Stochastic Process Algebras and Stochastic Model Checking

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PACO: Kick-off Meeting
Bertinoro, October 23-24 2008
Outline...

1. Motivations
2. PEPA
3. Stochastic CCS
4. MoSL: Mobile Stochastic Logic
5. Model Checking MoSL
6. Conclusions and Future Directions
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1. Motivations
2. PEPA
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6. Conclusions and Future Directions
A number of stochastic process algebras have been proposed in the last two decades. These are based on:

1. **Labeled Transition Systems (LTS)**
   - for providing compositional semantics of languages
   - for describing *qualitative properties*

2. **Continuous Time Markov Chains (CTMC)**
   - for analysing *quantitative properties*
Motivations...

A number of stochastic process algebras have been proposed in the last two decades. These are based on:

1. Labeled Transition Systems (LTS)
   - for providing compositional semantics of languages
   - for describing *qualitative properties*

2. Continuous Time Markov Chains (CTMC)
   - for analysing *quantitative properties*

Semantics of these calculi have been given by variants of the Structured Operational Semantics (SOS) approach but:

- they do not rely on any general framework
- it is rather difficult to appreciate differences and similarities of such semantics.
Examples of stochastic process algebras include: TIPP, PEPA, EMPA, stochastic $\pi$-calculus and STOKLAIM
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Common ingredients:

- Actions execution take time
- Execution times is described by means of Random Variables
- Random Variables are assumed to be Exponentially Distributed
- Random Variables are fully characterised by its rate
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Common ingredients:

- Actions execution take time
- Execution times is described by means of Random Variables
- Random Variables are assumed to be Exponentially Distributed
- Random Variables are fully characterised by its rate

If $X$ is exponentially distributed with parameter $\lambda \in \mathbb{R}_{>0}$:

- $\mathbb{P}\{X \leq d\} = 1 - e^{-\lambda \cdot d}$, for $d \geq 0$
- The average duration of $X$ is $\frac{1}{\lambda}$; the variance of $X$ is $\frac{1}{\lambda^2}$
- Memory-less: $\mathbb{P}\{X \leq t + d \mid X > t\} = \mathbb{P}\{X \leq d\}$
Continuous Time Markov Chains

Continuous Time Markov Chains are a successful mathematical framework for modeling and analysing performance and dependability of systems.

CTMCs provide well established Analysis Techniques and Tools such as:

- Steady State Analysis
- Transient Analysis
- Stochastic Timed/Temporal Logics
- Stochastic Model Checking

A CTMC is a pair \((\mathcal{S}, R)\)

- \(\mathcal{S}\): a countable set of states
- \(R: \mathcal{S} \times \mathcal{S} \to \mathbb{R}_{\geq 0}\), the rate matrix
A Continuous Time Markov Chain (CTMC) is associated to each term of the process algebras. This will be used for defining the stochastic behaviour of processes.
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We have to...

- compute *synchronizations rate*
- take into account transition multiplicity, for determining correct execution rate

Process Calculi:

\[
\alpha.P + \alpha.P = \alpha.P
\]

\[
\text{rec } X.\alpha.X \mid \text{rec } X.\alpha.X = \text{rec } X.\alpha.X
\]
Stochastic process calculi...

A Continuous Time Markov Chain (CTMC) is associated to each term of the process algebras. This will be used for defining the stochastic behaviour of processes.

We have to...

- compute \textit{synchronizations rate}
- take into account transition multiplicity, for determining correct execution rate

Stochastic Process Calculi:

\[ \alpha^\lambda \cdot P + \alpha^\lambda \cdot P \neq \alpha^\lambda \cdot P \]

\[ \text{rec } X \cdot \alpha^\lambda \cdot X \mid \text{rec } X \cdot \alpha^\lambda \cdot X \neq \text{rec } X \cdot \alpha^\lambda \cdot X \]
A Continuous Time Markov Chain (CTMC) is associated to each term of the process algebras. This will be used for defining the stochastic behaviour of processes.

We have to...
- compute *synchronizations rate*
- take into account transition multiplicity, for determining correct execution rate

Stochastic Process Calculi:

\[ \alpha \lambda . P + \alpha \lambda . P = \alpha^{2\lambda} . P \]

\[ \text{rec } X . \alpha \lambda . X \mid \text{rec } X . \alpha \lambda . X = \text{rec } X . \alpha^{2\lambda} . X \]
We introduce a variant of Rate Transition Systems (RTS), proposed by Klin and Sassone, and use it as the basis for defining stochastic behaviour of processes.

As in previous approaches to enhance process calculi with stochastic features, we will first define an enriched LTS by means of an SOS semantics and then use it to associate a CTMC to any term.

There are however two distinguishing aspects of our work:

- The transition relation we use associates terms and actions to functions from terms to rates
- We adapt the *apparent rate* approach, originally developed by Hillston for a process algebra with a CSP-like multi-party synchronisation paradigm, to calculi like CCS and $\pi$-calculus
Stochastic semantics of process calculi is defined by means of a transition relation $\rightarrow$ that associates to a process $P$ and a transition label $\alpha$ a function $(P, Q, \ldots)$ that maps each process into a non-negative real number. If $P(Q) = x \neq 0$ then $Q$ is reachable from $P$ via the execution of $\alpha$ with rate or weight $x$; if $P(Q) = 0$ then $Q$ is not reachable from $P$ via $\alpha$. We have that if $P$ then $\sum Q P(Q)$ represents the total rate/weight of $\alpha$ in $P$.
Semantics of stochastic process calculi. . .

Stochastic semantics of process calculi is defined by means of a transition relation $\alpha \rightarrow$ that associates to a process $P$ and a transition label $\alpha$ a function $(\mathcal{P}, \mathcal{Q}, \ldots)$ that maps each process into a non-negative real number.

$P \xrightarrow{\alpha} \mathcal{P}$ means that:
- if $\mathcal{P}(Q) = x \neq 0$ then $Q$ is reachable from $P$ via the execution of $\alpha$ with rate or weight $x$
- if $\mathcal{P}(Q) = 0$ then $Q$ is not reachable from $P$ via $\alpha$
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We have that if $P \xrightarrow{\alpha} \mathcal{P}$ then
- $\bigoplus \mathcal{P} = \sum_{Q} \mathcal{P}(Q)$ represents the total rate/weight of $\alpha$ in $P$. 
Definition (Rate Transition Systems)

A rate transition system is a triple $(S, A, \rightarrow)$ where:

- $S$ is a set of states;
- $A$ is a set of transition labels;
- $\rightarrow \subseteq S \times A \times [S \rightarrow \mathbb{R}_{\geq 0}]$

Notations:

- RTS will be denoted by $\mathcal{R}, \mathcal{R}_1, \mathcal{R}', \ldots$,
- Elements of $[S \rightarrow \mathbb{R}_{\geq 0}]$ are denoted by $\mathcal{P}, \mathcal{Q}, \mathcal{R}, \ldots$
- $\emptyset$ denotes the constant function $0$
- $[s_1 \mapsto v_1, \ldots, s_n \mapsto v_n]$ will denote a function associating $v_i$ to $s_i$ and $0$ to all the other states.
Definition

Let $\mathcal{R} = (S, A, \rightarrow)$ be an RTS, then:

- $\mathcal{R}$ is **fully stochastic** if and only if for each $s \in S$, $\alpha \in A$, $P$ and $Q$ we have: $s \xrightarrow{\alpha} P$, $s \xrightarrow{\alpha} Q \implies P = Q$

- $\mathcal{R}$ is **image finite** if and only if for each $s \in S$, $\alpha \in A$ and $P$ such that $s \xrightarrow{\alpha} P$ we have: $\{s' | P(s') > 0\}$ is finite

Mettere figure
For sets $C \subseteq S$ and $A' \subseteq A$, the set of derivatives of $C$ through $A'$, denoted $\text{Der}(C, A')$, is the smallest set such that:

- $C \subseteq \text{Der}(C, A')$,
- if $s \in \text{Der}(C, A')$ and there exists $\alpha \in A'$ and $Q \in \Sigma_S$ such that $s \xrightarrow{\alpha} Q$ then $\{s' \mid Q(s') > 0\} \subseteq \text{Der}(C)$

Let $\mathcal{R} = (S, A, \rightarrow)$ be a fully stochastics RTS, for $C \subseteq S$, the CTMC of $C$, when one considers only actions $A' \subseteq A$ is defined as $\text{CTMC}[C, A'] \overset{\text{def}}{=} (\text{Der}(C, A'), \mathcal{R})$ where for all $s_1, s_2 \in S$:

$$\mathcal{R}[s_1, s_2] \overset{\text{def}}{=} \sum_{\alpha \in A'} \mathbb{P}^\alpha(s_2) \text{ with } s_1 \xrightarrow{\alpha} \mathbb{P}^\alpha.$$
Rate aware bisimulation...

Definition (Rate Aware Bisimilarity)

- An equivalence relation $\mathcal{E}$ on $\mathcal{C}$ is a *rate aware* bisimulation if and only if, for all $(s_1, s_2) \in \mathcal{E}$, $C \in \mathcal{C}/\mathcal{E}$, and for all $\alpha$ and $\mathcal{P}$:

  $s_1 \xrightarrow{\alpha} \mathcal{P} \implies \exists \mathcal{Q} : s_2 \xrightarrow{\alpha} \mathcal{Q} \land \mathcal{P}(C) = \mathcal{Q}(C)$

- Two states $s_1, s_2 \in S$ are *rate aware bisimilar* ($s_1 \sim s_2$) if there exists a rate aware bisimulation $\mathcal{E}$ such that $(s_1, s_2) \in \mathcal{E}$.

Notice that *rate aware bisimilarity* and *strong bisimilarity* coincide when one does not take into account rates.

Theorem

Let $\mathcal{R} = (S, A, \longrightarrow)$, for each $A' \subseteq A$ and for each $s_1, s_2 \in S$ and $(S, R) = \text{CTMC}[\{s_1, s_2\}, A']$: $s_1 \sim s_2 \implies s_1 \sim_M s_2$
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PEPA: Performance Process Algebra

- In PEPA systems are be described as interactions of components that may engage in activities
  - Components reflect the behaviour of relevant parts of the system, while activities capture the actions that the components perform.
- Each PEPA activity consists of a pair \((\alpha, \lambda)\) where:
  - \(\alpha\) symbolically denotes the performed action;
  - \(\lambda > 0\) is the rate of the negative exponential distribution.
- If \(A\) is a set of actions, ranged over by \(\alpha, \alpha', \alpha_1, \ldots\), then \(\mathcal{P}_{PEPA}\) is the set of process terms \(P, P', P_1, \ldots\) defined according to the following grammar

\[
P ::= (\alpha, \lambda).P \mid P + P \mid P \otimes_L P \mid P/L \mid A
\]
PEPA Stochastic semantics...

\[ (\alpha, \lambda).P \xrightarrow{\alpha} [P \leftrightarrow \lambda] \quad \text{(Act)} \]

\[ P \xrightarrow{\alpha} \mathcal{P} \quad Q \xrightarrow{\alpha} \mathcal{Q} \quad \mathcal{P} + \mathcal{Q} \xrightarrow{\alpha} \mathcal{P} + \mathcal{Q} \quad \text{(SUM)} \]

\[ P \xrightarrow{\alpha} \mathcal{P} \quad Q \xrightarrow{\alpha} \mathcal{Q} \quad \mathcal{P} \not\in \mathcal{L} \quad \mathcal{P} \not\in \mathcal{L} + \mathcal{Q} \mathcal{P} \not\in \mathcal{L} \quad \mathcal{P} \not\in \mathcal{L} + \mathcal{Q} \mathcal{P} \not\in \mathcal{L} + \mathcal{Q} \quad \mathcal{P} \not\in \mathcal{L} + \mathcal{Q} \quad \mathcal{P} \not\in \mathcal{L} + \mathcal{Q} \quad \mathcal{P} \not\in \mathcal{L} + \mathcal{Q} \quad \text{(SUM)} \]

\[ \mathcal{P} \xrightarrow{\alpha} \mathcal{P} \quad Q \xrightarrow{\alpha} \mathcal{Q} \quad \mathcal{P} \mathcal{Q} \xrightarrow{\alpha} \mathcal{P} \mathcal{Q} \quad \mathcal{P} \not\in \mathcal{L} \quad \mathcal{P} \not\in \mathcal{L} + \mathcal{Q} \mathcal{P} \not\in \mathcal{L} + \mathcal{Q} \quad \mathcal{P} \not\in \mathcal{L} + \mathcal{Q} \quad \mathcal{P} \not\in \mathcal{L} + \mathcal{Q} \quad \mathcal{P} \not\in \mathcal{L} + \mathcal{Q} \quad \text{(SUM)} \]

\[ \mathcal{P} \xrightarrow{\alpha} \mathcal{P} \quad \mathcal{Q} \xrightarrow{\alpha} \mathcal{Q} \quad \mathcal{P} \alpha \xrightarrow{\alpha} \mathcal{P} \alpha \quad \mathcal{P} \not\in \mathcal{L} \quad \mathcal{P} \not\in \mathcal{L} + \mathcal{Q} \mathcal{P} \not\in \mathcal{L} + \mathcal{Q} \quad \mathcal{P} \not\in \mathcal{L} + \mathcal{Q} \quad \mathcal{P} \not\in \mathcal{L} + \mathcal{Q} \quad \mathcal{P} \not\in \mathcal{L} + \mathcal{Q} \quad \text{(SUM)} \]

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PEPA Stochastic semantics. . .

\[
(\alpha, \lambda).P \xrightarrow{\alpha} [P \leftrightarrow \lambda] \quad (\text{ACT})
\]

\[
(\alpha, \lambda).P \xrightarrow{\beta} \emptyset \quad (\emptyset\text{-ACT})
\]

\[
P \xrightarrow{\alpha} P \quad Q \xrightarrow{\alpha} Q \quad \text{(SUM)}
\]

\[
P + Q \xrightarrow{\alpha} P + Q
\]

where:

- \(P + Q\) denotes the function \(\mathcal{R}\) such that: \(\mathcal{R}(P) = P(P) + Q(P)\).
PEPA Stochastic semantics.

\[
\begin{align*}
(\alpha, \lambda).P \xrightarrow{\alpha} [P \leftrightarrow \lambda] \\
(\alpha, \lambda).P \xrightarrow{\beta} \emptyset \\
P \xrightarrow{\alpha} P \quad Q \xrightarrow{\alpha} Q \\
P + Q \xrightarrow{\alpha} P + Q
\end{align*}
\]

(SUM)

where:

- \( P + Q \) denotes the function \( R \) such that: \( R(P) = P(P) + Q(P) \).
PEPA Stochastic semantics... 

\[
\begin{align*}
\alpha, \lambda \cdot P & \xrightarrow{\alpha} [P \leftrightarrow \lambda] \quad \text{(Act)} \\
\alpha \neq \beta & \quad \text{(\emptyset-ACT)}
\end{align*}
\]

\[
\begin{align*}
P & \xrightarrow{\alpha} \mathcal{P} \\
Q & \xrightarrow{\alpha} \mathcal{Q} \text{ (SUM)}
\end{align*}
\]

\[
P + Q \xrightarrow{\alpha} \mathcal{P} + \mathcal{Q}
\]

where:

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\end{align*}
\]

\[
\begin{align*}
P &\xrightarrow{\alpha} \mathcal{P} \\
Q &\xrightarrow{\alpha} \mathcal{Q} \\
\mathcal{P} + \mathcal{Q} &\xrightarrow{\alpha} \mathcal{P} + \mathcal{Q} \quad \text{(SUM)}
\end{align*}
\]

where:

- \( \mathcal{P} + \mathcal{Q} \) denotes the function \( \mathcal{R} \) such that: \( \mathcal{R}(P) = \mathcal{P}(P) + \mathcal{Q}(P) \).
PEPA Stochastic semantics. . .

\[(\alpha, \lambda).P \xrightarrow{\alpha} [P \leftrightarrow \lambda] \quad (\text{Act}) \]

\[\alpha \neq \beta \quad (\emptyset-\text{Act})\]

\[P \xrightarrow{\alpha} \mathcal{P} \quad Q \xrightarrow{\alpha} \mathcal{Q} \quad (\text{SUM})\]

\[P + Q \xrightarrow{\alpha} \mathcal{P} + \mathcal{Q}\]

where:

- \(\mathcal{P} + \mathcal{Q}\) denotes the function \(\mathcal{R}\) such that: \(\mathcal{R}(P) = \mathcal{P}(P) + \mathcal{Q}(P)\).
PEPA Stochastic semantics... 

\[
\begin{align*}
\frac{(\alpha, \lambda).P \xrightarrow{\alpha} [P \leftrightarrow \lambda]}{(\text{Act})} & \quad \frac{(\alpha, \lambda).P \xrightarrow{\beta} \emptyset}{(\emptyset\text{-Act})} \\
\frac{P \xrightarrow{\alpha} \mathcal{P} Q \xrightarrow{\alpha} \mathcal{Q} \quad \text{(SUM)}}&
\end{align*}
\]

where:

- \( \mathcal{P} + \mathcal{Q} \) denotes the function \( \mathcal{R} \) such that: \( \mathcal{R}(P) = \mathcal{P}(P) + \mathcal{Q}(P) \).
PEPA Stochastic semantics...

\[
\begin{align*}
&P \xrightarrow{\alpha} P & Q \xrightarrow{\alpha} Q & \alpha \notin L \quad \text{(INT)} \\
&P \triangleright^L Q \xrightarrow{\alpha} P \triangleright^L Q + P \triangleright^L Q \\
&P \xrightarrow{\alpha} P & Q \xrightarrow{\alpha} Q & \alpha \in L \quad \text{(COOP)} \\
&P \triangleright^L Q \xrightarrow{\alpha} P \triangleright^L Q \cdot \min_{\oplus} \{P, Q\} & \oplus P \cdot \oplus Q
\end{align*}
\]

where:

- \( P \triangleright^L Q \) denotes the function \( R \) such that:
  \[
  R(R) = \begin{cases} 
  Q(R) & \text{if } Q = P \triangleright^L R \\
  \emptyset & \text{otherwise}
  \end{cases}
  \]

- \( P + Q \) denotes the function \( R \) such that:
  \[
  R(R) = \begin{cases} 
  P(P) \cdot Q(Q) & \text{if } R = P \triangleright^L Q \\
  \emptyset & \text{otherwise}
  \end{cases}
  \]
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**StoCCS: Stochastic CCS**

StoCCS is a Markovian extension of CCS where:

- **Output activities** are enriched with rates characterizing random variables with exponential distributions, modeling their duration;
- **Input activities** are equipped with weights characterizing the relative selection probability.

Other synchronisation patterns proposed in the literature can be easily dealt by using the proposed approach.

Continuous Time Markov Chains (CTMCs) for StoCCS specifications are obtained by considering only internal actions and channel interactions.
STOC CCS: Stochastic CCS

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- **input activities** are equipped with *weights* characterizing the relative selection probability;

Other synchronisation patterns proposed in the literature can be easily dealt by using the proposed approach.

Continuous Time Markov Chains (CTMCs) for STOC CCS specifications are obtained by considering only internal actions and channel interactions.
**STOCCS**: Stochastic CCS

STOCCS is a Markovian extension of CCS where:

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- *output activities* are enriched with *rates* characterizing random variables with exponential distributions, modeling their duration;

- *input activities* are equipped with *weights* characterizing the relative selection probability.

Other synchronisation patterns proposed in the literature can be easily dealt by using the proposed approach.

Continuous Time Markov Chains (CTMS) for STOCCS specifications are obtained by considering only internal actions and channel interactions.
SToCCS: Transitions rates

The rate of a transition basically depends on the rate of the triggering activity. The synchronization rate of $\alpha$ and $\alpha$ depends on the rate of $\alpha$, the weight of selected $\alpha$ and on the total weight of $\alpha$. The total weight of $\alpha$ is the sum of the weights of all $\alpha$-transitions $\alpha,\omega$ $\alpha,\omega$ $\alpha,\lambda$.

Two synchronizations can occur with rates:

$$\lambda \cdot \omega_1 + \omega_2$$

$$\lambda \cdot \omega_2 + \omega_1$$

Total synchronization rate does not depend on the number of available (input) partners.
The rate of a transition basically depends on the rate of the triggering activity.
The rate of a transition basically depends on the rate of the triggering *activity*

The synchronization rate of $\alpha$ and $\overline{\alpha}$ depends on the rate of $\overline{\alpha}$, the weight of selected $\alpha$ and on the total weight of $\alpha$.

- the *total weight* of $\alpha$ is the *sum* of the weights of all $\alpha$-transitions
The rate of a transition basically depends on the rate of the triggering *activity*.

The synchronization rate of $\bar{\alpha}$ and $\alpha$ depends on the rate of $\bar{\alpha}$, the weight of *selected* $\alpha$ and on the *total weight* of $\alpha$.

- the *total weight* of $\alpha$ is the *sum* of the weights of *all* $\alpha$-transitions.
The rate of a transition basically depends on the rate of the triggering activity.

The synchronization rate of $\overline{\alpha}$ and $\alpha$ depends on the rate of $\overline{\alpha}$, the weight of selected $\alpha$ and on the total weight of $\alpha$.
- the total weight of $\alpha$ is the sum of the weights of all $\alpha$-transitions.

Two synchronizations can occur with rates:

$$\lambda \cdot \frac{\omega_1}{\omega_1 + \omega_2} \quad \text{and} \quad \lambda \cdot \frac{\omega_2}{\omega_1 + \omega_2}$$

Total synchronization rate does not depend on the number of available (input) partners.
Synchronisation rule (SYNC)

\[
\begin{align*}
  P \xrightarrow{\vec{a}} P \quad P \xrightarrow{a} P_i \quad P \xrightarrow{\vec{a}} P_0 \quad Q \xrightarrow{\vec{a}} Q \quad Q \xrightarrow{a} Q_i \quad Q \xrightarrow{\vec{a}} Q_0 \\
  P|Q \xrightarrow{\vec{a}} P|Q + P_0|Q_0 + P_i|Q_i + P_0|Q_i
\end{align*}
\]

Next states of \( P|Q \) after \( \vec{a} \), i.e. after a synchronisation over channel \( a \), are:
**STOCSS: Stochastic semantics**

### Synchronisation rule (SYNC)

\[
P \xrightarrow{a} \mathcal{P} \quad P \xrightarrow{a} \mathcal{P}_i \quad P \xrightarrow{a} \mathcal{P}_o \quad Q \xrightarrow{a} \mathcal{Q} \quad Q \xrightarrow{a} \mathcal{Q}_i \quad Q \xrightarrow{a} \mathcal{Q}_o
\]

\[
P|Q \xrightarrow{a} \mathcal{P}|Q + P|\mathcal{Q} + \frac{\mathcal{P}_i|\mathcal{Q}_o}{\oplus \mathcal{P}_i} + \frac{\mathcal{P}_o|\mathcal{Q}_i}{\oplus \mathcal{Q}_i}
\]

Next states of \(P|Q\) after \(\xrightarrow{a}\), i.e. after a synchronisation over channel \(a\), are:

1. the next states of \(P\) after \(\xrightarrow{a}\) in parallel with \(Q\);
**STOCSS: Stochastic semantics**

### Synchronisation rule (SYNC)

\[
\begin{align*}
P & \xrightarrow{\overleftarrow{a}} P_i \quad P \xrightarrow{a} P_o \quad Q \xrightarrow{\overleftarrow{a}} \overline{Q} \quad Q \xrightarrow{a} \overline{Q}_i \quad \overline{P} | \overline{Q} \xrightarrow{\overleftarrow{a}} P | Q + P | \overline{Q} + \frac{P_i | Q_o + P_o | Q_i}{\oplus P_i + \oplus Q_i}
\end{align*}
\]

Next states of \( P|Q \) after \( \xleftarrow{a} \), i.e. after a synchronisation over channel \( a \), are:

1. the next states of \( P \) after \( \xleftarrow{a} \) in parallel with \( Q \);
2. the next states of \( Q \) after \( \xleftarrow{a} \) in parallel with \( P \);
**Synchronisation rule (SYNC)**

\[
P \xrightarrow{\overline{a}} P \quad P \xrightarrow{a} P_i \quad P \xrightarrow{\overline{a}} P_o \quad Q \xrightarrow{a} Q \quad a \quad Q \xrightarrow{\overline{a}} Q_o
\]

\[
P \mid Q \xrightarrow{\overline{a}} P \mid Q + P \mid Q + \frac{P_i \mid Q_o}{\oplus P_i} + \frac{P_o \mid Q_i}{\oplus Q_i}
\]

Next states of \( P \mid Q \) after \( \overline{a} \), i.e. after a synchronisation over channel \( a \), are:

1. the next states of \( P \) after \( \overline{a} \) in parallel with \( Q \);
2. the next states of \( Q \) after \( \overline{a} \) in parallel with \( P \);
3. the next states of \( P \) after \( \overline{a} \) in parallel with the next states of \( Q \) after \( a \);
Synchronisation rule (SYNC)

\[
P \xrightarrow{\bar{a}} P \quad P \xrightarrow{a} P_i \quad P \xrightarrow{\bar{a}} P_o \quad Q \xrightarrow{\bar{a}} Q_i \quad Q \xrightarrow{a} Q_o
\]

\[
P | Q \xrightarrow{\bar{a}} P | Q + P | Q + \frac{P_i | Q_o}{\oplus P_i} + \frac{P_o | Q_o}{\oplus Q_i}
\]

Next states of \( P | Q \) after \( \langle a \rangle \), i.e. after a synchronisation over channel \( a \), are:

1. the next states of \( P \) after \( \langle a \rangle \) in parallel with \( Q \);
2. the next states of \( Q \) after \( \langle a \rangle \) in parallel with \( P \);
3. the next states of \( P \) after \( \bar{a} \) in parallel with the next states of \( Q \) after \( a \);
4. the next states of \( P \) after \( a \) in parallel with the next states of \( Q \) after \( \bar{a} \).
Unfortunately, the proposed semantics, like in stochastic $\pi$, does not respect a standard and expected property of the CCS parallel composition. Indeed, using the above semantics, this operator is not associative.
Unfortunately, the proposed semantics, like in stochastic π, does not respect a standard and expected property of the CCS parallel composition. Indeed, using the above semantics, this operator is not associative.

For instance:

\[
\overline{a}^\lambda . P | (a^{\omega_1} . Q_1 | a^{\omega_2} . Q_2) \xrightarrow{a} \\
(\overline{a}^\lambda . P | a^{\omega_1} . Q_1) | a^{\omega_2} . Q_2 \xrightarrow{a}
\]
Unfortunately, the proposed semantics, like in stochastic π, does not respect a standard and expected property of the CCS parallel composition. Indeed, using the above semantics, this operator is not associative.

For instance:

\[
\begin{align*}
\overline{a}^{\lambda}.P|\langle a^{\omega_1}.Q_1 | a^{\omega_2}.Q_2 \rangle & \xrightarrow{a} [P|\langle Q_1 | a^{\omega_2}.Q_2 \rangle \mapsto \frac{\lambda \cdot \omega_1}{\omega_1+\omega_2}, P|\langle a^{\omega_1}.Q_1 | Q_2 \rangle \mapsto \frac{\lambda \cdot \omega_2}{\omega_1+\omega_2}] \\
(\overline{a}^{\lambda}.P|a^{\omega_1}.Q_1)|a^{\omega_2}.Q_2 & \xrightarrow{a} [(P|Q_1)|a^{\omega_2}.Q_2 \mapsto \lambda, (P|a^{\omega_1}.Q_1)|Q_2 \mapsto \lambda]
\end{align*}
\]
Computing the rate of a synchronization

\[ P \xrightarrow{\overline{s}, \lambda} P' \]

If \( \omega \) is the total weight of \( s \) in \( P \):

\[ \lambda' = \lambda \cdot \frac{\omega}{\omega + \lambda} \]

This is crucial to guarantee associativity of parallel composition in CCS-like synchronizations.
Computing the rate of a synchronization

If $\omega$ is the total weight of $s$ in $P$:

$$\lambda' = \lambda \cdot \omega$$

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Computing the rate of a synchronization

If $\bar{\omega}$ is the total weight of $s$ in $P$:

$$\lambda' = \lambda \cdot \frac{\bar{\omega}}{\bar{\omega} + \omega}$$
Computing the rate of a synchronization

If \( \overline{\omega} \) is the total weight of \( s \) in \( P \):

\[
\lambda' = \lambda \cdot \frac{\overline{\omega}}{\overline{\omega} + \omega}
\]

This is crucial to guarantee associativity of parallel composition in CCS-like synchronizations.
The synchronization rule:

\[
\begin{align*}
P \xrightarrow{s} P & \quad P \xrightarrow{s} P_d \quad P \xrightarrow{s} P_i \quad Q \xrightarrow{s} Q \quad Q \xrightarrow{s} Q_d \quad Q \xrightarrow{s} Q_i \\
\frac{P \mid Q \xrightarrow{s} P \mid Q \cdot \oplus P_d + Q \oplus Q_d}{\oplus P_d + \oplus Q_d} & + \frac{P \mid Q \cdot \oplus Q_d + Q \oplus Q_d}{\oplus P_d + \oplus Q_d} + \frac{(\nu r)(P \mid Q_d)}{\oplus P_d + \oplus Q_d} + \frac{(\nu r)(P_i \mid Q_d)}{\oplus P_d + \oplus Q_d}
\end{align*}
\]
The synchronization rule:

\[
P \xrightarrow{\bar{s}} P \quad \quad P \xrightarrow{s} P_d \quad \quad P \xrightarrow{\bar{s}} P_i \quad \quad Q \xrightarrow{\bar{s}} Q \quad \quad Q \xrightarrow{s} Q_d \quad \quad Q \xrightarrow{\bar{s}} Q_i
\]

\[
P|Q \xrightarrow{\bar{s}} P|Q \cdot \oplus P_d + \frac{P|Q \cdot \oplus P_d + (\nu r)(P_d|Q_i) + (\nu r)(P_i|Q_d)}{\oplus P_d + \oplus Q_d}
\]

Interactions on channel \( s \) in \( P|Q \) are determined by considering...
The synchronization rule:

\[ P \xrightarrow{s} P \quad P \xrightarrow{s} P_d \quad P \xrightarrow{s} P_i \quad Q \xrightarrow{s} Q \quad Q \xrightarrow{s} Q_d \quad Q \xrightarrow{s} Q_i \]

\[ P|Q \xrightarrow{s} P|Q \cdot \oplus P_d \oplus Q \cdot \oplus Q_d + \frac{P|Q \cdot \oplus P_d}{\oplus P_d + \oplus Q_d} + \frac{(\nu r)(P_d|Q)}{\oplus P_d + \oplus Q_d} + \frac{(\nu r)(P_i|Q)}{\oplus P_d + \oplus Q_d} \]

Interactions on channel \( s \) in \( P|Q \) are determined by considering:

- the synchronisations in \( P \), where synchronization rates are updated for considering input in \( Q \);
The synchronization rule:

\[
P \xrightarrow{\bar{s}} P \quad P \xrightarrow{s} P_d \quad P \xrightarrow{\bar{s}} P_i \quad Q \xrightarrow{\bar{s}} Q \quad Q \xrightarrow{s} Q_d \quad Q \xrightarrow{\bar{s}} Q_i
\]

\[
P|Q \xrightarrow{\bar{s}} P|Q \cdot \oplus P_d + \oplus P_d + \oplus Q_d + \oplus Q_d + (\nu r)(P_d|Q_i) + (\nu r)(P_i|Q_d)
\]

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STOCSS: stochastic semantics (2)

The synchronization rule:

\[
P \xrightarrow{s} P \quad P \xrightarrow{\bar{s}} P_d
\]
\[
P \xrightarrow{s} P_i \quad Q \xrightarrow{s} Q_d
\]
\[
Q \xrightarrow{\bar{s}} Q_i
\]

\[
P|Q \xrightarrow{s} P|Q \cdot \bigoplus P_d \quad \bigoplus P_d + \bigoplus Q_d
\]

Interactions on channel \( s \) in \( P|Q \) are determined by considering:

- the synchronisations in \( P \), where synchronization rates are updated for considering input in \( Q \);
- the synchronisations in \( Q \), where synchronization rates are updated for considering input in \( P \);
- interactions between input in \( P \) with output in \( Q \);
**STOCCS: stochastic semantics (2)**

The synchronization rule:

\[
\begin{align*}
P \xrightarrow{s} \overline{P} & \quad P \xrightarrow{s} \overline{P} \quad P \xrightarrow{s} \overline{P} \\
Q \xrightarrow{s} \overline{Q} & \quad Q \xrightarrow{s} \overline{Q} \quad Q \xrightarrow{s} \overline{Q}
\end{align*}
\]

\[
P|Q \xrightarrow{s} \overline{P} \cdot \overline{Q} + \overline{P} \cdot \overline{Q} + \nu r(\overline{P} \cdot \overline{Q}) + \nu r(\overline{P} \cdot \overline{Q})
\]

Interactions on channel \( s \) in \( P|Q \) are determined by considering:

- the synchronisations in \( P \), where synchronization rates are updated for considering input in \( Q \);
- the synchronisations in \( Q \), where synchronization rates are updated for considering input in \( P \);
- interactions between input in \( P \) with output in \( Q \);
- interactions between input in \( P \) with output in \( Q \).
Outline...

1. Motivations
2. PEPA
3. Stochastic CCS
4. MoSL: Mobile Stochastic Logic
5. Model Checking MoSL
6. Conclusions and Future Directions
MoSL: General

1. a *temporal logic* (dynamic evolution);
2. both *action*- and *state*-based;
3. a *real-time* logic (real-time bounds);
4. a *probabilistic logic* (performance and dependability aspects);
5. a *spatial logic* (spatial structure of the network).
MoSL: General (cont’d)

MoSL ← Site-awareness
/action binding variables

[De Nicola et al.] aCTL

[Clark et al.] CTL

[Aziz et al., Baier et al.] CSL

[aCSL]

[Clark et al.] CTMC

[De Nicola et al.] CSL

[Hermanns et al.]

M. Loreti (DSI@FI)
MoSL: Atomic propositions

\[ \mathcal{A} ::= \rho \rightarrow \Phi \mid \rho \leftarrow \Phi \]
MoSL: Atomic propositions

\[ \mathcal{N} ::= \rho \rightarrow \Phi \mid \rho \leftarrow \Phi \]

- \( \mathcal{B} \) is the set of resource predicate \( \rho, \rho_1, \rho' \ldots \)

- Let \( \mathcal{R} = (S, A, \to) \) be an RTS, we let:
  - \( \oplus \) denoting a total function \( S \times \mathcal{B} \to S \);
  - \( \ominus \) denoting a partial function \( S \times \mathcal{B} \to S \).
**MoSL: Atomic propositions**

\[ \mathcal{A} ::= \rho \rightarrow \Phi \mid \rho \leftarrow \Phi \]

- \( \mathcal{B} \) is the set of *resource predicate* \( \rho, \rho_1, \rho' \ldots \)

- Let \( \mathcal{R} = (S, A, \rightarrow) \) be an RTS, we let:
  - \( \oplus \) denoting a *total function* \( S \times \mathcal{B} \rightarrow S \);
  - \( \ominus \) denoting a *partial function* \( S \times \mathcal{B} \rightarrow S \).

\[ s \models_{\oplus, \ominus} b \rightarrow \Phi \iff \exists s' : s' = s \ominus b \land s' \models_{\oplus, \ominus} \phi \]

\[ s \models_{\oplus, \ominus} b \leftarrow \Phi \iff s \oplus b \models_{\oplus, \ominus} \phi \]
In CTL

$\Phi \cup \Psi$
MoSL: Action specifiers and action sets

In aCTL

\[ \Phi \; \Delta \cup \Omega \; \Psi \]

\( \Delta, \Omega \): Sets of actions (uninterpreted, atomic)
MoSL: Action specifiers and action sets

In MoSL

\[
\Phi \Delta \cup \Omega \Psi
\]

\(\Delta, \Omega\): Sets of *action specifiers*, to be matched against actions
Satisfied by those paths where eventually a $\Psi$-state is reached, by time $t$, via a $\Phi$-path, and, in addition, while evolving between $\Phi$ states, actions are performed satisfying $\Delta$ and the $\Psi$-state is entered via an action satisfying $\Omega$. 

\[
\Phi \quad \Delta \quad U^t \quad \Psi
\]
MoSL: Path formulae

\[ \Phi \, \Delta \, \mathcal{U}^{<t} \, \Psi \]

- Satisfied by those paths where eventually a \( \Psi \)-state is reached, by time \( t \), via a \( \Phi \)-path, and, in addition, while evolving between \( \Phi \) states, actions are performed satisfying \( \Delta \) and the \( \Psi \)-state is entered via an action satisfying \( \Omega \).

- Instantiations of variables in \( \Omega \) act as binders \( \Psi \).
MoSL: Path formulae

\[ \Phi \Delta \mathcal{U}^{<t} \Psi \]

- Satisfied by those paths where eventually a \( \Psi \)-state is reached, by time \( t \), via a \( \Phi \)-path, and, in addition, while evolving between \( \Phi \) states, actions are performed satisfying \( \Delta \) and the \( \Psi \)-state is entered via an action satisfying \( \Omega \).

- Instantiations of variables in \( \Omega \) act as binders \( \Psi \).

- Simpler operator: \( \Phi \Delta \mathcal{U}^{<t} \Psi \).
MoSL: Path formulae

\( \Phi \triangle U^<_{\Omega} \Psi \)

- Satisfied by those paths where eventually a \( \Psi \)-state is reached, by time \( t \), via a \( \Phi \)-path, \textit{and}, in addition, while evolving between \( \Phi \) states, actions are performed satisfying \( \Delta \) and the \( \Psi \)-state is entered via an action satisfying \( \Omega \).

- Instantiations of variables in \( \Omega \) act as binders \( \Psi \).

- Simpler operator: \( \Phi \triangle U^< \Psi \).

- Time \( t \) can be omitted (assumed as \( \infty \)).
MoSL: State formulae

Φ ::= tt | ω | ¬ Φ | Φ ∨ Φ
MoSL: State formulae

\[ \Phi ::= \text{tt} | \text{N} | \neg \Phi | \Phi \lor \Phi | \mathcal{P}_{\bowtie p}(\varphi) \]

with \(\bowtie \in \{<, >, \leq, \geq\}\) and \(p \in [0, 1]\)

CSL path-operator: \(\mathcal{P}_{\bowtie p}(\varphi)\)
Satisfied by a state \(s\) iff the total probability mass for all paths starting in \(s\) that satisfy \(\varphi\) meets the bound \(\bowtie p\);
MoSL: State formulae

\[ \Phi ::= \text{tt} \mid \mathbb{N} \mid \neg \Phi \mid \Phi \lor \Phi \mid P_{\triangleleft p}(\varphi) \mid S_{\triangleleft p}(\Phi) \]

with \(\triangleleft \in \{<, >, \leq, \geq\}\) and \(p \in [0, 1]\)

**CSL path-operator:** \(P_{\triangleleft p}(\varphi)\)
Satisfied by a state \(s\) iff the total probability mass for all paths starting in \(s\) that satisfy \(\varphi\) meets the bound \(\triangleleft p\);

**CSL Steady-state operator:** \(S_{\triangleleft p}(\Phi)\)
Satisfied by a state \(s\) iff the probability of reaching from \(s\), in the long run, a state which satisfies \(\Phi\) is \(\triangleleft p\).
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Model Checking MoSL

- We have defined an algorithm that given a finite RTS \((S, A, \rightarrow)\) and a MoSL formula \(\Phi\), yields the states in \(S\) satisfying \(\Phi\);
- Model-checking of RTSs is performed by relying on a CSL model checker.
  - the RTS to be model-checked is translated into an equivalent state-labelled CTMC that can be analysed by making use of existing (state-based) CSL model checkers.
Definition

For finite and fully stochastic RTS $\mathcal{R} = (S, A, \rightarrow)$ let $\mathcal{K}(\mathcal{R})$ be the CTMC $(W, \mathcal{R})$ such that:

- $W = S \times A \cup \{\bot\}$;
- Let $s_1 \xrightarrow{\alpha} \mathcal{P}: \mathcal{R}((s_1, \beta), (s_2, \alpha)) \overset{\text{def}}{=} \mathcal{P}(\alpha)$
Definition

For CTMC $\mathcal{M} = (S, R)$, $S_1, S_2 \subseteq S$, $t \in \mathbb{R}_{\geq 0}$, $p \in [0, 1]$, and $\sqin \in \{<, >, \leq, \geq\}$:

- $\text{until}(\sqin, p, t, S_1, S_2, \mathcal{M}) \overset{\text{def}}{=} \{ s \in S | \mathbb{P}\{\pi \in \text{Paths}(s) | \exists t' < t. \pi(t') \in S_2 \text{ and } \forall t'' < t'. \pi(t'') \in S_1\} \sqin p\}$

- $\text{steady}(\sqin, p, S_1, \mathcal{M}) \overset{\text{def}}{=} \{ s \in S | \lim_{t \to \infty} \mathbb{P}\{\pi \in \text{Paths}(s) | \pi(t) \in S_1\} \sqin p\}$
Let $\mathcal{R} = (S, A, \rightarrow)$:

- $\text{Sat}^{\oplus, \ominus}(\texttt{tt}, \mathcal{R}) \overset{\text{def}}{=} S$
- $\text{Sat}^{\oplus, \ominus}(\neg \Phi, \mathcal{R}) \overset{\text{def}}{=} S \setminus \text{Sat}^{\oplus, \ominus}(\Phi, \mathcal{R})$
- $\text{Sat}^{\oplus, \ominus}(\Phi_1 \lor \Phi_2, \mathcal{R}) \overset{\text{def}}{=} \text{Sat}^{\oplus, \ominus}(\Phi_1, \mathcal{R}) \cup \text{Sat}^{\oplus, \ominus}(\Phi_2, \mathcal{R})$
- $\text{Sat}^{\oplus, \ominus}(\rho \leftarrow \Phi, \mathcal{R}) \overset{\text{def}}{=} \{ s \mid s \oplus \rho \in \text{Sat}^{\oplus, \ominus}(\Phi, \mathcal{R}) \}$
- $\text{Sat}^{\oplus, \ominus}(\rho \rightarrow \Phi, \mathcal{R}) \overset{\text{def}}{=} \{ s \mid s \ominus \rho \in \text{Sat}^{\oplus, \ominus}(\Phi, \mathcal{R}) \}$
Let $R = (S, A, \rightarrow)$:

- $\text{Sat}^{\oplus, \ominus}(\mathcal{P}_x \alpha \Phi \Delta \mathcal{U}_t^< \Psi), R) \overset{\text{def}}{=} $
  
  let $X = \{ \alpha | \alpha \in A : \alpha \models \Delta \}$ in
  
  let $Y = \{ \beta | \beta \in A : \beta \models \Omega \}$ in
  
  let $S_1 = \text{Sat}^{\oplus, \ominus}(\Phi, R) \times (X \cup \{ \bot \})$ in
  
  let $S_2 = \text{Sat}^{\oplus, \ominus}(\Psi, R) \times Y$ in

  $\{ s | (s, \bot) \in \text{until}(\nabla, p, t, S_1, S_2, \mathcal{K}(R)) \}$

- $\text{Sat}^{\oplus, \ominus}(S_x \alpha \Phi), R) \overset{\text{def}}{=} $

  $\{ s \in S | (s, \bot) \in \text{steady}(\nabla, p, \text{Sat}^{\oplus, \ominus}(\Phi, R) \times (A \cup \{ \bot \}), \mathcal{K}(R)) \}$
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Conclusions:

- We have introduced Rate Transition Systems and have used them as the basic model for defining stochastic behaviour of processes.
- We have then shown how RTS can be used to provide the stochastic operational semantics of PEPA and CCS.
- We have also introduced a natural notion of bisimulation over RTS that is finer than Markovian bisimulation and useful for reasoning about stochastic behaviours.
- We have introduced a stochastic logic, \( \text{MoSL} \), that permits specifying quantitative properties of RTS.
- We have presented a model checking algorithm for \( \text{MoSL} \).
Future Work...

Ongoing work

- Consider alternative semantics synchronization rates:
  - based on *phase type* distributions
  - based on *Interactive Markov Chains*

- Develop an on-the-fly model checker for MoSL.
Thank you for your attention!