Strict Divergence for Probabilistic Timed Automata

Angelo Troina

Dipartimento di Informatica, University of Turin, Italy

Work by J. Sproston

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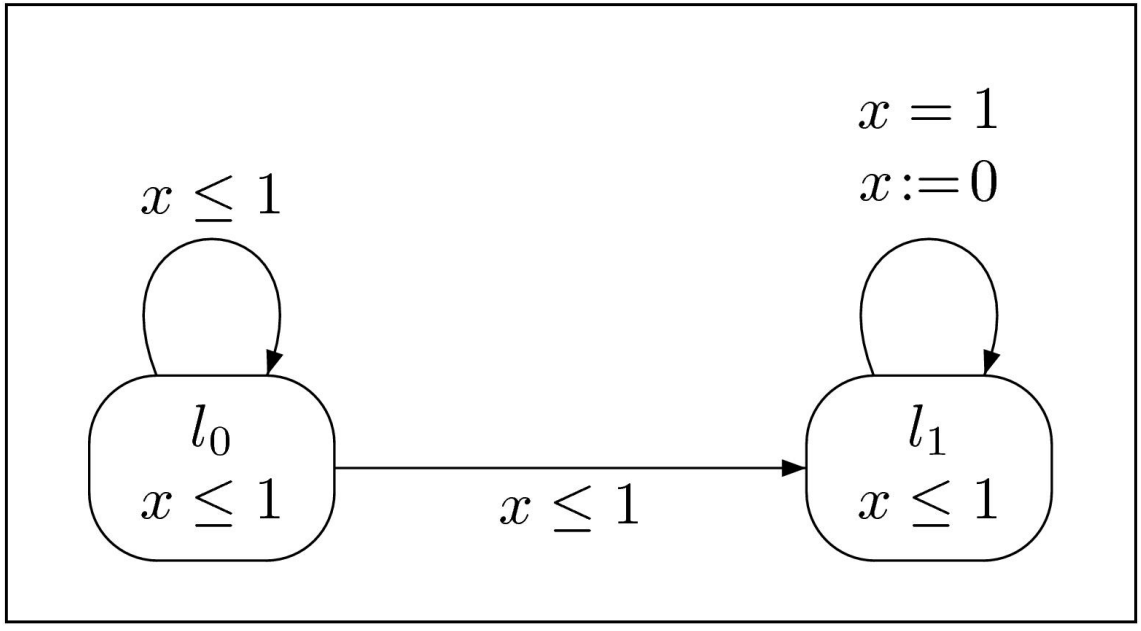
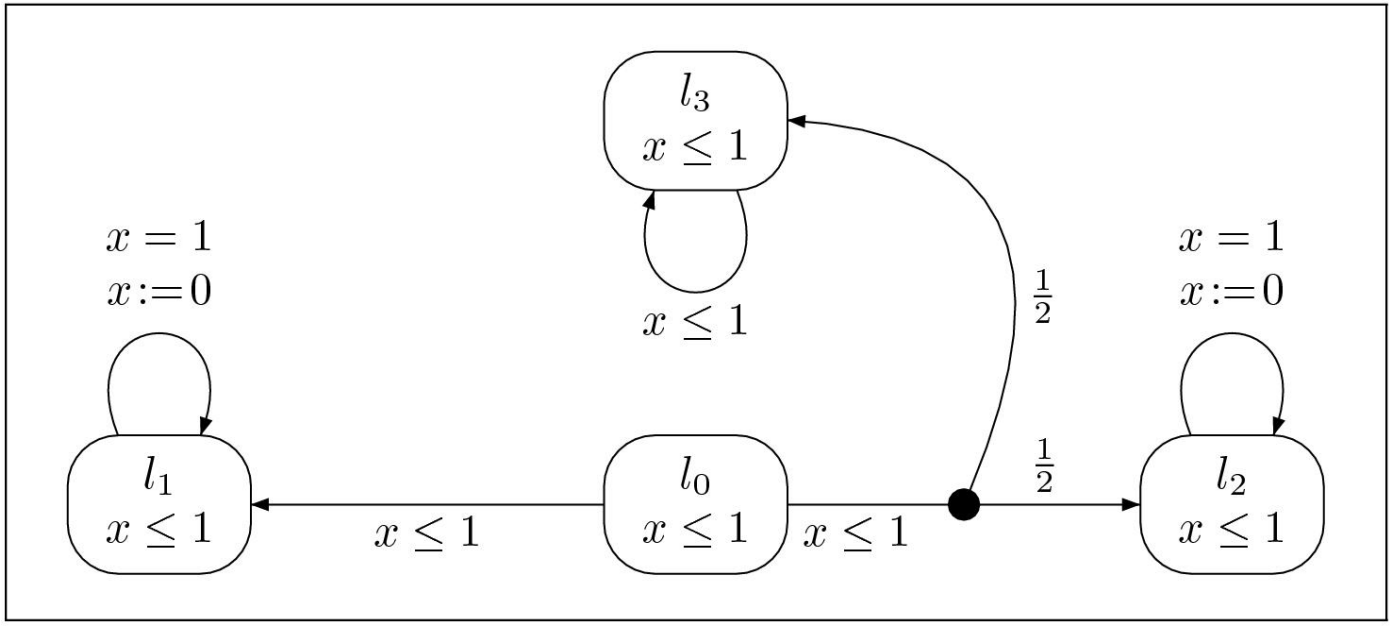
Timed automata

- Timed automata [AlurDill94]:
  - Finite-state graph
  - Finite set of *clocks*: real-valued variables increasing at the same rate as real-time
  - Clock constraints (*invariants* in nodes, *guards* on edges)
  - Clock resets (set some clocks to 0 when an edge is traversed)



Probabilistic timed automata

- Probabilistic timed automata [Jen96,KNSS02]:
  - Finite-state Markov decision process (probabilistic and nondeterministic choice)
  - Finite set of *clocks*: real-valued variables increasing at the same rate as real-time
  - Clock constraints (*invariants* in nodes, *guards* on edges)
  - Clock resets (set some clocks to 0 when an edge is traversed)



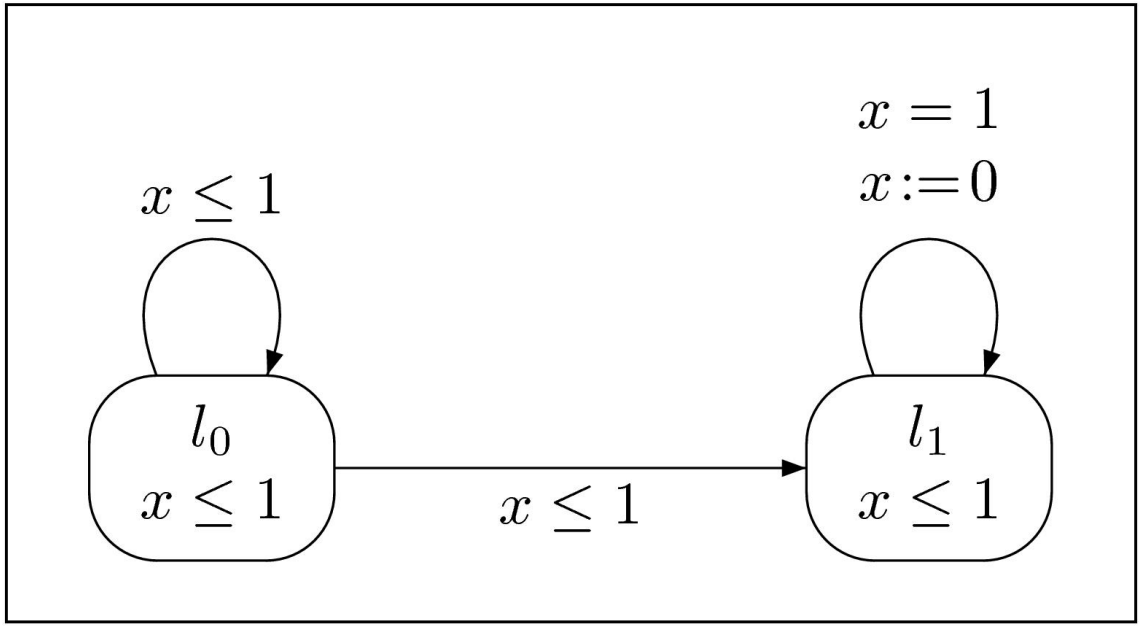
Probabilistic timed automata

- Nondeterminism is resolved by strategies:
  - Strategy: function from the finite behaviour (history of the system) to the next nondeterministic choice to take.
  - A strategy induces a probability space over behaviours.
  - Have pessimistic and optimistic reasoning about correctness properties:
    - Pessimistic: corresponds to a strategy with the minimum probability of satisfying the property.
    - E.g., is the probability of reaching a goal state within 6000ns at least 0.8, no matter how unfavourable the nondeterministic choices are?
    - Optimistic: corresponds to a strategy with the maximum probability of satisfying the property.
    - E.g., can nondeterminism be resolved (in a favourable way) so that the probability of reaching a goal state within 6000ns is greater than 0.99?

Probabilistic timed automata

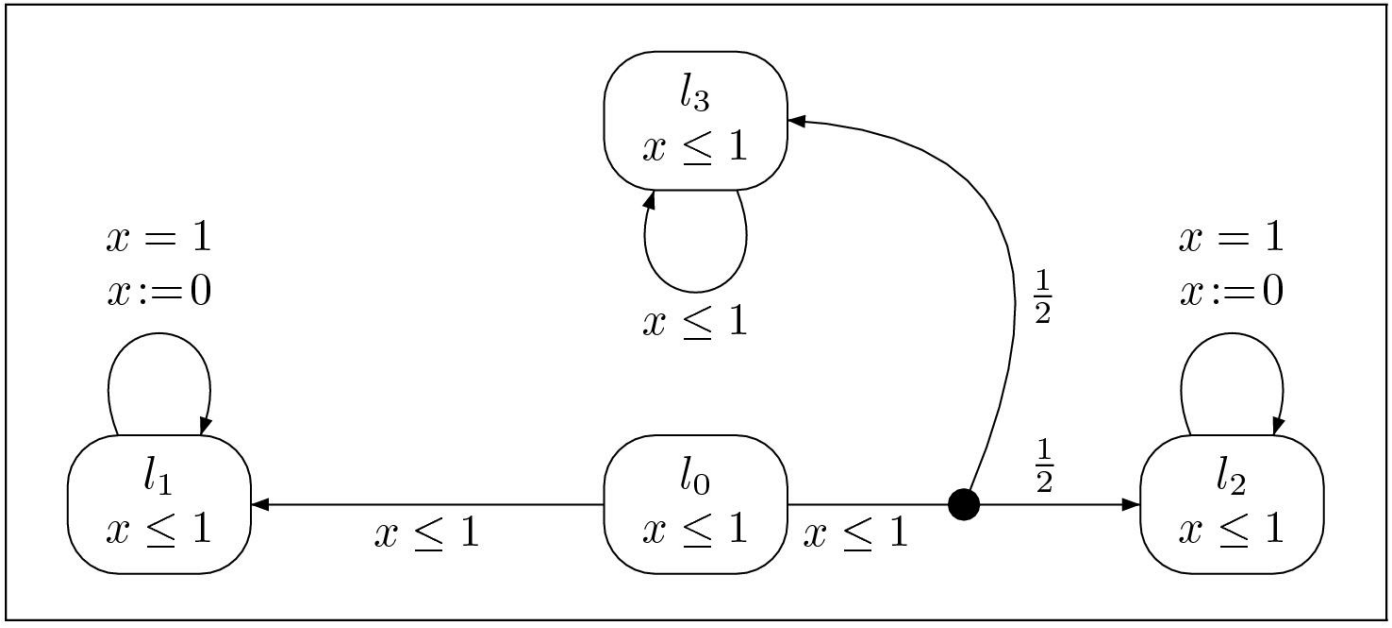
- Typical verification method:
  - Transform to a finite-state Markov decision process (e.g., using region equivalence).
  - Use a probabilistic model checking tool (e.g., PRISM).

What is (time) divergence?



- Models of timed systems can (erroneously) contain unrealistic behaviour.
- E.g., loop in l_0 forever: elapsed time in the system is 1.
- Timed automata: exist techniques to include only divergent behaviours (time exceeds any bound).

What is probabilistic divergence?

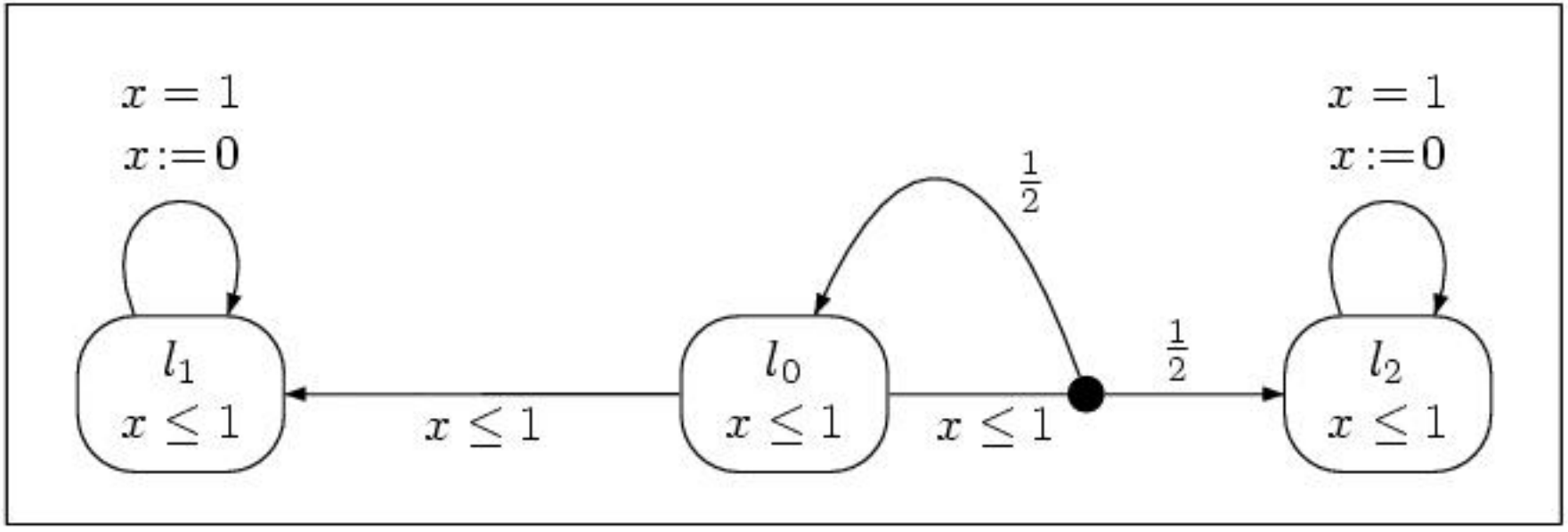


- E.g., take rightmost distribution from l_0: with probability $\frac{1}{2}$ elapsed time in the system is ∞ (good), but with probability $\frac{1}{2}$ elapsed time in the system is 1 (bad).
- A solution: only consider those strategies that let time diverge with probability 1 [KNSS02].

What is probabilistic divergence?

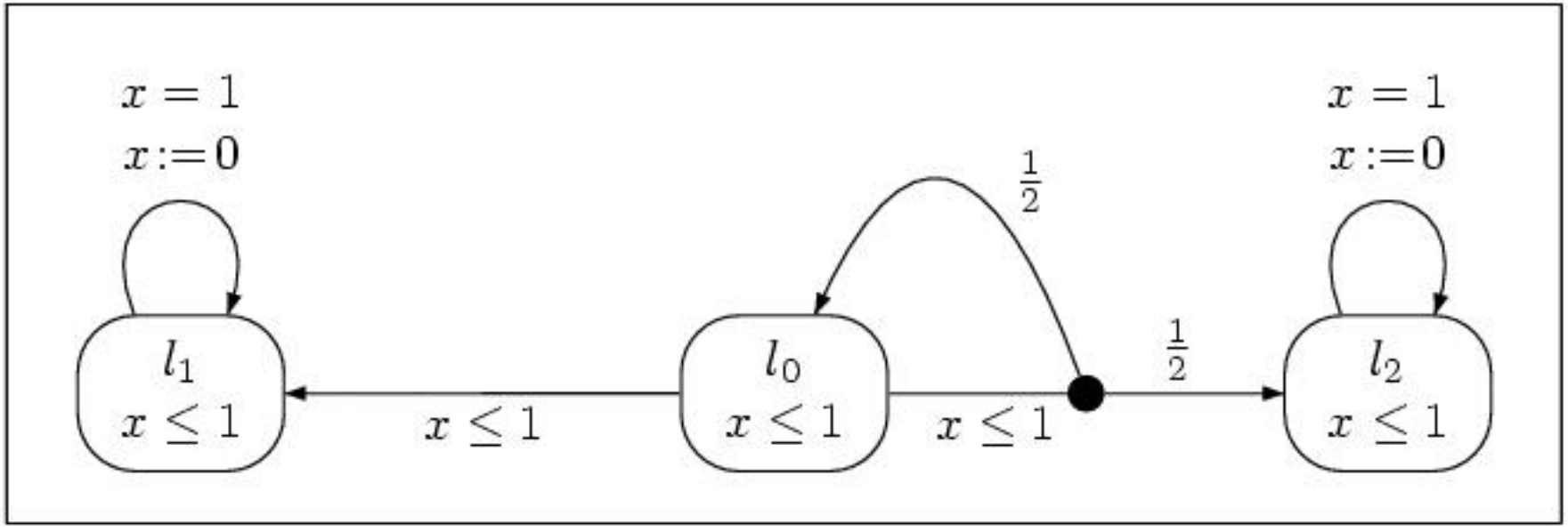
- Probabilistically divergent strategies: let time diverge with probability 1.
- Model checking algorithms under probabilistically divergent strategies:
  - EXPTIME algorithm if we know *a priori* (e.g., syntactically) that all strategies are probabilistic time divergent [KNSS02].
  - General case subsequently considered, but algorithm not optimal (2EXPTIME) [KNSW07] .
  - Therefore provided an optimal EXPTIME algorithm in the general case.

Probabilistic divergence



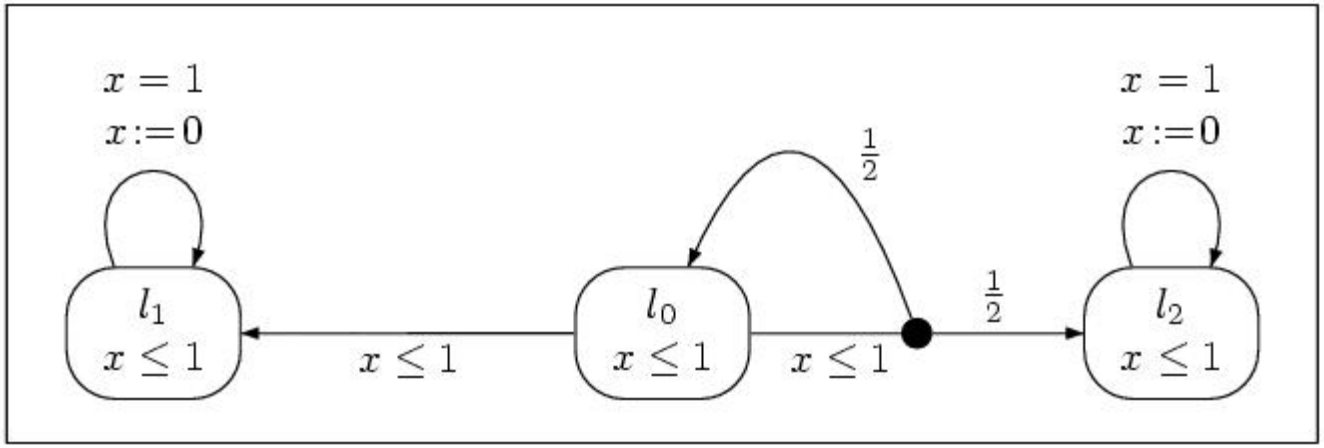
What is the maximum probability of reaching l_2 from $(l_0, x = 0)$ under probabilistically divergent strategies?

Probabilistic divergence



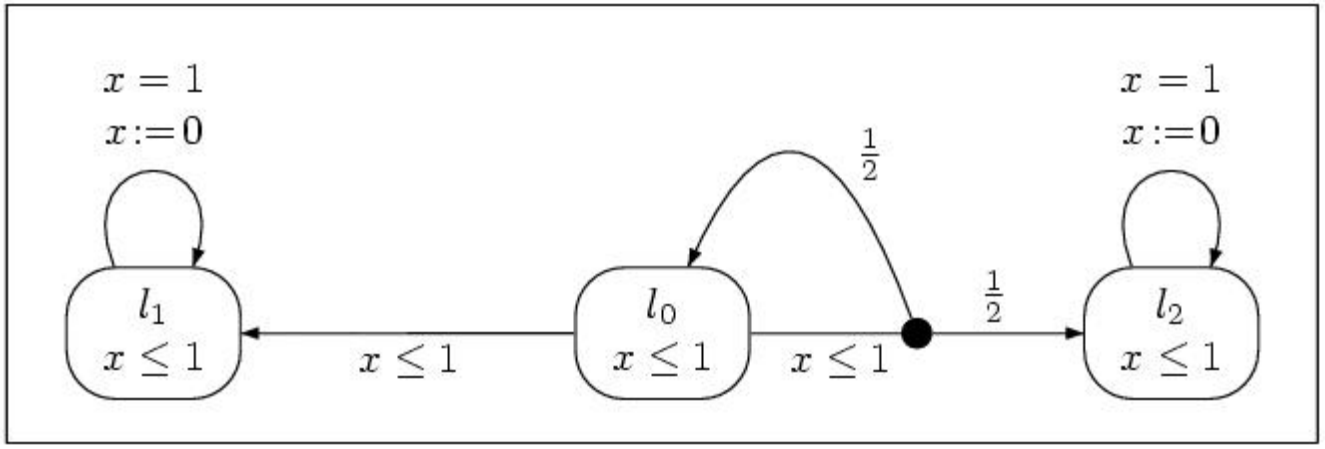
The maximum probability of reaching l_2 from $(l_0, x = 0)$ under probabilistically divergent strategies is 1.

Probabilistic divergence



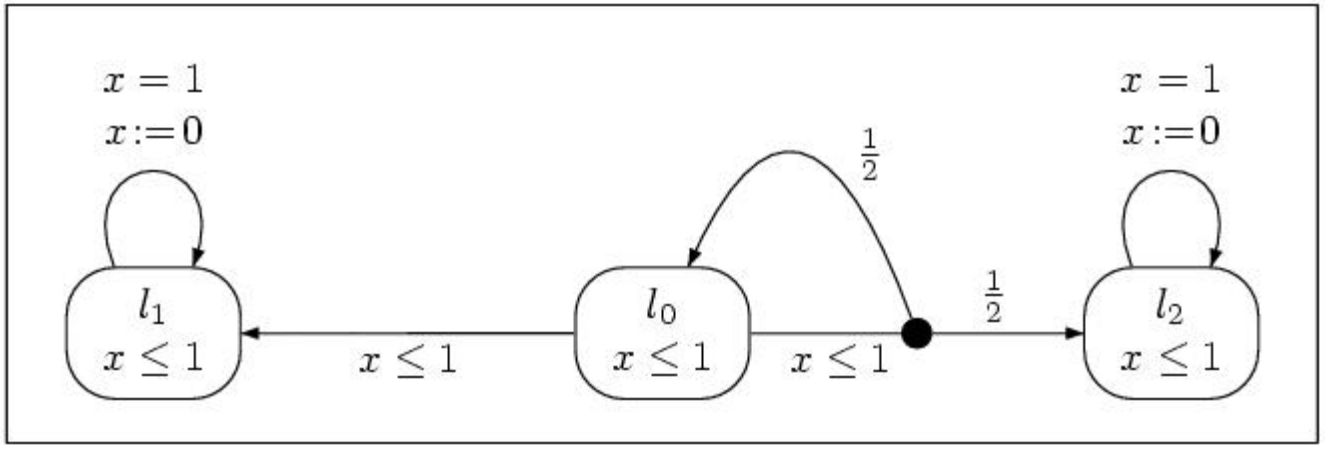
- But what does a strategy have to do to achieve probability 1 of reaching l_2 from $(l_0, x = 0)$? Consider the following probabilistically divergent strategy:
  - Let $\frac{1}{2}$ time units elapse in l_0, then take rightmost transition.
  - If return to l_0, let $\frac{1}{4}$ time units elapse, then take rightmost transition.
  - If return to l_0, let $\frac{1}{8}$ time units elapse, then take rightmost transition.
  - ...

Strict divergence



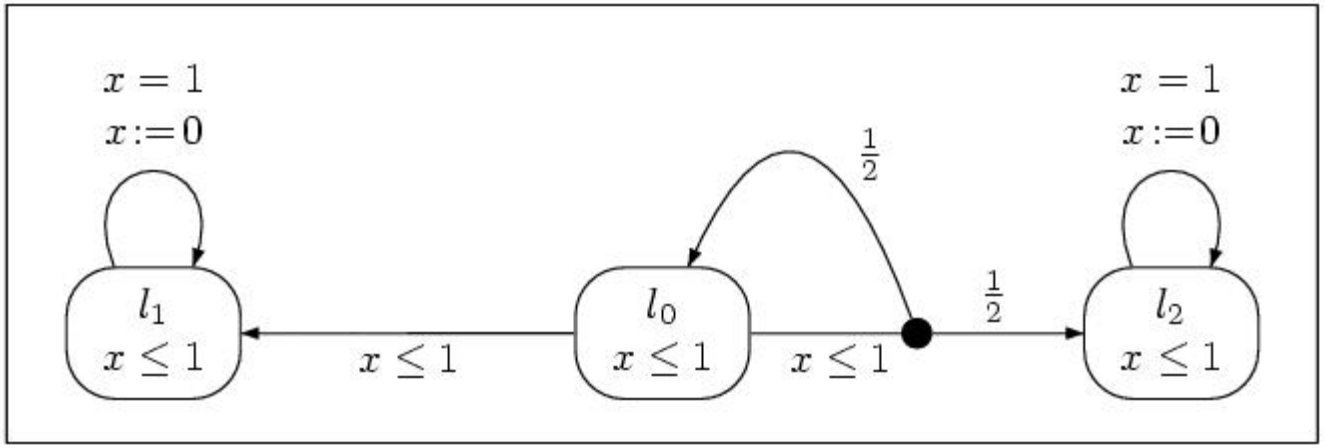
- To achieve probability 1 of reaching l_2 from $(l_0, x = 0)$, a strategy must take the rightmost transition an infinite number of times before 1 time units elapse.
- Realistic? Not if the rightmost transition corresponds to a physical action.
- Strictly divergent strategy: time must diverge on *all* of the strategy's paths.

Strict divergence



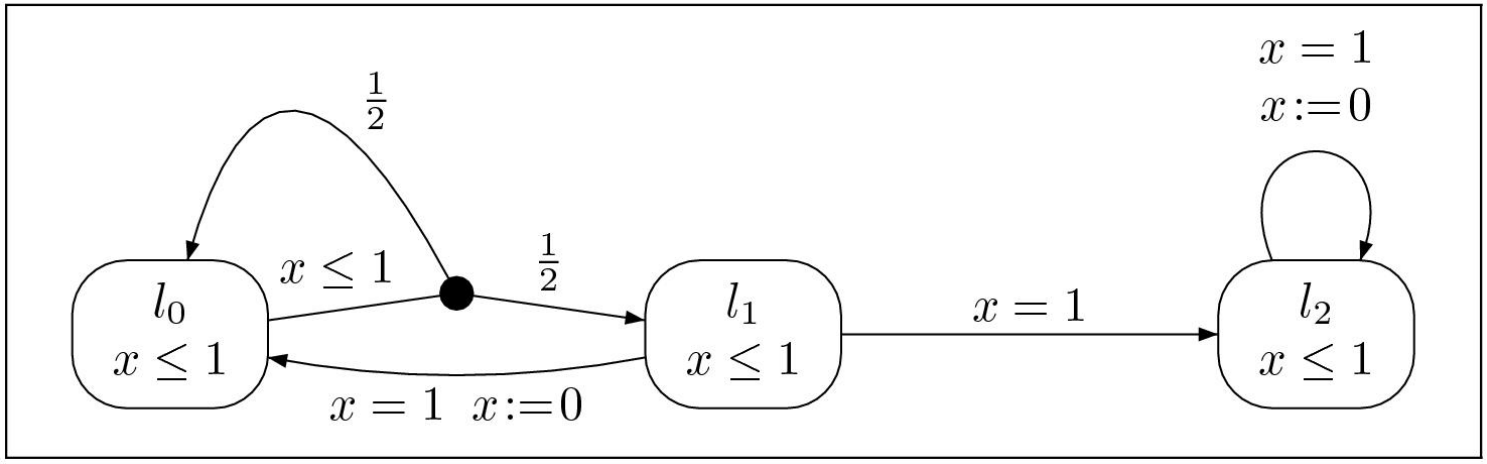
- What is the maximum probability of reaching l_2 from $(l_0, x = 0)$ under *strictly divergent strategies*?
- A strictly divergent strategy tries to reach l_2 via the rightmost transition, but cannot take this transition an infinite number of times.
- Intuitively, a strictly divergent strategy must "give up" after a finite but arbitrary number of attempts to reach l_2.

Strict divergence



- There does not exist any strictly divergent strategy reaching l_2 from l_0 when $x = 0$ with probability 1.
- But, for any $\epsilon > 0$, we can find strictly divergent strategy such that the probability of l_2 from l_0 when $x = 0$ is greater than $1 - \epsilon$.
- In general: strictly divergent strategies can approximate arbitrarily closely the maximum reachability probability attained by probabilistcially divergent strategies.

Strict divergence



- What is the minimum probability of reaching l_2 from $(l_1, x = 0)$?
  - The path that loops forever in l_0 is not time divergent, but has probability 0.
  - Probabilistically divergent strategies: min. prob. is 0.
  - Strictly divergent strategies: min. prob. is 1.
- Minimum reachability probabilities may differ arbitrarily much between probabilistically and strictly divergent strategies.

Model-checking algorithms: strict divergence

- Present a model-checking algorithm which considers only strictly divergent strategies.
- Consider maximum reachability properties.
- Non-strict bounds in properties (e.g., maximum probability of reaching l_2 is $\leq \frac{99}{100}$): similar to the algorithm for probabilistically divergent strategies.

Model-checking algorithms: strict divergence

- Strict bounds in properties (e.g., maximum probability of reaching l_2 is < 1):
  - For each state, distinguish between the following two cases:
    1. The supremum probability can be attained by a strictly divergent strategy.
    2. Strictly divergent strategies can only approximate the supremum probability.
  - Compute the set of states in Case 1.
  - E.g., from $(l_0, x = 0)$, the maximum probability of reaching l_2 is < 1?
    - Compute the supremum probability of reaching l_2 from $(l_0, x = 0)$; say this is 1.
    - If $(l_0, x = 0)$ is in the set of Case 1 states, answer No, otherwise Yes.

Model-checking algorithms: strict divergence

- Minimum reachability properties: relies on:
  1. Reasoning about non-strict/strict bounds similar to that for maximum reachability properties.
  2. Reasoning about infinite sojourns in *end components* (MDP analogue of bottom strongly connected components).
- Not limited to reachability properties: can apply all results to PTCTL (probabilistic, timed temporal logic).
- Algorithms are EXPTIME-complete.