobolev embeddings for kinetic Fokker-Planck equations, *Journal of Functional Analysis*, 286 (2024).

[2] Pagliarani S., Pascucci P., Pignotti M.: Intrinsic Taylor formula for Kolmogorov-type homogeneous groups, *Journal of Mathematical Analysis and Applications*, 435 (2016).

[3] Pesce A., Portaro S.: Fractional Sobolev spaces related to an ultraparabolic operator, *Journal of Evolution Equations*, 25 (2025).