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Phenomena**

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A regularizing effect in elliptic problems

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Abstract.

We show [1] that the interplay between coefficients has a regularizing effect on the solution of some nonlinear Dirichlet problems. The simplest example is the linear problem

$$\begin{cases} \operatorname{div}(M(x)\nabla u) + a(x)u = f, & x \in \Omega \\ u = 0, & x \in \partial\Omega \end{cases}$$

where Ω is a bounded open set of \mathbf{R}^N , M is a bounded elliptic matrix and $0 \leq a(x) \in L^1(\Omega)$. Even if $f(x)$ only belongs to $L^1(\Omega)$, the assumption

there exists $Q > 0$ such that $|f(x)| \leq Q a(x)$

implies the existence of a weak solution u belonging to $W_0^{1,2}(\Omega)$ and to $L^\infty(\Omega)$.

References

- [1] D. Arcoya and L. Boccardo *Regularizing effect of the interplay between coefficients in some elliptic equations* - *Jornal of Functional Analysis* **268** (2015) 1153–1166.

Two-phase functionals: regularity, functional properties and borderline cases

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Abstract.

The aim of the talk is to present some recent regularity results for local minimizers of non-autonomous functionals; their energy density changes its ellipticity and growth properties according to the size of a “modulating coefficient”, switching between two different phases. Special emphasis will be put on the interplay between the regularity of the coefficient, the difference of ellipticity between the phases and the underlying functional properties. The talk is based on joint works with M. Colombo and G. Mingione.

Variational methods in the study of solitons

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Abstract.

Roughly speaking a soliton is a solution of a field equation whose energy travels as a localized packet and which preserves this localization in time. In the first part of this talk we present the main ideas which lie behind this notion. In the second part, we discuss an abstract theorem (obtained in collaboration with Fortunato) which allows to prove the existence of solitons in several situations. We conclude this talk showing and discussing some equations to which this theorem can be applied:

- Nonlinear Klein-Gordon equation;
- Nonlinear Schrödinger equation;
- The Yang–Mill equations;
- The Korteweg-deVries equation;
- The Nonlinear beam equation;
- The Benjamin-Ono equation;
- The Fractional Korteweg-deVries equation.

Approximation of planar Orlicz-Sobolev homeomorphisms by diffeomorphisms

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Abstract.

We will review some known results for approximating planar homeomorphisms by diffeomorphisms and introduce a new result on approximation of homeomorphisms in Orlicz-Sobolev spaces.

Non homogeneous boundary value problems for the stationary Navier-Stokes equations in 2-d domains with semi-infinite outlets

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Abstract.

We would like to present existence results for the stationary non homogeneous Navier-Stokes problem in domains having semi-infinite outlets. We assume for this Leray problem a very general outflow condition.

Change of variables for Orlicz-Sobolev and Lorentz-Sobolev spaces

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Abstract.

Let Ω and Ω' be bounded domains of \mathbf{R}^n , $n \geq 2$. We consider homeomorphisms $f: \Omega \rightarrow \Omega'$ in the Sobolev space $W^{1,p}(\Omega, \Omega')$ for $p > n - 1$ and we introduce the composition operator $C_{f^{-1}}: u \mapsto u \circ f^{-1}$ induced by the inverse of f . The differentiability properties of $C_{f^{-1}}(u)$ are investigated and we focus on the cases where both the gradients of u and f both belong to suitable Orlicz classes. Our conclusions will always read as $C_{f^{-1}}(u) \in \text{BV}(\Omega')$ or $C_{f^{-1}}(u) \in W^{1,1}(\Omega')$ depending upon the fact that f is a mapping of finite inner or outer distortion respectively. Analogous results are obtained in the setting of Lorentz spaces.

The elastica problem under area constraint

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Abstract.

We discuss the problem of minimizing the elastic energy $E(\gamma)$ of a closed curve γ among all plane simple regular closed curves of given enclosed area $A(\gamma)$, and we show that the minimum is attained for a circumference. Possible extensions of the result to more general functionals of the curvature will also be presented. This is a joint work with B. Kawohl and C. Nitsch.

Distributional Jacobian equal to H^1 measure

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Abstract.

Let $1 \leq p < 2$. We construct a Hölder continuous $W^{1,p}$ mapping of a square into \mathbf{R}^2 such that the distributional Jacobian equals to one dimensional Hausdorff measure on a line segment. This answers an open problem by S. Müller from 1993. This is a joint result with Zhuomin Liu and Jan Malý.

Asymptotic analysis and sign-changing bubble towers for Lane-Emden problems

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Abstract.

We analyze the asymptotic behavior as $p \rightarrow +\infty$ of families of solutions of the semilinear Lane-Emden problem

$$\begin{aligned} -\Delta u &= |u|^{p-1}u && \text{in } \Omega \\ u &= 0 && \text{on } \partial\Omega \end{aligned} \tag{1}$$

where $\Omega \subset \mathbf{R}^2$ is a smooth bounded domain. We show that any family $(u_p)_p$ of finite energy solutions concentrates at a finite number of points. In the case of positive solutions we can also give a complete description of this concentration phenomenon. For sign-changing solutions the analysis is more delicate, and we show that, under suitable symmetry assumptions, the positive and negative parts of families of symmetric solutions concentrate at the same point, and that the limit profile looks like a tower of two different bubbles given by the superposition of a regular and a singular solution of the Liouville problem in the plane.

The results presented are obtained in collaboration with F. De Marchis and F. Pacella (Roma Sapienza).

A variational approach to parabolic equations and systems via the calculus of variations

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Abstract.

We introduce the notion of “variational solutions”, also named “parabolic minimizers”, to a class of evolutionary problems possessing a variational structure. The advantage of these variational solutions stems from the fact that they might exist even in situations where the associated nonlinear parabolic system makes no sense. We should compare with the stationary case, where it is possible to establish the existence of minimizers by the direct methods of the calculus

of variations in fairly general situations, whereas additional stronger assumptions are needed to guarantee that the minimizers fulfill the Euler-Lagrange system. Under reasonable assumptions, such as coercivity and convexity of the energy function, the variational solution exists and is unique. Under further natural growth conditions, the variational solution is also the unique weak solution to the associated Cauchy-Dirichlet problem. This is a joint collaboration with V.Böglein and F.Duzaar.

Second order ordinary differential systems with nonlocal Neumann conditions at resonance

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Abstract.

Using topological degree arguments, several existence theorems are proved for the following system of nonlocal resonant boundary value problem

$$x'' = f(t, x, x'), \quad x'(0) = 0, \quad x'(1) = \int_0^1 x'(s) dg(s)$$

where $f : [0, 1] \times \mathbf{R}^k \times \mathbf{R}^k \rightarrow \mathbf{R}^k$ is continuous and bounded, $g = \text{diag}(g_1, \dots, g_k)$ $g_j : [0, 1] \rightarrow \mathbf{R}$ has bounded variation ($j = 1, \dots, k$).

The vector field f satisfies sign, Villari or Nirenberg-type conditions at infinity.

This is a joint work with K. Szymańska-Dębowska.

Nonlinear potential theory of elliptic systems

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Abstract.

Nonlinear potential theory deals with the extension to nonlinear elliptic and parabolic equations of the classical potential theory of harmonic functions and solutions to linear problems. One of the main achievements is a set of pointwise estimates for solutions and their gradients via linear and nonlinear potentials, eventually implying basic fine properties. The theory works in the case of single equations. In a couple of recent papers, Tuomo Kuusi (Helsinki) and myself have obtained a few basic results aimed at laying the foundations of a vectorial nonlinear potential theory. I will give a brief summary of these.

Continuity estimates for p -Laplace type equations

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Abstract.

We study the Dirichlet problem for a p -Laplacian type operator in the setting of the Orlicz-Zygmund space $L^q \log^{-\alpha} L$, $q > 1$ and $\alpha > 0$. More precisely, our aim is to establish under which assumptions on $\alpha > 0$ existence and uniqueness of the solution are assured and to prove continuity and stability of the associated nonlinear operator.

Fully nonlinear elliptic equations: symmetry and spectral properties

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Abstract.

We will present some new results about partial symmetry of solutions of some fully nonlinear elliptic equations in bounded domains with Dirichlet boundary conditions. The main ingredients are maximum principles and the properties of the principal eigenvalue of the maximal Pucci operator in particular subdomains. Some interesting consequences for the eigenvalues of these operators in symmetric domains will be discussed.

Higher integrability for minimizers of asymptotically convex functionals with discontinuous coefficients

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Abstract.

We present some regularity results of vectorial minimizers of integral functionals with standard p -growth, contained in [2, 1]. We assume that the non-homogeneous densities are uniformly convex and have a radial structure, with respect to the gradient variable, only at infinity. Under a $W^{1,n}$ -Sobolev dependence on the spatial variable of the integrand, n being the space dimension, we show that the minimizers u have gradient that belongs locally to L^q for every $q > p$. As a consequence, they are locally α -Hölder continuous for every $\alpha < 1$.

References

- [1] A. Passarelli di Napoli. *Higher differentiability of minimizers of variational integrals with Sobolev coefficients: the case $p = n = 2$* - Pot. Anal. (2014)
- [2] G. Cupini, F. Giannetti, R. Giova & A. Passarelli di Napoli. *Higher integrability estimates for minimizers of asymptotically convex integrals with discontinuous coefficients*. Preprint (2015)

Sharp solvability conditions for a fractional parabolic problem involving Hardy potential¹

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Abstract.

In this talk we study the influence of the Hardy potential in the fractional heat equation. In particular, we consider the problem

$$(P_\theta) \quad \begin{cases} u_t + (-\Delta)^s u = \lambda \frac{u}{|x|^{2s}} + \theta u^p + cf & \text{in } \Omega \times (0, T), \\ u(x, t) > 0 & \text{in } \Omega \times (0, T), \\ u(x, t) = 0 & \text{in } (\mathbf{R}^N \setminus \Omega) \times [0, T), \\ u(x, 0) = u_0(x) & \text{if } x \in \Omega, \end{cases}$$

where $N > 2s$, $0 < s < 1$, $(-\Delta)^s$ is the fractional Laplacian of order $2s$, $p > 1, c, l > 0, \theta = \{0, 1\}$, and $u_0, f \geq 0$ are in a suitable class of functions. The main results are:

- (1) Optimal summability of the data with respect to the data and the spectral value to have positive solution. Here is necessary a Harnack inequality for an associated operator.
- (2) *Instantaneous and complete blow up* in the linear problem (P_0)
- (3) The existence of a critical power $p_+(s, \lambda)$ in the semilinear problem (P_1) .

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Multiplicity of nonnegative solutions for quasilinear Schrödinger problems

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Abstract.

The talk deals with existence and multiplicity of nonnegative (weak) solutions for quasilinear Schrödinger Dirichlet problems involving nonlinearities with possibly supercritical growth at infinity and indefinite sign. An appropriate change of variables reduces the quasilinear problem into a semilinear one. Variational and sub-super methods are applied in order to obtain the main existence results.

Evolution and Selection in Living Systems: a mathematical model

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Abstract.

This talk is devoted to the modeling and qualitative analysis of Darwinian selection phenomena and evolution. The approach takes advantage of the mathematical tools of the kinetic theory of active particles applied to describe the selective dynamics of evolution processes. We present a specific example which illustrates the mathematical structures proposed. A general qualitative analysis is developed. In particular, under suitable weak assumptions, covered by the example, we show that the underlying non linear Cauchy problem admits a non-negative bounded unique global solution.

References

- [1] M. Delitala, P. Pucci and M.C. Salvatori, Modelling epidemics and virus mutations by methods of the mathematical kinetic theory for active particles, *Math. Models Methods Appl. Sci.*, **21**(2011), 843-870.
- [2] P. Pucci and M.C. Salvatori, On an initial value problem modeling evolution and selection in living systems, *Discrete and Continuous Dynamical Systems Series S*, **7**, (2014), 807-821.

Reverse Hölder inequalities and new applications

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Abstract.

Reverse Hölder inequalities with the same support:

$$\left(\int_Q v^q \right)^{\frac{1}{q}} \leq K \int_Q v$$

for any cube $Q \subset \mathbf{R}^n$, have interesting applications in the regularity theory of differential equations (quasiminimizers of integrals, quasiconformal Jacobians, Dirichlet problems with L^p - boundary data, Hamilton– Jacobi equations).

Sharp results are discussed.

Lusin (N) condition and the distributional determinant

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Abstract.

In Geometric Function Theory one usually assumes that $f \in W^{1,n}(\Omega, \mathbf{R}^n)$ (or that f belongs to some function space very close to $W^{1,n}$ like the closure of the grand-Sobolev space $W^{1,n}$) (see [2]) and using this shows that the Jacobian is equal to the distributional Jacobian, (see [1]), i.e.

$$\int_{\Omega} \varphi(x) J_f(x) dx = - \int_{\Omega} f_1(x) J(\varphi(x), f_2(x), \dots, f_n(x)) dx$$

for every $\varphi \in C_c^\infty(\Omega)$. Our main aim is to show that Sobolev regularity $f \in W^{1,n}$ can be essentially weakened if we a priori know that f is continuous and satisfies the Lusin (N) condition, i.e. for every $N \subset \Omega$ with $|N| = 0$ we have $|f(N)| = 0$.

As corollary we obtain various positive properties which were known before only with the stronger assumption $f \in W^{1,n}$.

This is a joint work with L. D’Onofrio and S. Hencl.

References

- [1] L. Greco. *A remark on the equality $\det Df = \text{Det } Df$* , *Differential Integral Equations* **6** (1993) 1089–1100
- [2] T. Iwaniec and C. Sbordone, *On the integrability of the Jacobian under minimal hypotheses*, *Arch. Rational Mech. Anal.* **119** 129–143 (1992)
- [3] S. Hencl, P. Koskela, *Lectures on mappings of finite distortion*, *Lecture Notes in Mathematics* 2096, Springer, (2014)

Strong competition versus fractional diffusion

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Abstract.

Different phenomena can be modeled by elliptic systems involving a number of densities subject to reaction, diffusion and competitive interaction. Relevant particular cases include the Lotka-Volterra competition, widely used in population dynamics and ecology, and the Gross-Pitaevskii one, which appears in the theory of Bose-Einstein condensation. When competition prevails, the densities tend to segregate each other, and a typical question regards the common shared regularity (i.e. uniform Hölder bounds), uniformly w.r.t. the competition parameter. In this talk, I will first review the theory addressing such issue in the case of standard diffusion. Next I will report on some recent results, obtained in collaboration with Susanna Terracini (Università di Torino) and Alessandro Zilio (CAMS, EHESS, Paris), concerning the case of anomalous diffusion, i.e. when the Laplace operator is replaced by the (nonlocal) fractional laplacian.

Some questions related to elliptic fractional Laplacian equations with Neumann boundary conditions

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Abstract.

In this talk we consider the following fractional semilinear Neumann problem on a smooth bounded domain $\Omega \subset \mathbf{R}^n$, $n \geq 2$,

$$\begin{cases} (-\varepsilon\Delta)^{1/2}u + u = u^p, & \text{in } \Omega, \\ \partial_\nu u = 0, & \text{on } \partial\Omega, \\ u > 0, & \text{in } \Omega, \end{cases}$$

where $\varepsilon > 0$ and $1 < p < (n+1)/(n-1)$. This is the fractional version of the semilinear Neumann problem studied by Lin–Ni–Takagi in the late 80's. The problem arises by considering steady states of the Keller–Segel model with non-local chemical concentration diffusion. Using the semigroup language for the extension method and variational techniques, we will comment the variational techniques involved in obtaining the existence of nonconstant smooth solutions for small ε . In the case of large ε we give results about nonexistence of nonconstant solutions, and it will be also shown that as $\varepsilon \rightarrow 0$ the solutions u_ε tend to zero in measure on Ω , while they form spikes in $\bar{\Omega}$. The regularity estimates of the fractional Neumann Laplacian that we develop here are essential for the analysis. These results are based on a joint work with Pablo Raúl Stinga.

On some mean field equations with probability measure in $2D$ -turbulence

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Abstract.

We outline some results obtained in recent years concerning a class of semilinear elliptic equations of the mean field type containing a probability measure, motivated by the statistical mechanics description of $2D$ turbulence. In particular, we exhibit the optimal Moser-Trudinger constants, we study the blow-up of solution sequences, we prove their mass quantization under suitable conditions and we consequently derive the existence of solutions by variational methods. Those results are obtained in collaboration with T. Ricciardi.

References

- [1] T. Ricciardi, G. Zecca, Mass quantization and minimax solutions for Neris mean field equation in $2D$ -turbulence, *J. Differential Equations*, (2015)
- [2] T. Ricciardi, R. Takahashi, G. Zecca, X. Zhang, On the blow-up, mass quantization and existence for a class of sinh-Poisson equations in $2D$ -turbulence. (Submitted)
- [3] T. Ricciardi, G. Zecca, On the blow-up of solutions to Liouville type equations, *Advanced Nonlinear Studies*, (2015) (to appear)
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