3RD CONFERENCE ON RECENT TRENDS IN NONLINEAR PHENOMENA

Organized by

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Honorary Chairwoman: **Patrizia Pucci**

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Aims and scope

The study of nonlinear problems is a relevant topic in Mathematics, both for the pure mathematical research and in view of concrete real-world applications. Indeed, nonlinear phenomena arise in a quite natural way in many different contexts. The aim of this conference is to bring together leading academic scientists in the field of nonlinear phenomena in order to discuss recent results, related problems (in particular in relation to concrete applications) and future perspectives.

Talks:

- Carlo Bardaro *Università degli Studi di Perugia, Italy*
- Lucio Boccardo *Università 'La Sapienza' di Roma, Italy*
- Matteo Bonforte *Universidad Autónoma de Madrid, Spain*
- Denis Bonheure *Université Libre de Bruxelles, Belgium*
- Alberto Farina *Université de Picardie Jules Verne, France*
- Filippo Gazzola *Politecnico di Milano, Italy*
- Gabriele Grillo *Politecnico di Milano, Italy*
- Andrea Malchiodi *Scuola Normale Superiore, Italy*
- Jan Maly *Univerzita Karlova, Czech Republic*
- Paolo Marcellini *Università degli Studi di Firenze, Italy*
- Jean Mawhin *Université Catholique de Louvain, Belgium*
- Giuseppe Mingione *Università degli Studi di Parma, Italy*
- Enzo Mitidieri *Università degli Studi di Trieste, Italy*
- Vicentiu Radulescu *University of Craiova and Romanian Academy, Romania*
- Dusan Repovs *University of Ljubljana, Slovenia*
- Marco Rigoli *Università degli Studi di Milano, Italy*
- Sandro Salsa *Politecnico di Milano, Italy*
- Carlo Sbordone *Università degli Studi di Napoli 'Federico II', Italy*
- Gabriella Tarantello *Università degli Studi di Roma 'Tor Vergata', Italy*
- Susanna Terracini *Università degli Studi di Torino, Italy*
- Gianluca Vinti *Università degli Studi di Perugia, Italy*
- Binlin Zhang *Heilongjiang Institute of Technology, China*

Talks

Mellin analysis and its associated metric: applications to sampling theory

Carlo Bardaro
Università degli Studi di Perugia, Italy

In this talk, I will present some recent results about the Paley-Wiener theorem in Mellin analysis frame and the corresponding Mellin-Bernstein spaces. Also, applications to the exponential sampling formula are described and its generalizations to not Mellin band-limited functions, with estimates of the error of approximation in terms of a new concept of distance involving the inverse Mellin transform.

Minima and critical points for some integral (vectorial) functionals

Lucio Boccardo
Università 'La Sapienza' di Roma, Italy

The study of some simple examples of integral (vectorial) functionals gives Euler-Lagrange equations not studied in the classical books. I will present some results (joint papers with Luigi Orsina) concerning the study of these cases; one of problems concerns the Schrödinger-Maxwell equation.

Nonlinear and Nonlocal Degenerate Diffusions on Bounded Domains

Matteo Bonforte
Universidad Autónoma de Madrid, Spain

We investigate quantitative properties of nonnegative solutions $u(t,x) \geq 0$ to the nonlinear fractional diffusion equation, $\partial_t u + \mathcal{L}F(u) = 0$ posed in a bounded domain, $x \in \Omega \subset \mathbb{R}^N$, with appropriate homogeneous Dirichlet boundary conditions. As $\mathcal{L}$ we can use a quite general class of linear operators that includes the three most common versions of the fractional Laplacian $(-\Delta)^s$, $0 < s < 1$, in a bounded domain with zero Dirichlet boundary conditions, but it also includes many other examples, since our theory only needs some basic properties that are typical of "linear heat semigroups". The nonlinearity F is assumed to be increasing and is allowed to be degenerate, the prototype being $F(u) = |u|^{m-1}u$, with $m > 1$.

We will present some recent results about existence, uniqueness and a priori estimates for a quite large class of very weak solutions, that we call weak dual solutions. We then show sharper lower and upper estimates up to the boundary, which fairly combine into various forms of Harnack type inequalities. The standard Laplacian case $s = 1$ is included and the linear case $m = 1$ can be recovered in the limit in most of the results. When the nonlinearity is of the form $F(u) = |u|^{m-1}u$, with $m > 1$, global Harnack estimates are the key tool to understand the sharp asymptotic behavior of the solutions.

We finally show that solutions are classical, and even C^∞ in space in the interior of the domain, when the operator $\mathcal{L}$ is the (restricted) fractional Laplacian. The above results are contained on a series of recent papers with J. L. Vazquez, and also with A. Figalli, Y. Sire and X. Ros-Oton.

Some questions arising from the nonlinear theory of electromagnetism of Born and Infeld

Denis Bonheure
Université Libre de Bruxelles, Belgium

In this talk, I will discuss some questions related to the nonlinear theory of electromagnetism formulated by Born and Infeld in 1934. I will discuss the link between this theory and the curvature operators in the Euclidean and in the Lorentz-Minkowski space. I will address the solvability of the electrostatic Born-Infeld equation with sources (and another model driven by the nonlinear Klein-Gordon equation) emphasizing the open questions and the partial recent progress we made. Finally, I will present some results on a finite order approximation of the model. In this approximation, the operator is a finite sum of p-Laplacians. The talk is based on joint works with P. D'Avenia, A. Pomponio, J. Foldes and F. Colasuonno.

A Bernstein-type result for the minimal surface equation

Alberto Farina
Université de Picardie Jules Verne, France

We prove the following Bernstein-type theorem: *if u is an entire solution to the minimal surface equation, such that $N-1$ partial derivatives $\frac{\partial u}{\partial x_j}$ are bounded on one side (not necessarily the same), then u is an affine function.* Besides its novelty, our theorem also provides a new, simple and self-contained proof of celebrated results of Moser and of Bombieri & Giusti.

Energy transfer between modes in a nonlinear nonlocal beam equation

Filippo Gazzola
Politecnico di Milano, Italy

We consider the nonlinear nonlocal beam evolution equation introduced by Woinowsky-Krieger. We first study the stationary problem and we discuss possible multiple solutions due to the effects of compression. Then we study the existence and behavior of periodic solutions for the evolution problem: these are called nonlinear modes. Some solutions of the beam equation only have two active modes and we investigate whether there is an energy transfer between them. The answer depends on the geometry of the energy function which, in turn, depends on the amount of compression compared to the spatial frequencies of the involved modes. Our results are complemented with numerical experiments; overall, they give a complete picture of the instabilities that may occur in the beam. We expect these results to hold also in more complicated dynamicalsystems such as suspension bridges. This talk is based on joint works with U. Battisti, E. Berchio, A. Ferrero, C. Gasparetto.

On the porous medium equation on negatively curved manifolds

Gabriele Grillo
Politecnico di Milano, Italy

We report on some recent results concerning existence, uniqueness and asymptotic behavior of solutions to the porous medium equation on classes of negatively curved manifolds. In particular we shall deal with the following topics:

- existence and uniqueness for (finite) measure data;
- upper and lower pointwise bounds on solutions;
- large data and blow-up in finite time.

This is joint work with M. Muratori, F. Punzo, J.L. Vazquez.

On the Sobolev quotient in CR geometry

Andrea Malchiodi
Scuola Normale Superiore, Italy

We consider a class of three-dimensional CR manifolds which are modeled on the Heisenberg group. We prove positivity of the mass under the condition that the Webster curvature is positive and that the manifold is embeddable. We apply this result to the CR Yamabe problem, and we discuss the properties of Sobolev-type quotients, giving some counterexamples for Rossi spheres. This is joint work with J.H.Cheng and P.Yang.

Jacobians and Hessians of Sobolev mappings

Jan Maly
Univerzita Karlova, Czech Republic

Recently, new results appeared which compare the topological orientation of a Sobolev mappings with the sign of its Jacobian. Surprisingly, the topological and analytical orientation of a Sobolev homeomorphism can somewhat differ. This fact has consequences to nonlinear approximation problems. Also, there are examples of Sobolev homeomorphism which have vanishing Jacobian a.e., so that the sign of the Jacobian cannot give any information on the topological orientation. Such an example can be a gradient, which links the research to the degenerate Monge-Ampère equation. Examples of functions with low rank Hessians complement recent positive results on rigidity of isometric immersions.

A variational approach to parabolic equations via the calculus of variations and the parabolic bounded slope condition

Paolo Marcellini
Università degli Studi di Firenze, Italy

The notion of 'variational solutions' to a class of Cauchy-Dirichlet problem for evolutionary partial differential equations possessing a variational structure is the main tool for the existence of global Lipschitz-continuous solutions. We give conditions for the existence of a unique classical solution, with Lipschitz continuous spatial gradient, to a Cauchy-Dirichlet problem with a given Lipschitz-continuous initial datum whose restriction to the boundary satisfies the 'bounded slope condition'.

As already said, a main step is the existence of a 'variational solution' to the parabolic Cauchy-Dirichlet problem. The advantage of these variational solutions relies on the fact that they might exist even in situations where the associated nonlinear parabolic equation makes no sense. We should compare with the stationary case, where it is possible to establish the existence of minimizers by the direct methods of the calculus of variations in fairly general situations, whereas additional stronger assumptions are needed to guarantee that the minimizers fulfill the Euler-Lagrange equation. Under reasonable assumptions, such as coercivity and convexity of the energy function, the variational solution exists and is unique. Under further natural growth conditions, the variational solution is also the unique weak solution to the associated Cauchy-Dirichlet problem.

This is a joint collaboration with Verena Bögelein, Frank Duzaar and Stefano Signoriello.

Convex sets and second order systems with nonlocal boundary conditions at resonance

Jean Mawhin
Université Catholique de Louvain, Belgium

The solvability of the resonant nonlocal boundary value problem

$$x'' = f(t, x, x'), \quad x(0) = 0, \quad x'(1) = \int_0^1 x'(s) dg(s),$$

with $f : [0,1] \times \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}^n$ continuous, $g = \mathrm{diag}(g_1, \ldots, g_n)$, $g_i : [0,1] \to \mathbb{R}$ of bounded variation, $\int_0^1 dg_i(s) = 1$ $(i = 1, \ldots, n)$, is studied using Leray-Schauder continuation theorem.

The requested a priori estimates follow from the existence of an open bounded convex subset $C \subset \mathbb{R}^n$, such that, for each $t \in [0,1]$ and $x \in \overline{C}$, the vector fields $f(t, x, \cdot)$ satisfy suitable geometrical conditions on ∂C.

The special cases where C is a ball or a parallelotope are considered.

Joint work with K. Szymańska-Dębowska (Technical University of Lodz).

TBA

Giuseppe Mingione
Università degli Studi di Parma, Italy

Quasilinear elliptic equations with singular potentials

Enzo Mitidieri
Università degli Studi di Trieste, Italy

We analyse some classes of quasilinear Schrödinger equations related to Hardy and Gagliardo-Nirenberg inequalities.

A characterization property for a class of fractional equations

Vicentiu Radulescu
Institute of Mathematics of the Romanian Academy, Bucharest, Romania

In this talk, we report on some a recent result in collaboration with Giovanni Molica Bisci. We are concerned with the qualitative analysis of positive weak solutions of a sublinear problem that involves the fractional Laplace operator. The main result establishes a characterization property, which points out the relationship between the behaviour of the nonlinear term near the origin and the existence of a solution with a controlled norm, provided that the parameter belongs to a certain interval.

References

[1] G. Molica Bisci, V. Radulescu, A sharp eigenvalue theorem for fractional elliptic equations, Israel Journal of Mathematics, in press.
[2] G. Molica Bisci, V. Radulescu, R. Servadei, Variational Methods for Nonlocal Fractional Problems, Encyclopedia of Mathematics and its Applications, Cambridge University Press, Cambridge, 2016.

Stationary Schrödinger-type equations with variable exponent

Dusan Repovs
University of Ljubljana, Slovenia

We report on the some recent results [1, 2] concerning the qualitative analysis of some classes of nonlinear Schrödinger-type equations with variable exponent. We are first concerned with the existence of stationary waves for a nonlinear Dirichlet problem involving a Laplace operator with variable exponent and we provide sufficient conditions for the existence of one or multiple solutions. Next, we consider a fourth-order eigenvalue problem with several variable exponents. Taking into account multiple competition effects, we establish a concentration property of the spectrum in a neighbourhood of infinity. The proofs combine variational and topological methods, as developed in the recent monograph [3].

References

[1] D. Repovš, Stationary waves of Schrödinger-type equations with variable exponent, *Anal. Appl. (Singap.)* **13** (2015), 645-661.

[2] D. Repovš, Concentration phenomena for higher-order equations with variable exponent, submitted.

[3] V. Rădulescu, D. Repovš, *Partial Differential Equations with Variable Exponents: Variational Methods and Qualitative Analysis*, CRC Press, Taylor & Francis Group, Boca Raton FL, 2015.

Generalized mean curvature flow solitons

Marco Rigoli
Università degli Studi di Milano, Italy

We study a very general class of mean curvature flow solitons depending on the existence of a closed conformal vector field in the ambient space, both from the point of view of geometry and of analysis.

On the boundary behavior of solution to parabolic equations of p-Laplacian type

Sandro Salsa
Politecnico di Milano, Italy

We describe some result obtained in a joint paper with Gianazza and Avelin on the behavior at the lateral part of a cylinder of solution to parabolic singular/degenerate paraboliceequation, whose prototype is the parabolic p-Laplace equation. In particular we examine the validity of the so called Carleson estimate and of the boundary Harnack principle.

Optimal regularity for the gradient of solutions to Dirichlet problem for planar quasilinear elliptic equations

Carlo Sbordone
Università degli Studi di Napoli 'Federico II', Italy

We establish the maximal regularity of the gradient of solution u to Dirichlet problem for a planar quasilinear elliptic equation of Leray-Lions type:

$$\begin{cases} \operatorname{div} A(x, \nabla u) = f & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases} \tag{1}$$

for f an integrable function. Here $A = A(x, z)$ is a Caratheodory function with a linear growth at infinity, with a merely measurable dependance on the x-variable. For f in a given rearrangement-invariant space $X(\Omega)$ of integrable functions, we determine the optimal rearrangement-invariant space $Y(\Omega)$ such that

$$||Du||_{X(\Omega)} \leq c||f||_{Y(\Omega)}$$

for every f in $Y(\Omega)$ where u is the solution to (1).

These results are obtained in a joint paper with Angela Alberico and Andrea Cianchi (Nonlinear Analysis, to appear).

The role of singular Liouville systems in the study of non-abelian Chern-Simons vortices

Gabriella Tarantello
Università degli Studi di Roma 'Tor Vergata', Italy

We describe recent results about the construction on non-abelian Chern-Simons vortices of non-topological type in terms of entire solutions for a class of singular Liouville systems in the plane.

TBA

Susanna Terracini
Università degli Studi di Torino, Italy

Approximation by sampling type operators with applications to Digital Image Processing

Gianluca Vinti
Università degli Studi di Perugia, Italy

The aim of this talk is to show that the theory of multivariate sampling type operators plays a fundamental role in signal analysis and it has important applications to Digital Image Processing. In particular, we will discuss some recent applications to computer tomography images for the study of vascular diseases. We will show how the implementation of the theory developed along with other Digital Image Processing algorithms, can be useful for the diagnosis of abdominal aortic aneurysms.

Some recent results on fractional Kirchhoff-type problems

Binlin Zhang
Heilongjiang Institute of Technology, China

In this talk, we review some existence results on fractional $p-$Kirchhoff problems which stem from various aspects in physics. The fractional Kirchhoff problems have been studied extensively in recent years. Most of the known results are on the non-degenerate case. It is worthy pointing out that the Kirchhoff problems involving the fractional Laplacian possess significant difficulties due to the presence of the Kirchhoff function and of the nonlocal feature of the fractional Laplacian. In this context, some recent results on some kinds of fractional Kirchhoff problems in the non-degenerate and degenerate cases are presented. Finally, we will mention some related subjects on fractional problems with magnetic field, such as Kirchhoff-Schrödinger type, Kirchhoff-Choquard type and so on.