Uniform Logical Characterizations of Testing Equivalences for Nondeterministic, Probabilistic and Markovian Processes

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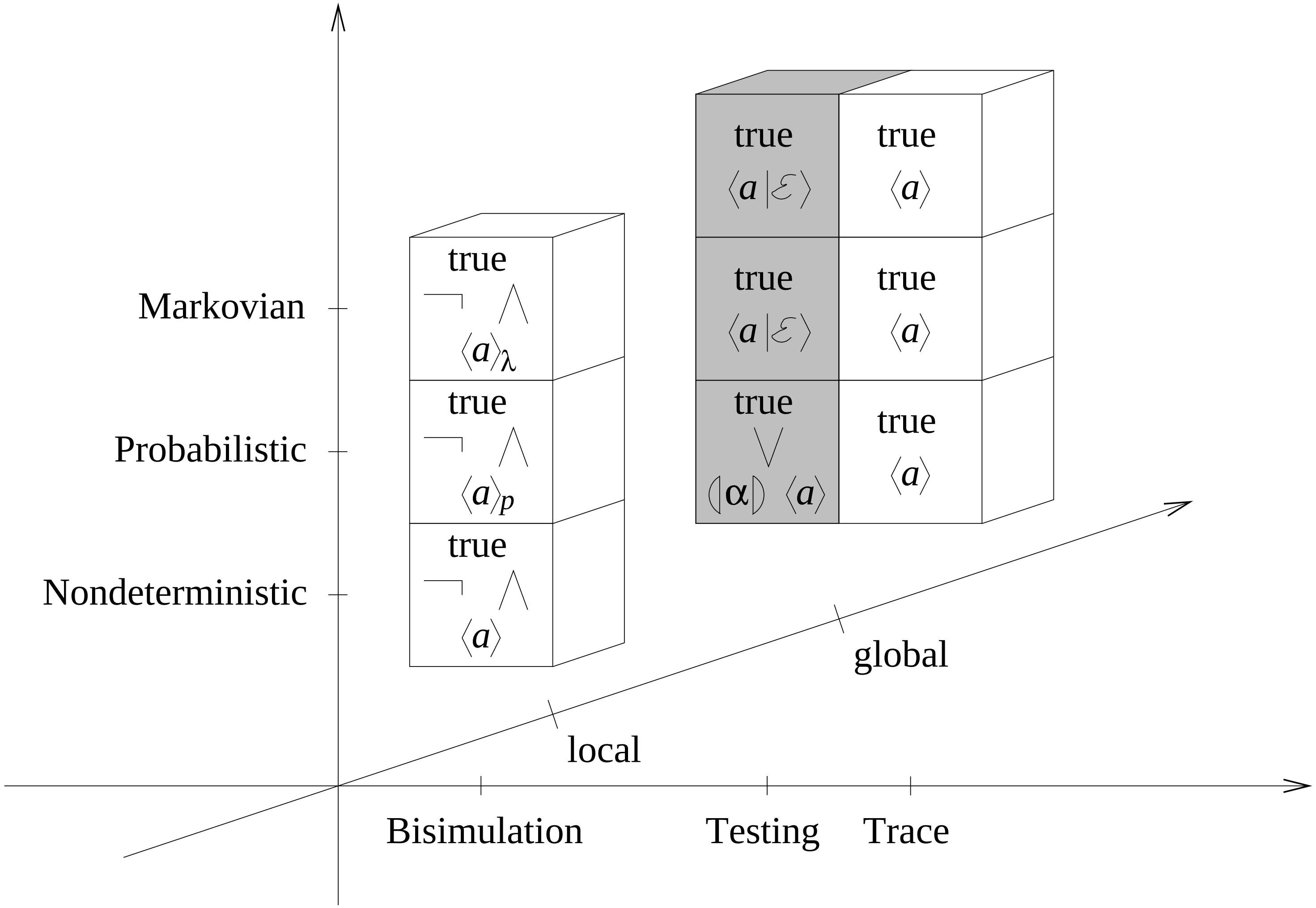
Modal Logics for Behavioral Equivalences

- Behavioral equivalences establish whether computing systems possess the same behavioral properties.
- The specific set of properties that are preserved depends on the specific behavioral equivalence (bisimulation, testing, trace, ...).
- These properties can usually be characterized by means of a modal logic.
- Notable example: *Hennessy-Milner logic (HML) and bisimilarity*.

- Comparative study conducted in 2006 (operator set, quantitative info):



Sets of Processes

- Minimal syntax generating all finite-state processes without silent moves.

- Nondeterministic processes:

$$\boxed{P \ ::= \ \underline{0} \mid a\,.\,P \mid P + P \mid A}$$

- Probabilistic processes (generative):

$$\boxed{P \ ::= \ \underline{0} \mid \sum_{i \in I} {<}a_i, p_i{>}.P_i \mid A \qquad\qquad p_i \in \mathbb{R}_{]0,1]}, \sum_{i \in I} p_i = 1}$$

- Markovian processes (generative):

$$\boxed{P \ ::= \ \underline{0} \mid {<}a, \lambda{>}.P \mid P + P \mid A \qquad\qquad \lambda \in \mathbb{R}_{>0}}$$

Sets of Tests

- Introduction of a success state s.
- Nondeterministic tests:

$$\boxed{\begin{array}{rcl} T & ::= & \mathrm{s} \mid T' \\ T' & ::= & a\,.\,T \mid T' + T' \end{array}}$$

- Reactive tests ($w \in \mathbb{R}_{>0}$):

$$\boxed{\begin{array}{rcl} T & ::= & \mathrm{s} \mid T' \\ T' & ::= & <a, *_w>.T \mid T' + T' \end{array}}$$

Testing Equivalences

- Nondeterministic testing equivalence: $P_1 \sim_{\mathrm{NT}} P_2$ if and only if for all nondeterministic tests T

$$\boxed{\begin{array}{rcl} P_1 \text{ may pass } T & \Longleftrightarrow & P_2 \text{ may pass } T \\ P_1 \text{ must pass } T & \Longleftrightarrow & P_2 \text{ must pass } T \end{array}}$$

- Probabilistic testing equivalence: $P_1 \sim_{\mathrm{PT}} P_2$ if and only if for all reactive tests T

$$\boxed{\mathit{prob}(\mathcal{SC}(P_1, T)) \;=\; \mathit{prob}(\mathcal{SC}(P_2, T))}$$

- Markovian testing equivalence: $P_1 \sim_{\mathrm{MT}} P_2$ if and only if for all reactive tests T and sequences $\theta \in (\mathbb{R}_{>0})^*$ of average amounts of time

$$\boxed{\mathit{prob}(\mathcal{SC}_{\leq \theta}(P_1, T)) \;=\; \mathit{prob}(\mathcal{SC}_{\leq \theta}(P_2, T))}$$

New Modal Characterization of $\sim_{\mathrm{NT}}$

- Modal language syntax:

$$\boxed{\begin{array}{rcl} \phi & ::= & \mathrm{true} \mid \phi' \\ \phi' & ::= & \langle a \rangle \phi \mid \phi' \vee \phi' \end{array}}$$

- Actions initially occurring in a formula:

$$\boxed{\begin{array}{rcl} \mathit{init}(\mathrm{true}) & = & \emptyset \\ \mathit{init}(\phi_1 \vee \phi_2) & = & \mathit{init}(\phi_1) \cup \mathit{init}(\phi_2) \\ \mathit{init}(\langle a \rangle \phi) & = & \{a\} \end{array}}$$

- May-satisfy relation:

$$\boxed{\begin{array}{lll} P & \models_{\mathrm{may}} & \mathrm{true} \\ P & \models_{\mathrm{may}} & \phi_1 \vee \phi_2 \qquad \text{if } P \models_{\mathrm{may}} \phi_1 \text{ or } P \models_{\mathrm{may}} \phi_2 \\ P & \models_{\mathrm{may}} & \langle a \rangle \phi \qquad \text{if there exists } P' \text{ such that } P \xrightarrow{a}_{\mathrm{N}} P' \text{ and } P' \models_{\mathrm{may}} \phi \end{array}}$$

- Must-satisfy relation:

$$\boxed{\begin{array}{llll} P & \models_{\mathrm{must}} & \mathrm{true} & \\ P & \models_{\mathrm{must}} & \phi_1 \vee \phi_2 & \text{if } \mathit{init}(P) \cap (\mathit{init}(\phi_1) \cup \mathit{init}(\phi_2)) \neq \emptyset \\ & & & \quad \text{and } \mathit{init}(P) \cap \mathit{init}(\phi_1) \neq \emptyset \text{ implies } P \models_{\mathrm{must}} \phi_1 \\ & & & \quad \text{and } \mathit{init}(P) \cap \mathit{init}(\phi_2) \neq \emptyset \text{ implies } P \models_{\mathrm{must}} \phi_2 \\ P & \models_{\mathrm{must}} & \langle a \rangle \phi & \text{if there exists } P' \text{ such that } P \xrightarrow{a}_{\mathrm{N}} P' \\ & & & \quad \text{and each such } P' \models_{\mathrm{must}} \phi \end{array}}$$

- Non-standard interpretation of disjunction and diamond (must case):
  - Should it be $P \models_{\mathrm{must}} \phi_1 \vee \phi_2$ if $P \models_{\mathrm{must}} \phi_1$ or $P \models_{\mathrm{must}} \phi_2$, then we would have $a.\underline{0} + b.\underline{0} \models_{\mathrm{must}} \langle a \rangle \mathrm{true} \vee \langle b \rangle \langle c \rangle \mathrm{true}$ because $a.\underline{0} + b.\underline{0} \models_{\mathrm{must}} \langle a \rangle \mathrm{true}$ ...
  - ... but it is not the case that $a.\underline{0} + b.\underline{0}$ must pass $a.\mathrm{s} + b.c.\mathrm{s}$.
  - Should it be $P \models_{\mathrm{must}} \langle a \rangle \phi$ if for all P' whenever $P \xrightarrow{a}_{\mathrm{N}} P'$ then $P' \models_{\mathrm{must}} \phi$, then we would have $\underline{0} \models_{\mathrm{must}} \langle a \rangle \mathrm{true}$ because there is no P' reachable from $\underline{0}$ via a ...
  - ... but it is not the case that $\underline{0}$ must pass $a.\mathrm{s}$.

New Modal Characterization of $\sim_{\mathrm{PT}}$

- $\phi_1 \vee \phi_2$ now obeys $\mathit{init}(\phi_1) \cap \mathit{init}(\phi_2) = \emptyset$.
- Quantitative interpretation: $[\![\phi]\!]_{\mathrm{PT}}(P) = 0$ for $\phi \not\equiv \mathrm{true}$ whenever $\mathit{init}(P) \cap \mathit{init}(\phi) = \emptyset$, otherwise

$$\boxed{\begin{array}{rcl} [\![\mathrm{true}]\!]_{\mathrm{PT}}(P) & = & 1 \\ [\![\phi_1 \vee \phi_2]\!]_{\mathrm{PT}}(P) & = & p_1 \cdot [\![\phi_1]\!]_{\mathrm{PT}}(P) + p_2 \cdot [\![\phi_2]\!]_{\mathrm{PT}}(P) \\ [\![\langle a \rangle \phi]\!]_{\mathrm{PT}}(P) & = & \sum\limits_{P \xrightarrow{a,p}_{\mathrm{P}} P'} \frac{p}{\mathit{prob}_{\mathrm{c}}(P \mid \{a\})} \cdot [\![\phi]\!]_{\mathrm{PT}}(P') \end{array}}$$

where (avoiding an overestimate):

$$p_j = \frac{\mathit{prob}_{\mathrm{c}}(P \mid \mathit{init}(\phi_j))}{\mathit{prob}_{\mathrm{c}}(P \mid \mathit{init}(\phi_1 \vee \phi_2))}$$

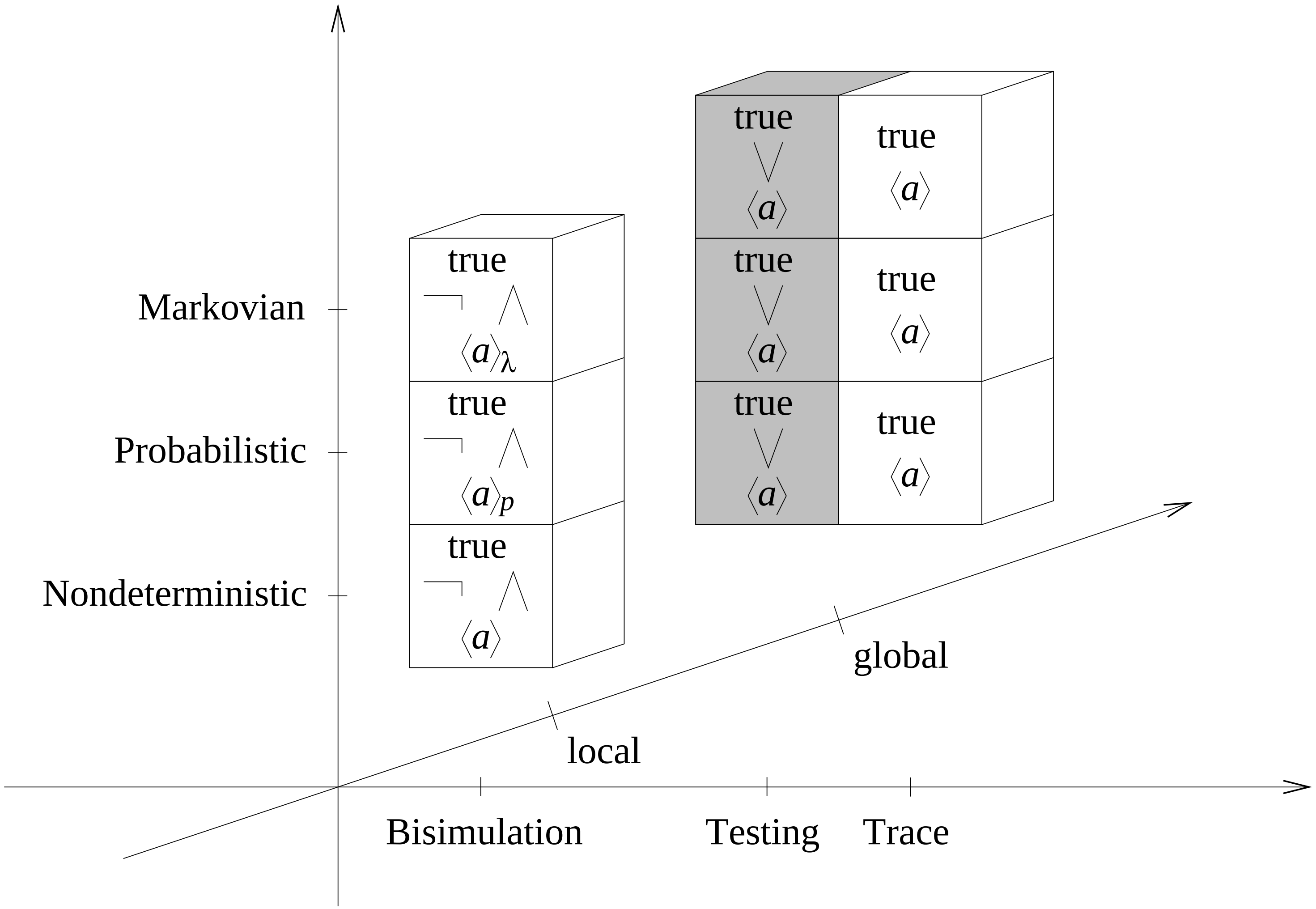
New Modal Characterization of $\sim_{\mathrm{MT}}$

- $\phi_1 \vee \phi_2$ again obeys $init(\phi_1) \cap init(\phi_2) = \emptyset$.
- Quantitative interpretation: $[\![\phi]\!]_{\mathrm{MT}}(P, \theta) = 0$ for $\phi \not\equiv \mathrm{true}$ whenever $init(P) \cap init(\phi) = \emptyset$ or $\theta = \varepsilon$, otherwise

$$
\begin{aligned}
[\![\mathrm{true}]\!]_{\mathrm{MT}}(P, \theta) &= 1 \\
[\![\phi_1 \vee \phi_2]\!]_{\mathrm{MT}}(P, t \circ \theta) &= p_1 \cdot [\![\phi_1]\!]_{\mathrm{MT}}(P, t_1 \circ \theta) + p_2 \cdot [\![\phi_2]\!]_{\mathrm{MT}}(P, t_2 \circ \theta) \\
[\![\langle a \rangle \phi]\!]_{\mathrm{MT}}(P, t \circ \theta) &= \begin{cases} \sum\limits_{P \xrightarrow{a, \lambda}_{\mathrm{M}} P'} \frac{\lambda}{rate_{\mathrm{c}}(P | \{a\})} \cdot [\![\phi]\!]_{\mathrm{MT}}(P', \theta) & \text{if } \frac{1}{rate_{\mathrm{c}}(P | \{a\})} \leq t \\ 0 & \text{if } \frac{1}{rate_{\mathrm{c}}(P | \{a\})} > t \end{cases}
\end{aligned}
$$

where (avoiding an overestimate/underestimate):

$$
\begin{aligned}
p_j &= \frac{rate_{\mathrm{c}}(P | init(\phi_j))}{rate_{\mathrm{c}}(P | init(\phi_1 \vee \phi_2))} \\
t_j &= t + \left(\frac{1}{rate_{\mathrm{c}}(P | init(\phi_j))} - \frac{1}{rate_{\mathrm{c}}(P | init(\phi_1 \vee \phi_2))}\right)
\end{aligned}
$$



Markovian
Probabilistic
Nondeterministic
true
¬ ∧ $\langle a \rangle_\lambda$
true
¬ ∧ $\langle a \rangle_p$
true
¬ ∧ $\langle a \rangle$
true
∨ $\langle a \rangle$
true
$\langle a \rangle$
global
local
Bisimulation
Testing
Trace