4TH CONFERENCE ON RECENT TRENDS IN NONLINEAR PHENOMENA

organized by

Reza Aftabizadeh, Giovanni Molica Bisci, Giovanni Anello, Filippo D. Cammaroto, Luigi D'Onofrio, Franco Oliveri, Raffaella Servadei, Rosa Di Salvo and Matteo Gorgone

at the

Accademia Peloritana dei Pericolanti
Palazzo Università
Piazza Pugliatti, 1

on September 18-20, 2017

Aims and scope

In the literature a wide interest has been always shown in nonlinear phenomena, both for pure mathematical interest and for the various applications in many fields such as, for instance, physics, chemistry, engineering. The aim of this conference is to present and discuss recent results about nonlinear PDEs and their applications in Mathematical Physics and to bring together leading academic scientists in these fields.

Talks:

- Vincenzo Ambrosio (Università degli Studi di Urbino 'Carlo Bo', Italy)
- Vieri Benci (Università di Pisa, Italy)
- Isabeau Birindelli (Università 'La Sapienza' di Roma, Italy)
- Lucio Boccardo (Università 'La Sapienza' di Roma, Italy)
- Daniel Campbell (University of Hradec Kralove, Czech Republic)
- Italo Capuzzo Dolcetta (Università 'La Sapienza' di Roma, Italy)
- Michel Chipot (Universitat Zurich, Switzerland)
- Giuseppe Di Fazio (Università degli Studi di Catania, Italy)
- Giorgio Fusco (Università degli Studi de L'Aquila, Italy)
- Nicola Fusco (Università degli Studi di Napoli 'Federico II', Italy)
- Teresa Isernia (Università Politecnica delle Marche, Italy)
- Andrea Malchiodi (Scuola Normale Superiore, Italy)
- Paolo Marcellini (Università degli Studi di Firenze, Italy)
- Jean Mawhin (Université Catholique de Louvain, Belgium)
- Filomena Pacella (Università 'La Sapienza' di Roma, Italy)
- Giampiero Palatucci (Università degli Studi di Parma, Italy)
- Patrizia Pucci (Università degli Studi di Perugia, Italy)
- Vicentiu Radulescu (University of Craiova, Romania)
- Dusan Repovs (University of Ljubljana, Slovenia)
- Biagio Ricceri (Università degli Studi di Catania, Italy)
- Sandro Salsa (Politecnico di Milano, Italy)
- Carlo Sbordone (Università degli Studi di Napoli 'Federico II', Italy)
- Simone Secchi (Università di Milano 'Bicocca', Italy)
- Gabriella Tarantello (Università degli Studi di Roma 'Tor Vergata', Italy)

Talks

Some recent results for fractional periodic problems

Vincenzo Ambrosio
Università degli Studi di Urbino 'Carlo Bo', Italy

In this talk we present some existence and multiplicity results for a class of fractional periodic problems involving nonlinearities with subcritical, critical and supercritical growth. Such equations are characterized by the nonlocal pseudo-differential operator $(-\Delta + m^s)^s$, with $s \in (0,1)$ and $m \geq 0$, defined through the spectral decomposition of the elliptic operator $-\Delta + m^2$ subject to periodic boundary conditions. We investigate these problems using critical point theory after transforming them into degenerate elliptic equations in the half-cylinder $(0,T)^N \times (0,\infty)$, with periodic conditions on the lateral boundary $\partial(0,T)^N \times [0,\infty)$, and a nonlinear Neumann boundary condition on the basis $(0,T)^N \times \{0\}$, via a variant of the extension method adapted in a periodic setting.

Hylomorphic solitons

Vieri Benci
Università di Pisa, Italy

Roughly speaking a solitary wave is a solution of a field equation whose energy travels as a localized packet and which preserves this localization in time. A soliton is a solitary wave which exhibits some strong form of stability so that it has a particle-like behavior. The solitons whose existence can be established via the ratio energy/charge will be called hylomorphic solitons. In the first part of the talk, I will present these general ideas. In the second part, I will show a very general abstract theorem which allow to prove the existence of hylomorphic soliton in many different situations. Finally I will show the application of this theorem to the Nonlinear Schroedinger equation, to the Nonlinear Klein-Gordon eq., to the generalized KdV eq., to the Benjamin-Ono eq. and to a model of suspended bridge.

Toward a Faber Krahn inequality

Isabeau Birindelli
Università 'La Sapienza' di Roma, Italy

We shall see how symmetry plays a role on the estimates of the principal eigenvalue. We shall briefly recall some results for the Pucci operators and then show some new surprising results for the Harvey Lawson truncated laplacian.

Dirichlet problems with singular convection terms

Lucio Boccardo
Università 'La Sapienza' di Roma, Italy

1. Recent results

In a paper ([1]), dedicated to the memory of Guido Stampacchia in the thirtieth anniversary of his death, I improved some of his results (see [6]) concerning the linear Dirichlet problem

$$(1) \qquad \begin{cases} -\operatorname{div}(M(x)\nabla u) = -\operatorname{div}(u\,E(x)) + f(x) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

Here Ω is a bounded, open subset of $\mathbb{R}^N$, $N > 2$,

$$E \in (L^N(\Omega))^N, \tag{2}$$

$$f \in L^m(\Omega), \quad 1 \le m < \frac{N}{2}, \tag{3}$$

and $M(x)$ is a bounded and measurable matrix such that

$$\alpha|\xi|^2 \le M(x)\xi\xi, \quad |M(x)| \le \beta, \quad \text{a.e. } x \in \Omega, \quad \forall\, \xi \in \mathbb{R}^N. \tag{4}$$

To be more precise, in [1] is proved the existence of u

$$\begin{cases} \text{weak solution belonging to } W_0^{1,2}(\Omega) \cap L^{m^{**}}(\Omega), \text{ if } m \ge \frac{2N}{N+2}; \\ \text{distributional solution belonging to } W_0^{1,2}(\Omega)m, \text{ if } 1 < m < \frac{2N}{N+2}; \end{cases} \tag{5}$$

the result u bounded needs a slightly stronger assumption. Note that the above existence results are exactly the results proved with $E = 0$ in [6] and [5].

The main difficulty is due to the noncoercivity in $W_0^{1,2}(\Omega)$ of the differential operator $-\operatorname{div}(M(x)\nabla v) + \operatorname{div}(v\,E(x))$.

An important case where the main assumption of [1] (that is (2) here) is not satisfied, is the case of the assumption

$$|E(x)| \le \frac{A}{|x|}, \; A > 0, \quad 0 \in \Omega, \tag{6}$$

which is slightly weaker than (2). We will see the importance of the size of A: the existence results below strongly depend on A.

Observe that the function $u_A = r^{-A} - r^2$, $D \in \mathbb{R}$, is solution of the boundary value problem

$$\begin{cases} -\Delta u = A \operatorname{div}\left(u \dfrac{x}{|x|^2}\right) + (2 + D)N & \text{in } \{x : |x| < 1\}, \\ u = 0 & \text{on } \{x : |x| = 1\}. \end{cases}$$

If $A > 0$, u_A is an unbounded solution of a Dirichlet problem with bounded datum the real number $(2 + A)N$; u_A is a weak solution if $A < 1 + N/2$, since it belongs to $W_0^{1,2}(\Omega)$, and u_D is a distributional solution if $1 + N/2 \le A < N - 2$.

Then in [2] are proved the two following theorems.

Theorem 1. *Assume (4), (3) with* $\frac{2N}{N+2} < m < \frac{N}{2}$, *(6) with*

$$|A| < \frac{\alpha(N - 2m)}{m}. \tag{7}$$

Then there exists a weak solution $u \in W_0^{1,2}(\Omega) \cap L^{m^{**}}(\Omega)$ *of the Dirichlet problem (1).*

Theorem 2. *Assume (4), (3) with* $1 < m < \frac{2N}{N+2}$, *(6) with* $|A| < \frac{\alpha(N-2m)}{m}$. *Then there exists a distributional solution* $u \in W_0^{1,m*}(\Omega)$ *of the Dirichlet problem (1).*

Then in [2] the case $E \in (L^r(\Omega))^N$, $r < N$, or in general

$$E \in (L^2(\Omega))^N$$

is studied. As claimed by the above example and theorems, this assumption "pushes" the solutions outside of the standard Sobolev spaces and distributional solutions framework, so that a more general definition of solution is useful in order to show the existence.

In all the cases, an important (recall that our problem is noncoercive) starting point is the a priori (formal) estimate

$$\int_\Omega |\nabla \log(1 + |u|)|^2 \le \frac{1}{\alpha^2}\int_\Omega |E|^2 + \frac{2}{\alpha}\int_\Omega |f|. \tag{8}$$

The presence of a zero order term helps a little bit; in

$$\begin{cases} -\operatorname{div}(M(x)\nabla u) + \alpha_0 \, u = -\operatorname{div}(u\, E(x)) + f(x) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega, \end{cases}$$

with $\alpha_0 > 0$, it is possible to prove the a priori estimate (poor, but stronger than the previous log-inequality)

$$\alpha_0 \|u\|_{L^1(\Omega)} \le \|f\|_{L^1(\Omega)},$$

where the proof needs $E \in (L^2(\Omega))^N$, but it does not depend on $\|E\|_{L^2}$.

In all the above existence results, we use a **nonlinear approach** to the linear non-coercive boundary value problem (1).

2. Work in progress

We are working on the boundary value problem

$$\begin{cases} -\operatorname{div}(M(x)\nabla u) = E(x)\cdot\nabla u + f(x) & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

Even if this problem is the dual of (1), our **nonlinear approach** and our nonlinear estimates (see (8)) do not allow a direct study with the of the duality theory.

References

[1] L. Boccardo: Some developments on Dirichlet problems with discontinuous coefficients; Boll. Unione Mat. Ital. (9) 2 (2009), 285–297.

[2] L. Boccardo: Dirichlet problems with singular convection terms and applications; J. Differential Equations 258 (2015), 2290–2314.

[3] L. Boccardo: Stampacchia-Calderon-Zygmund theory for linear elliptic equations with discontinuous coefficients and singular drift; preprint.

[4] L. Boccardo: prprint.

[5] L. Boccardo, T. Gallouët, *Nonlinear elliptic equations with right-hand side measures*, Comm. Partial Differential Equations **17** (1992), 641–655.

[6] G. Stampacchia: Le problème de Dirichlet pour les équations elliptiques du second ordre à coefficients discontinus; Ann. Inst. Fourier (Grenoble) 15 (1965), 189–258.

Approximation of Sobolev monotone maps

Daniel Campbell

University of Hradec Kralove, Czech Republic

In the study of variational models for non-linear elasticity in the context of proving regularity we are lead to the challenging so-called Ball-Evan's problem of approximating a Sobolev homeomorphism with diffeomorphisms in its Sobolev space. In some cases however we are not able to guarantee that the limit of a minimizing sequence is a homeomorphism and so the closure of Sobolev homeomorphisms comes into the game. For $p \ge 2$ they are exactly Sobolev monotone maps and for $1 \le p < 2$ contain monotone maps. In our paper we prove that monotone maps can be approximated by diffeomorphisms in their Sobolev (or Orlicz-Sobolev) space including the case $p = 1$ not proven by Iwaniec and Onninen.

A few recent results on the weak maximum principle

Italo Capuzzo Dolcetta
Università 'La Sapienza' di Roma, Italy

I will discuss a few topics concerning the weak maximum principle for second order operators with nonnegative characteristic form and, more generally, for fully nonlinear degenerate elliptic ones. The first part of the talk will be focused on a characterization of the validity the weak maximum principle in the spirit of previous results of Donsker-Varadhan and Berestycki-Nirenberg-Varadhan. In the second part, I will present some recent research, steaming from a series of papers by Caffarelli-Li-Nirenberg, about the validity of the weak maximum principle for one-directional elliptic operators on some specific classes of unbounded domains, possibly of infinite measure.

Minimal solutions to some variational inequalities

Michel Chipot
Universitat Zurich, Switzerland

Applying an asymptotic method, we will establish the existence of the minimal solution to some variational elliptic inequalities defined on bounded or unbounded domains. The minimal solution is obtained as limit of solutions to some classical variational inequalities defined on domains becoming unbounded when some parameter tends to infinity (joint work with S. Guesmia and S. Harkat).

Fefferman–Poincaré inequality and regularity for quasilinear subelliptic equations

Giuseppe Di Fazio
Università degli Studi di Catania, Italy

We study conditions on the lower order terms that ensure a Harnack inequality for non negative weak solutions of the subelliptic equation

$$\sum_{j=1}^{m} X_j^* A_j(x, u(x), Xu(x)) + B(x, u(x), Xu(x)) = 0\,,$$

where $X_1, \ldots, X_m$ are a system of non commuting vector fields.

Assuming suitable conditions on the functions A and B we obtain an invariant Harnack inequality and regularity results for weak solutions. The conditions we find are also necessary, at least in some cases. The results follow from an embedding involving a suitable generalization of the Stummel – Kato class to the setting of degenerate vector fields.

Periodic orbits near heteroclinics

Giorgio Fusco
Università degli Studi de L'Aquila, Italy

Let $W : \mathbb{R}^m \to \mathbb{R}$, $m \geq 1$ be a nonnegative potential with exactly two distinct zeros $a_\pm \in \mathbb{R}^m$. In the scalar case $m = 1$ phase plane analysis shows that the Newton equation

$$u'' = W_u(u) \tag{9}$$

possesses a heteroclinic solution u^∞ that connects a_- to a_+ and a family of periodic solutions u^T that converge in compacts to u^∞ as $T \to +\infty$.

We prove that, under the assumption that W is invariant under the reflection γ that exchange a_- to a_+, the same is true in the vector case $m > 1$.

We also extend this result to an infinite dimensional setting. We assume that W is invariant under a reflection σ that fixes $a_\pm$, $\sigma a_\pm = a_\pm$, and that there exist exactly two distinct heteroclinic solutions of (9) $\bar{u}_-$ and $\bar{u}_+$ that satisfy

$$\bar{u}_- = \sigma \bar{u}_+.$$

Under a non degeneracy assumption on $\bar{u}_\pm$ we show that, for $L > L_0$, the PDE system

$$\Delta u = W_u(u),$$

has a solution $u^L : \mathbb{R}^2 \to \mathbb{R}^m$ which is L-periodic in x, $u^L(x+L, y) = u^L(x, y)$, and such that the restriction of u^L to $(-\frac{L}{4}, \frac{L}{4}) \times \mathbb{R}$, converges along a subsequence to a heteroclinic connection $u^\infty : \mathbb{R}^2 \to \mathbb{R}^m$ between $\bar{u}_-$ and $\bar{u}_+$:

$$\lim_{L \to +\infty} u^L(x, \cdot) = u^\infty(x, \cdot),$$
$$\lim_{x \to \pm\infty} u^\infty(x, \cdot) = \bar{u}_\pm.$$

A stability result for the first eigenvalue of the p-Laplacian

Nicola Fusco
Università degli Studi di Napoli Federico II, Italy

The Faber-Krahn inequality states that balls are the unique minimizers of the first eigenvalue of the p-Laplacian among all sets with fixed volume. We shall present a sharp quantitative form of this inequality. This extends to the case p¿1 a recent result proved by Brasco, De Philippis and Velichkov for the Laplacian.

Sign-changing solutions for fractional Schrödinger equations with vanishing potentials

Teresa Isernia
Università Politecnica delle Marche, Italy

We consider the following class of fractional Schrödinger equations

$$(-\Delta)^s u + V(x)u = K(x)f(u) \text{ in } \mathbb{R}^N$$

where $s \in (0, 1)$, $N > 2s$, $(-\Delta)^s$ is the fractional Laplacian, the potentials V, K are positive and continuous functions, allowed for vanishing behavior at infinity, and f is a subcritical continuous function.
By using a minimization argument and a quantitative deformation lemma, we obtain the existence of a sign-changing solution.
Moreover, when f is odd, the problem admits infinitely many nontrivial solutions (not necessarily sign-changing).
The results in this talk are based on joint works with V. Ambrosio, G.M. Figueiredo and G. Molica Bisci.

Entire solutions of the Cahn-Hilliard equation

Andrea Malchiodi
Scuola Normale Superiore, Italy

It is well known that the Allen-Cahn equation arises as Euler-Lagrange equation for energies that Gamma-converge to the perimeter functional. The Cahn-Hilliard equation is a fourth-order counterpart, modeling phase separation in binary fluids. Somehow similar Gamma-converge results hold, with limiting energy being the Willmore's. We will discuss existence and qualitative properties of entire solutions in Euclidean space, related to this limiting characterization. This is joint work with R.Mandel and M.Rizzi.

Regularity and existence for elliptic and parabolic equations and systems under general and p, q growth conditions

Paolo Marcellini
Università degli Studi di Firenze, Italy

We give some recent *existence* and *interior regularity results* - partly obtained in collaboration with *Giovanni Cupini, Michela Eleuteri* and *Elvira Mascolo* - for elliptic partial differential *equations* in divergence form, or elliptic *systems* of m partial differential equations in divergence form of the type

$$\sum_{i=1}^{n} \frac{\partial}{\partial x_i} a_\alpha^i \left(x, u\left(x\right), Du\left(x\right)\right) = b_\alpha \left(x, u\left(x\right), Du\left(x\right)\right), \quad \alpha = 1, 2, \ldots, m\,,$$

for maps $u : \Omega \subset \mathbb{R}^n \to \mathbb{R}^m$. Here the vector field $\left(a_\alpha^i \left(x, s, \xi\right)\right)$ assumes values in the set of $m \times n$ matrices and it satisfies some *general growth conditions* with respect to the gradient variable $\xi \in \mathbb{R}^{m\times n}$, sometine *$p, q$ growth conditions*.

As a part of a joint research-project started in 2013 with *Verena Bögelein and Frank Duzaar*, we consider the evolution problem associated with a convex integrand $f\colon \mathbb{R}^{m\times n} \to [0, \infty)$ satisfying - for instance - some p, q-growth assumption. To establish the existence of solutions we introduce the concept of *variational solutions*. In contrast to weak solutions, i.e. mappings $u\colon \Omega_T \subset \mathbb{R}^{n+1} \to \mathbb{R}^m$ which solve

$$\partial_t u - \mathrm{div} Df(Du) = 0$$

weakly in Ω_T, variational solutions in general exist under a much weaker assumption on the gap $q - p$.

In particular, if $2 \leq p \leq q < p + \min\left\{1, \frac{4}{n}\right\}$, we obtain the existence of variational solutions and we also show that they are actually - in this case - weak solutions. This means that any solution u automatically admits the necessary higher integrability of the spatial derivative Du to satisfy the parabolic system in the weak sense, i.e. we prove that

$$u \in L^q_{\mathrm{loc}}\left(0, T; W^{1,q}_{\mathrm{loc}}(\Omega, \mathbb{R}^m)\right).$$

The sharpness of existence conditions for ordinary differential systems with nonlocal boundary conditions

Jean Mawhin
Université Catholique de Louvain, Belgium

An existence result from the years 1960 (Krasnosel'skii) tells that if $f : [0, 1] \times \mathbb{R}^n \to \mathbb{R}^n$ is continuous and if there exists $R > 0$ such that

$$\text{either } \langle x, f(t, x)\rangle \geq 0 \quad \text{or} \quad \langle x, f(t, x)\rangle \leq 0, \;\; \forall\, (t, x) \in [0, 1] \times \partial B(R), \tag{10}$$

where $B(R) \subset \mathbb{R}^n$ denotes the open ball of center 0 and radius R, then the periodic problem

$$x' = f(t, x),\; x(0) = x(1) \tag{11}$$

has at least one solution such that $|x(t)| \leq R$ for all $t \in [0, 1]$. The two results are equivalent, the second one being deduced from the first one through the change of variable $\tau = 1 - t$. Condition (10) has been generalized in several directions for problem (11).

Recently, with K. Szymańska-Dębowska, some extensions of those results have been obtained for nonlocal boundary conditions of the type

$$x(0) = \int_0^1 dg(s)x(s) \quad \text{or} \quad x(1) = \int_0^1 dg(s)x(s) \tag{12}$$

where g is a function with bounded variation from $[0,1]$ into diagonal $(n \times n)$-matrices, satisfying some conditions, and containing the periodic boundary conditions as special cases.

The situations (11) and (12) are compared with the use of counterexamples, showing the singularity of the periodic case.

This is a joint work with K. Szymańska-Dębowska.

Constant mean curvature surfaces and partially overdetermined problems in cones

Filomena Pacella
Università 'La Sapienza' di Roma, Italy

We will present some recent results in collaboration with G.Tralli about:

i) characterization of constant mean curvature surfaces with boundary in cones
ii) domains in cones which admit a solution of a partial overdetermined problem of Serrin type.

It will be shown that, as in the classical case of surfaces without boundary, the above questions are strictly related. Finally, connections with a relative isoperimetric inequality in cones proved in [Lions-Pacella, 1990] will be emphasized.

The Perron Method for nonlinear integro-differential equations

Giampiero Palatucci
Università degli Studi di Parma, Italy

In this talk we consider nonlinear integro-differential operators of differentiability order $s \in (0,1)$ and summability growth $p > 1$, whose model is the fractional p-Laplacian $(-\Delta)_p^s$. We present several results for the corresponding *weak supersolutions*, as comparison principles, a priori bounds, lower semicontinuity, and many others. We then discuss the good definition of (s,p)-*superharmonic functions*, and we introduce the fractional counterpart of the celebrated *Perron method* in nonlinear Potential Theory.

Entire solutions of nonlocal elasticity models for composite materials

Patrizia Pucci
Università degli Studi di Perugia, Italy

Many structural materials, which are preferred for the developing of advanced constructions, are inhomogeneous ones. Composite materials have complex internal structure and properties, which make them to be more effectual in the solution of special problems required for civil and environmental engineering. As a consequence of this internal heterogeneity, they exhibit complex mechanical properties. In this work, the analysis of some features of the behavior of composite materials under different loading conditions is carried out. The dependence of nonlinear elastic response of composite materials on loading conditions is studied. Several approaches to model elastic nonlinearity such as different stiffness for particular type of loadings and nonlinear shear stress–strain relations are considered. Instead of a set of constant anisotropy coefficients, the anisotropy functions are introduced. Eventually, the combined constitutive relations are proposed to describe simultaneously two types of physical nonlinearities. The first characterizes the nonlinearity of shear stress–strain dependency and the latter determines the stress state susceptibility of material properties. Quite satisfactory correlation between the theoretical dependencies and the results of experimental studies is demonstrated, as described in [1, 2] as well as in the references therein.

In [3] we continue the study performed in [1], expanding the investigation to more general equations, involving the fractional p–Laplacian operator.

This research has been developed within a scientific project *Nonlocal Elasticity Models for Composite Materials* with Professors F. Cluni and V. Gusella of the *Dipartimento di Ingegneria Civile ed Ambientale* of the *Università degli Studi di Perugia*.

References

[1] G. Autuori, F. Cluni, V. Gusella and P. Pucci, *Mathematical models for nonlocal elastic composite materials*, Adv. Nonlinear Anal., pages 39, https://doi.org/10.1515/anona-2016-0186.

[2] G. Autuori, F. Cluni, V. Gusella and P. Pucci, *Effects of the fractional Laplacian order on the nonlocal elastic rod response*, ASCE-ASME Journal of Risk and Uncertainty in Engineering Systems, Part B: Mechanical Engineering, 131 (2017), 030902-1 - 030902-5.

[3] G. Autuori and P. Pucci, *Entire solutions of nonlocal elasticity models for composite materials*, to appear in Discrete Contin. Dyn. Syst. Ser. S, Special Issue in "Recent progresses in the theory of nonlinear nonlocal problems" Ed. M. Squassina, pages 21.

Nonhomogeneous problems with singular weights

Vicenţiu D. Rădulescu
University of Craiova, Romania

Recent systematic study of partial differential equations with variable exponents was motivated by the description of several relevant models in electro-rheological and thermo-rheological fluids, image processing, or robotics. These nonlinear processes are usually modeled by PDEs with one or more several variable exponents. The presence of a nonhomogeneous framework creates several striking phenomena, such as staircase effect, Lavrentiev phenomenon, Winslow effect, etc.

In this talk, we are concerned with the qualitative analysis and stabilization properties for some classes of problems with variable exponent driven either by the nonhomogenous biharmonic operator or by Leray-Lions operators with variable exponent. The proofs combine variational and topological arguments, as introduced and developed in [3, 4, 5].

This talk reports on some recent results established in [1] and [2].

References

[1] K. Kefi, V. Rădulescu, On a $p(x)$-biharmonic problem with singular weights, *ZAMP* **68** (2017), 68:80.

[2] J. Giacomoni, V. Rădulescu, G. Warnault, Quasilinear parabolic problem with variable exponent: qualitative analysis and stabilization, *Commun. Contemp. Math.*, in press.

[3] P. Pucci, Q. Zhang, Existence of entire solutions for a class of variable exponent elliptic equations, *J. Differential Equations* **257** (2014), 1529-1566.

[4] V. Rădulescu, D. Repovš, *Partial Differential Equations with Variable Exponents: Variational Methods and Qualitative Analysis*, CRC Press, Taylor & Francis, Boca Raton, 2015.

[5] V. Rădulescu, Nonlinear elliptic equations with variable exponent: old and new, *Nonlinear Anal.* **121** (2015), 336-369.

Nonlinear problems on the Sierpiński gasket

Dusan Repovs
University of Ljubljana, Slovenia

We report on some recent results established in [1] in collaboration with G. Molica Bisci and R. Servadei. The Sierpiński gasket in the plane is a connected set obtained from an equilateral triangle by removing the open middle inscribed equilateral triangle of 1/4 the area, removing the corresponding open triangle from each of the three constituent triangles, and continuing this way. The mathematical analysis of various phenomena on self-similar domains had a strong development after the pioneering contributions of Mandelbrot [2], Falconer [3] and Strichartz [4].

We are concerned with a class of elliptic equations on fractal domains depending on a real parameter. By studying various competition effects, we establish sufficient conditions for the existence of at least two strong solutions. Our approach combines the Pucci-Serrin theorem [5], Ricceri's variational principle [6] and the mountain pass theorem in the fractal setting [7].

References

[1] G. Molica Bisci, D. Repovš, R. Servadei, Nonlinear problems on the Sierpiński gasket, *J. Math. Anal. Appl.* **452** (2017), 883-895.

[2] B. Mandelbrot, How Long Is the Coast of Britain? Statistical Self-Similarity and Fractional Dimension, *Science* **156** (1967), 636-638.

[3] K. Falconer, *The geometry of Fractal Sets*, Cambridge Tracts in Mathematics, Cambridge University Press, Cambridge, 1986.

[4] R. Strichartz, Analysis on fractals, *Notices Amer. Math. Soc.* **46** (1999), 1199-1208.

[5] P. Pucci, J. Serrin, A mountain pass theorem, *J. Differential Equations* **60** (1985), 142-149.

[6] B. Ricceri, A general variational principle and some of its applications, *J. Comput. Appl. Math.* **113** (2000), 401-410.

[7] K. Falconer, J. Hu, Nonlinear elliptic equations on the Sierpiński gasket, *J. Math. Anal. Appl.* **240** (1999), 552-573.

Four conjectures in Nonlinear Analysis

Biagio Ricceri
Università degli Studi di Catania, Italy

In this lecture, I will propose and highlight four conjectures: one on the Monge-Ampère equation; one on an eigenvalue problem; one on a non-local problem; one on disconnectedness versus infinitely many solutions.

Regularity of higher order in two-phase free boundary problems

Sandro Salsa
Politecnico di Milano, Italy

We present new results on the regularity for free boundary problems governed by uniformly elliptic equations with distributed sources. Joint work with Daniela de Silva and Fausto Ferrari.

Some remarks on BMO–type spaces of Bourgain, Brezis and Mironescu

Carlo Sbordone
Università degli Studi di Napoli 'Federico II', Italy

A new BMO–type space $B \subset L^1(Q)$ on the unit cube $Q \subset \mathbb{R}^n$ has been recently introduced by Bourgain, Brezis and Mironescu, by mean of the seminorm

$$\|f\|_B = \sup_{0<\varepsilon<1} [f]_\varepsilon$$

where $[f]_\varepsilon$ is defined with a suitable maximization procedure. The space B contains BMO and the space BV of functions of bounded variation.

Its subspace B_0

$$B_0 = \left\{ f \in B : \limsup_{\varepsilon\to 0} [f]_\varepsilon = 0 \right\}$$

which contains VMO and $W^{1,1}$, has the interesting property

$$\chi_A \in B_0,\ A \subset Q \qquad \Longrightarrow \qquad |A| \in \{0,1\}\,,$$

where χ_A is the characteristic function of A and $|A|$ is the Lebesgue measure of A.

Later, Ambrosio, Bourgain, Brezis and Figalli found connections between B and the notion of perimeter of sets. Further results characterizing total variation of BV functions and norm of Sobolev functions, independent of theory of distributions, were given by Fusco, Moscariello and myself.

We describe recent work in progress with L.D'Onofrio, L. Greco and R. Schiattarella to explore more properties of these spaces.

Standing waves for the NLS on the double-bridge graph and a rational-irrational dichotomy

Simone Secchi
Università degli Studi di Milano 'Bicocca', Italy

We study standing waves of NLS equation posed on the double-bridge graph: two semi-infinite half-lines attached at a circle. At the two vertices Kirchhoff boundary conditions are imposed. The configuration of the graph is characterized by two lengths, L_1 and L_2. We study the solutions with possibly nontrivial components on the half-lines and a cnoidal component on the circle.

Concentration-compactness aspects of Liouville differential problems

Gabriella Tarantello
Università degli Studi di Roma 'Tor Vergata', Italy

We discuss various aspects of the concentration-compactness principle for Liouville type systems over compact surfaces and their novelty with respect to the case (well understood) of a single equations.