Stochastic Process Algebras and Stochastic Model Checking

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PACO: Kick-off Meeting
Bertinoro, October 23-24 2008

Outline…

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1. Motivations
2. PEPA
3. Stochastic CCS
4. MoSL: Mobile Stochastic Logic
5. Model Checking MoSL
6. Conclusions and Future Directions

Motivations…

A number of stochastic process algebras have been proposed in the last two decades. These are based on:

1. Labeled Transition Systems (LTS)
    - for providing compositional semantics of languages
    - for describing *qualitative properties*
2. Continuous Time Markov Chains (CTMC)
    - for analysing *quantitative properties*

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Semantics of these calculi have been given by variants of the Structured Operational Semantics (SOS) approach but:

- they do not rely on any general framework
- it is rather difficult to appreciate differences and similarities of such semantics.

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- Random Variables are assumed to be Exponentially Distributed
- Random Variables are fully characterised by its rate

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If X is *exponentially distributed* with *parameter* $\lambda \in \mathbf{R}_{>0}$:

- $\mathbb{P}\{X \leq d\} = 1 - e^{-\lambda \cdot d}$, for $d \geq 0$
- The average duration of X is $\frac{1}{\lambda}$; the variance of X is $\frac{1}{\lambda^2}$
- *Memory-less*: $\mathbb{P}\{X \leq t + d \mid X > t\} \; = \; \mathbb{P}\{X \leq d\}$

Continuous Time Markov Chains

Continuous Time Markov Chains are a successful mathematical framework for modeling and analysing performance and dependability of systems

CTMCs provide well established Analysis Techniques and Tools such as:

- Steady State Analysis
- Transient Analysis
- Stochastic Timed/Temporal Logics
- Stochastic Model Checking

A CTMC is a pair $(\S, \mathbf{R})$

- $\S$: a countable set of states
- $\mathbf{R} : \S \times \S \rightarrow \mathrm{R}_{\geq 0}$, the rate matrix

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Process Calculi:

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Semantics of stochastic process calculi…

We introduce a variant of Rate Transition Systems (RTS), proposed by Klin and Sassone, and use it as the basis for defining stochastic behaviour of processes.

As in previous approaches to enhance process calculi with stochastic features, we will first define an enriched LTS by means of an SOS semantics and then use it to associate a CTMC to any term.

There are however two distinguishing aspects of our work:

- The transition relation we use associates terms and actions to functions from terms to rates
- We adapt the *apparent rate* approach, originally developed by Hillston for a process algebra with a CSP-like multi-party synchronisation paradigm, to calculi like CCS and π-calculus

Semantics of stochastic process calculi…

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$P \xrightarrow{\alpha} \mathscr{P}$ means that:

- if $\mathscr{P}(Q) = x$ ($\neq 0$) then *Q* is reachable from *P* via the execution of α with rate or weight *x*
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We have that if $P \xrightarrow{\alpha} \mathscr{P}$ then

- $\oplus\mathscr{P} = \sum_Q \mathscr{P}(Q)$ represents the total rate/weight of α in *P*.

Rate transition systems...

Definition (Rate Transition Systems)

A rate transition system is a triple $(S, A, \longrightarrow)$ where:

- S is a set of states;
- A is a set of transition labels;
- $\rightarrow \subseteq S \times A \times [S \rightarrow \mathbf{R}_{\geq 0}]$

Notations:

- RTS will be denoted by $\mathcal{R}, \mathcal{R}_1, \mathcal{R}', \ldots,$
- Elements of $[S \rightarrow \mathbf{R}_{\geq 0}]$ are denoted by $\mathscr{P}, \mathscr{Q}, \mathscr{R}, \ldots$
- $\emptyset$ denotes the constant function 0
- $[s_1 \mapsto v_1, \ldots, s_n \mapsto v_n]$ will denote a function associating v_i to s_i and 0 to all the other states.

Rate transition systems…

Definition

Let $\mathcal{R} = (S, A, \rightarrow)$ be an RTS, then:

- $\mathcal{R}$ is *fully stochastic* if and only if for each $s \in S$, $\alpha \in A$, $\mathscr{P}$ and $\mathscr{Q}$ we have: $s \xrightarrow{\alpha} \mathscr{P}, s \xrightarrow{\alpha} \mathscr{Q} \Longrightarrow \mathscr{P} = \mathscr{Q}$
- $\mathcal{R}$ is *image finite* if and only if for each $s \in S$, $\alpha \in A$ and $\mathscr{P}$ such that $s \xrightarrow{\alpha} \mathscr{P}$ we have: $\{s' | \mathscr{P}(s') > 0\}$ is finite

Mettere figure

From RTS to CTMC…

For sets $C \subseteq S$ and $A' \subseteq A$, the set of derivatives of C *through* A', denoted $Der(C, A')$, is the smallest set such that:

- $C \subseteq Der(C, A')$,
- if $s \in Der(C, A')$ and there exists $\alpha \in A'$ and $\mathscr{Q} \in \Sigma_S$ such that $s \xrightarrow{\alpha} \mathscr{Q}$ then $\{s' \mid \mathscr{Q}(s') > 0\} \subseteq Der(C)$

Let $\mathcal{R} = (S, A, \rightarrow)$ be a *fully stochatics* RTS, for $C \subseteq S$, the CTMC of C, when one considers only actions $A' \subseteq A$ is defined as $CTMC[C, A'] \overset{def}{=} (Der(C, A'), \mathbf{R})$ where for all $s_1, s_2 \in S$:

$$\mathbf{R}[s_1, s_2] \overset{def}{=} \sum_{\alpha \in A'} \mathscr{P}^{\alpha}(s_2) \qquad \text{with } s_1 \xrightarrow{\alpha} \mathscr{P}^{\alpha}.$$

Rate aware bisimulation…

Definition (Rate Aware Bisimilarity)

- An equivalence relation $\mathcal{E}$ on $\mathcal{C}$ is a *rate aware* bisimulation if and only if, for all $(s_1, s_2) \in \mathcal{E}$, $C \in \mathcal{C}_{/\mathcal{E}}$, and for all α and $\mathscr{P}$:

$$s_1 \xrightarrow{\alpha} \mathscr{P} \Longrightarrow \exists \mathscr{Q} : \; s_2 \xrightarrow{\alpha} \mathscr{Q} \wedge \mathscr{P}(C) = \mathscr{Q}(C)$$

- Two states $s_1, s_2 \in S$ are *rate aware bisimilar* ($s_1 \sim s_2$) if there exists a rate aware bisimulation $\mathcal{E}$ such that $(s_1, s_2) \in \mathcal{E}$.

Notice that *rate aware bisimilarity* and *strong bisimilarity* coincide when one does not take into account rates.

Theorem

Let $\mathcal{R} = (S, A, \longrightarrow)$, *for each* $A' \subseteq A$ *and for each* $s_1, s_2 \in S$ *and* $(S, \mathbf{R}) = CTMC[\{s_1, s_2\}, A']$: $s_1 \sim s_2 \Longrightarrow s_1 \sim_M s_2$

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PEPA: Performance Process Algebra

- In PEPA systems are be described as interactions of *components* that may engage in *activities*
  - Components reflect the behaviour of relevant parts of the system, while activities capture the actions that the components perform.
- Each PEPA activity consists of a pair (α, λ) where:
  - α symbolically denotes the performed action;
  - $\lambda > 0$ is the rate of the negative *exponential* distribution.
- If $\mathcal{A}$ is a set of *actions*, ranged over by $\alpha, \alpha', \alpha_1, \ldots$, then $\mathcal{P}_{PEPA}$ is the set of process terms $P, P', P_1, \ldots$ defined according to the following grammar

$$P ::= (\alpha, \lambda).P \mid P + P \mid P \bowtie_L P \mid P/L \mid A$$

PEPA Stochastic semantics. . .

$$\frac{}{(\alpha,\lambda).P \xrightarrow{\alpha} [P \mapsto \lambda]}\ (\textsc{Act}) \qquad \frac{\alpha \neq \beta}{(\alpha,\lambda).P \xrightarrow{\beta} \emptyset}\ (\emptyset\text{-}\textsc{Act})$$

$$\frac{P \xrightarrow{\alpha} \mathscr{P} \quad Q \xrightarrow{\alpha} \mathscr{Q}}{P + Q \xrightarrow{\alpha} \mathscr{P} + \mathscr{Q}}\ (\textsc{Sum}) \qquad \frac{P \xrightarrow{\alpha} \mathscr{P} \quad Q \xrightarrow{\alpha} \mathscr{Q} \quad \alpha \notin L}{P \bowtie_L Q \xrightarrow{\alpha} \mathscr{P} \bowtie_L Q + P \bowtie_L \mathscr{Q}}\ (\textsc{Int})$$

$$\frac{P \xrightarrow{\alpha} \mathscr{P} \quad Q \xrightarrow{\alpha} \mathscr{Q} \quad \alpha \in L}{P \bowtie_L Q \xrightarrow{\alpha} \mathscr{P} \bowtie_L \mathscr{Q} \cdot \frac{\min\{\oplus\mathscr{P}, \oplus\mathscr{Q}\}}{\oplus\mathscr{P} \cdot \oplus\mathscr{Q}}}\ (\textsc{Coop})$$

$$\frac{P \xrightarrow{\alpha} \mathscr{P} \quad \alpha \notin L}{P/L \xrightarrow{\alpha} \mathscr{P}/L}\ (\text{P-}\textsc{Hide}) \qquad \frac{\alpha \in L}{P/L \xrightarrow{\alpha} \emptyset}\ (\emptyset\text{-}\textsc{Hide})$$

$$\frac{P \xrightarrow{\tau} \mathscr{P}_\tau \quad \forall \alpha \in L. P \xrightarrow{\alpha} \mathscr{P}_\alpha}{P/L \xrightarrow{\tau} \mathscr{P}_\tau/L + \sum_{\alpha \in L} \mathscr{P}_\alpha/L}\ (\textsc{Hide}) \qquad \frac{P \xrightarrow{\alpha} \mathscr{P} \quad A \triangleq P}{A \xrightarrow{\alpha} \mathscr{P}}\ (\textsc{Call})$$

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where:

- $\mathscr{P} + \mathscr{Q}$ denotes the function $\mathscr{R}$ such that: $\mathscr{R}(P) = \mathscr{P}(P) + \mathscr{Q}(P)$.

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Continuous Time Markov Chains(CTMS) for StoCCS specifications are obtained by considering only internal actions and channel interactions.

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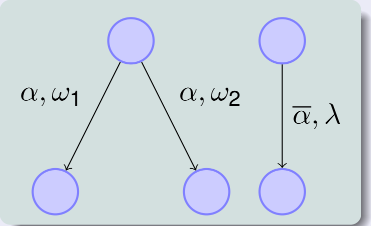

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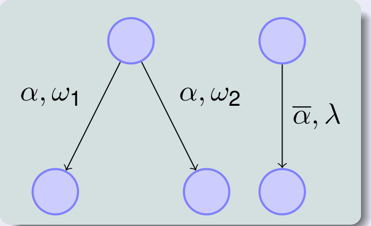

- Two synchronizations can occur with rates:

$$\lambda \cdot \frac{\omega_1}{\omega_1 + \omega_2} \qquad \lambda \cdot \frac{\omega_2}{\omega_1 + \omega_2}$$

- *Total synchronization rate* does not depend on the number of available (input) partners

STOCCS: Stochastic semantics

Synchronisation rule (SYNC)

$$\frac{P \xrightarrow{\overleftrightarrow{a}} \mathscr{P} \quad P \xrightarrow{a} \mathscr{P}_i \quad P \xrightarrow{\bar{a}} \mathscr{P}_o \quad Q \xrightarrow{\overleftrightarrow{a}} \mathscr{Q} \quad Q \xrightarrow{a} \mathscr{Q}_i \quad Q \xrightarrow{\bar{a}} \mathscr{Q}_o}{P|Q \xrightarrow{\overleftrightarrow{a}} \mathscr{P}|Q + P|\mathscr{Q} + \frac{\mathscr{P}_i|\mathscr{Q}_o}{\oplus \mathscr{P}_i} + \frac{\mathscr{P}_o|\mathscr{Q}_i}{\oplus \mathscr{Q}_i}}$$

Next states of $P|Q$ after $\overleftrightarrow{a}$, i.e. after a synchronisation over channel a, are:

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1. the next states of P after $\overleftrightarrow{a}$ in parallel with Q;
2. the next states of Q after $\overleftrightarrow{a}$ in parallel with P;
3. the next states of P after $\bar{a}$ in parallel with the next states of Q after a;

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Next states of $P|Q$ after $\overleftrightarrow{a}$, i.e. after a synchronisation over channel a, are:

1. the next states of P after $\overleftrightarrow{a}$ in parallel with Q;
2. the next states of Q after $\overleftrightarrow{a}$ in parallel with P;
3. the next states of P after $\bar{a}$ in parallel with the next states of Q after a;
4. the next states of P after a in parallel with the next states of Q after $\bar{a}$.

STOCCS: Stochastic semantics

Unfortunately, the proposed semantics, like in stochastic π, does not respect a standard and expected property of the CCS parallel composition. Indeed, using the above semantics, this operator is not associative.

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For instance:

$$\overline{a}^{\lambda}.P|(a^{\omega_1}.Q_1|a^{\omega_2}.Q_2) \quad \xrightarrow{\overleftrightarrow{a}}$$

$$(\overline{a}^{\lambda}.P|a^{\omega_1}.Q_1)|a^{\omega_2}.Q_2 \quad \xrightarrow{\overleftrightarrow{a}}$$

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For instance:

$$\begin{array}{lcl} \overline{a}^{\lambda}.P|(a^{\omega_1}.Q_1|a^{\omega_2}.Q_2) & \xrightarrow{\overleftrightarrow{a}} & [P|(Q_1|a^{\omega_2}.Q_2) \mapsto \frac{\lambda\cdot\omega_1}{\omega_1+\omega_2}, P|(a^{\omega_1}.Q_1|Q_2) \mapsto \frac{\lambda\cdot\omega_2}{\omega_1+\omega_2}] \\ (\overline{a}^{\lambda}.P|a^{\omega_1}.Q_1)|a^{\omega_2}.Q_2 & \xrightarrow{\overleftrightarrow{a}} & [(P|Q_1)|a^{\omega_2}.Q_2 \mapsto \lambda, (P|a^{\omega_1}.Q_1)|Q_2 \mapsto \lambda] \end{array}$$

Computing the rate of a synchronization

$$\begin{tikzcd}
P \arrow[d, "{\overleftrightarrow{s}\, , \lambda}"] \\
P'
\end{tikzcd}$$

Computing the rate of a synchronization

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P \arrow[d, "{\overleftrightarrow{s}, \lambda}"] & s^{\omega}.Q \arrow[d, "{s, \omega}"] \\
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P' & Q & & P' \mid Q
\end{tikzcd}$$

If $\overline{\omega}$ is the total weight of s in P:

$$\lambda' =$$

Computing the rate of a synchronization

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This is crucial to guarantee associativity of parallel composition in CCS-like synchronizations.

STOCCS: stochastic semantics (2)

The synchronization rule:

$$\frac{P \xrightarrow{\overleftrightarrow{s}} \mathscr{P} \quad P \xrightarrow{s} \mathscr{P}_d \quad P \xrightarrow{\bar{s}} \mathscr{P}_i \quad Q \xrightarrow{\overleftrightarrow{s}} \mathscr{Q} \quad Q \xrightarrow{s} \mathscr{Q}_d \quad Q \xrightarrow{\bar{s}} \mathscr{Q}_i}{P|Q \xrightarrow{\overleftrightarrow{s}} \frac{\mathscr{P}|Q \cdot \oplus\mathscr{P}_d}{\oplus\mathscr{P}_d + \oplus\mathscr{Q}_d} + \frac{P|\mathscr{Q} \cdot \oplus\mathscr{P}_d}{\oplus\mathscr{P}_d + \oplus\mathscr{Q}_d} + \frac{(\nu r)(\mathscr{P}_d|\mathscr{Q}_i)}{\oplus\mathscr{P}_d + \oplus\mathscr{Q}_d} + \frac{(\nu r)(\mathscr{P}_i|\mathscr{Q}_d)}{\oplus\mathscr{P}_d + \oplus\mathscr{Q}_d}}$$

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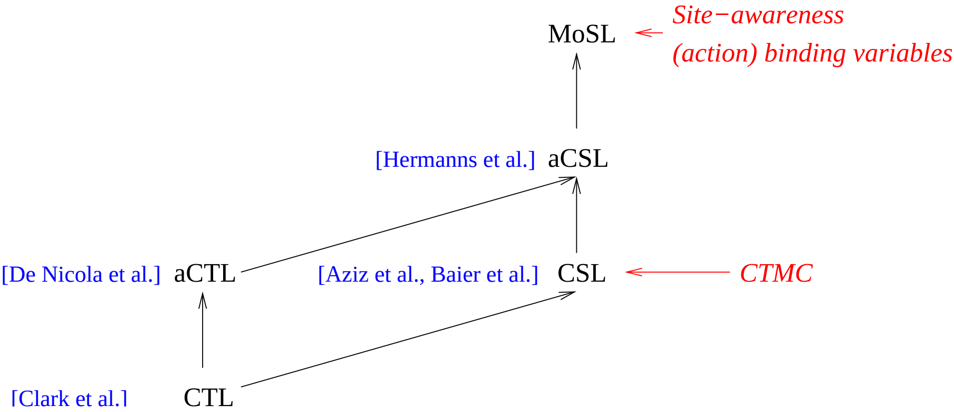
Outline. . .

1. Motivations
2. PEPA
3. Stochastic CCS
4. **MoSL: Mobile Stochastic Logic**
5. Model Checking MoSL
6. Conclusions and Future Directions

MoSL: General

1. a *temporal logic* (dynamic evolution);
2. both *action*- and *state*-based;
3. a *real-time* logic (real-time bounds);
4. a *probabilistic logic* (performance and dependability aspects);
5. a *spatial logic* (spatial structure of the network).

MoSL:General (cont'd)



MoSL ← *Site−awareness*

(action) binding variables

[Hermanns et al.] aCSL

[De Nicola et al.] aCTL

[Aziz et al., Baier et al.] CSL ← *CTMC*

[Clark et al.] CTL

MoSL: Atomic propositions

$$\aleph ::= \rho \rightarrow \Phi \mid \rho \leftarrow \Phi$$

MoSL: Atomic propositions

$$\aleph ::= \rho \rightarrow \Phi \mid \rho \leftarrow \Phi$$

- $\mathcal{B}$ is the set of *resource predicate* $\rho, \rho_1, \rho' \ldots$
- Let $\mathcal{R} = (S, A, \longrightarrow)$ be an RTS, we let:
  - $\oplus$ denoting a *total function* $S \times \mathcal{B} \rightarrow S$;
  - $\ominus$ denoting a *partial function* $S \times \mathcal{B} \rightarrow S$.

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- $s \models_{\oplus,\ominus} b \rightarrow \Phi \Leftrightarrow \exists s' : s' = s \ominus b \wedge s' \models_{\oplus,\ominus} \phi$
- $s \models_{\oplus,\ominus} b \leftarrow \Phi \Leftrightarrow s \oplus b \models_{\oplus,\ominus} \phi$

MoSL: Action specifiers and action sets

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In CTL

$$\Phi \;\; \mathcal{U} \; \Psi$$

MoSL: Action specifiers and action sets

In aCTL

$$\Phi \ {}_{\Delta}\mathcal{U}_{\Omega} \ \Psi$$

Δ, Ω: Sets of actions (uninterpreted, atomic)

MoSL: Action specifiers and action sets

In MoSL

$$\Phi \;{}_{\Delta}\mathcal{U}_{\Omega}\; \Psi$$

Δ, Ω: Sets of *action specifiers*, to be matched against actions

MoSL: Path formulae

$$\Phi \; {}_{\Delta}\mathcal{U}^{<t}_{\Omega} \; \Psi$$

- Satisfied by those paths where eventually a Ψ-state is reached, by time t, via a Φ-path, *and*, in addition, while evolving between Φ states, actions are performed satisfying Δ and the Ψ-state is entered via an action satisfying Ω.

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- Time t can be omitted (assumed as ∞).

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MoSL: State formulae

$$\Phi ::= \mathrm{tt} | \aleph \mid \neg\, \Phi \mid \Phi \ \vee \ \Phi \mid \mathcal{P}_{\bowtie p}(\varphi)$$

with $\bowtie \in \{<, >, \leq, \geq\}$ and $p \in [0, 1]$

CSL path-operator: $\mathcal{P}_{\bowtie p}(\varphi)$
Satisfied by a state s iff the total probability mass for all paths starting in s that satisfy φ meets the bound $\bowtie p$;

MoSL: State formulae

$$\Phi ::= \mathrm{tt} | \aleph \mid \neg\, \Phi \mid \Phi\ \vee\ \Phi \mid \mathcal{P}_{\bowtie p}(\varphi) \mid \mathcal{S}_{\bowtie p}(\Phi)$$

with $\bowtie \in \{<, >, \leq, \geq\}$ and $p \in [0, 1]$

CSL path-operator: $\mathcal{P}_{\bowtie p}(\varphi)$
Satisfied by a state *s* iff the total probability mass for all paths starting in *s* that satisfy φ meets the bound $\bowtie p$;

CSL Steady-state operator: $\mathcal{S}_{\bowtie p}(\Phi)$
Satisfied by a state *s* iff the probability of reaching from *s*, in the long run, a state which satisfies Φ is $\bowtie\ p$.

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Model Checking MoSL

- We have defined an algorithm that given a *finite* RTS $(S, A, \longrightarrow)$ and a MoSL formula Φ, yields the states in S satisfying Φ;
- Model-checking of RTSs is performed by relying on a CSL model checker.
  - the RTS to be model-checked is translated into an *equivalent* state-labelled CTMC that can be analysed by making use of existing (state-based) CSL model checkers.

Model Checking MoSL

Definition

For finite and fully stochastic RTS $\mathcal{R} = (S, A, \longrightarrow)$ let $\mathcal{K}(\mathcal{R})$ be the CTMC $(W, \mathbf{R})$ such that:

- $W = S \times A \cup \{\perp\}$;
- Let $s_1 \xrightarrow{\alpha} \mathscr{P}$: $\mathbf{R}((s_1, \beta), (s_2, \alpha)) \overset{def}{=} \mathscr{P}(\alpha)$

Model Checking MoSL

Definition

For CTMC $\mathcal{M} = (S, \mathbf{R})$, $S_1, S_2 \subseteq S$, $t \in \mathrm{R}_{\geq 0}$, $p \in [0, 1]$, and $\bowtie \in \{<, >, \leq, \geq\}$:

- $\mathit{until}(\bowtie, p, t, S_1, S_2, \mathcal{M}) \overset{def}{=}$

$$\begin{aligned}\{s \in S| \quad & \mathbb{P}\{\pi \in \mathit{Paths}(s) | \exists t' < t.\pi(t') \in S_2 \\ & \text{and } \forall t'' < t'.\pi(t'') \in S_1\} \bowtie p\}\end{aligned}$$

- $\mathit{steady}(\bowtie, p, S_1, \mathcal{M}) \overset{def}{=}$

$$\{s \in S | \mathit{lim}_{t \to \infty} \mathbb{P}\{\pi \in \mathit{Paths}(s) | \pi(t) \in S_1\} \bowtie p\}$$

Model Checking MoSL

Let $\mathcal{R} = (S, A, \longrightarrow)$:

- $Sat^{\oplus,\ominus}(\mathrm{tt}, \mathcal{R}) \overset{def}{=} S$
- $Sat^{\oplus,\ominus}(\neg\,\Phi, \mathcal{R}) \overset{def}{=} S \setminus Sat^{\oplus,\ominus}(\Phi, \mathcal{R})$
- $Sat^{\oplus,\ominus}(\Phi_1 \vee \Phi_2, \mathcal{R}) \overset{def}{=} Sat^{\oplus,\ominus}(\Phi_1, \mathcal{R}) \cup Sat^{\oplus,\ominus}(\Phi_2, \mathcal{R})$
- $Sat^{\oplus,\ominus}(\rho \leftarrow \Phi, \mathcal{R}) \overset{def}{=} \{s | s \oplus \rho \in Sat^{\oplus,\ominus}(\Phi, \mathcal{R})\}$
- $Sat^{\oplus,\ominus}(\rho \rightarrow \Phi, \mathcal{R}) \overset{def}{=} \{s | s \ominus \rho \in Sat^{\oplus,\ominus}(\Phi, \mathcal{R})\}$

Model Checking MoSL

Let $\mathcal{R} = (S, A, \longrightarrow)$:

- $Sat^{\oplus,\ominus}(\mathcal{P}_{\bowtie p}(\Phi \;{}_{\Delta}\mathcal{U}_{\Omega}^{<t}\; \Psi), \mathcal{R}) \overset{def}{=}$

  $$\begin{array}{l}\text{let } X = \{\alpha \mid \alpha \in A : \alpha \models \Delta\} \text{ in}\\ \text{let } Y = \{\beta \mid \beta \in A : \beta \models \Omega\} \text{ in}\\ \text{let } S_1 = Sat^{\oplus,\ominus}(\Phi, \mathcal{R}) \times (X \cup \{\bot\}) \text{ in}\\ \text{let } S_2 = Sat^{\oplus,\ominus}(\Psi, \mathcal{R}) \times Y \text{ in}\end{array}$$

  $$\{s \mid (s, \bot) \in until(\bowtie, p, t, S_1, S_2, \mathcal{K}(\mathcal{R}))$$

- $Sat^{\oplus,\ominus}(\mathcal{S}_{\bowtie p}(\Phi), \mathcal{R}) \overset{def}{=}$

  $$\{s \in S \mid (s, \bot) \in steady(\bowtie, p, Sat^{\oplus,\ominus}(\Phi, \mathcal{R}) \times (A \cup \{\bot\}), \mathcal{K}(\mathcal{R}))\}$$

Outline. . .

1 Motivations

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Conclusions. . .

Conclusions:

- We have introduced Rate Transition Systems and have used them as the basic model for defining stochastic behaviour of processes.
- We have then shown how RTS can be used to provide the stochastic operational semantics of PEPA and CCS.
- We have also introduced a natural notion of bisimulation over RTS that is finer than Markovian bisimulation and useful for reasoning about stochastic behaviours.
- We have introduced a stochastic logic, MoSL, that permits specifying quantitative properties of RTS.
- We have presented a model checking algorithm for MoSL.

Future Work. . .

Ongoing work

- Consider alternative semantics synchonization rates:
  - based on *phase type* distributions
  - based on *Interactive Markov Chains*
- Develop an an *on-the-fly* model checker for MoSL.

Thank you for your attention!