TOPICS IN NONLINEAR ANALYSIS AND APPLICATIONS

Dipartimento di Matematica e Applicazioni
Università di Milano 'Bicocca'
March 15-16, 2017

Abstracts of the talks

Spectral stability under removal of small capacity sets and applications to Aharonov-Bohm operators

Laura Abatangelo
Università di Milano 'Bicocca'

When a compact set concentrating to a point is removed from the domain, the asymptotic expansion of simple Dirichlet eigenvalues variation can be provided. It relies on the order of vanishing of the related eigenfunctions at the limit point. This topic has relevant applications to the study of the asymptotic behaviour of eigenvalues of Aharonov–Bohm operators with two colliding poles moving on one of the symmetry axis of the domain.
These recent results are obtained in collaboration with Veronica Felli (Milano 'Bicocca'), Luc Hillairet (Orleans) and Corentin Lena (Torino).

Periodic solutions for superlinear fractional problems

Vincenzo Ambrosio
Università degli Studi di Urbino 'Carlo Bo'

In this talk we focus our attention on the existence of T-periodic solutions to the following problem

$$(1) \qquad \begin{cases} [(-\Delta_x + m^2)^s - m^{2s}]u = f(x,u) & \text{in } (0,T)^N \\ u(x+Te_i) = u(x) & \text{for all } x \in \mathbb{R}^N, i = 1, \dots, N \end{cases}$$

where $s \in (0,1)$, $N > 2s$, $T > 0$, $m > 0$ and $f(x,u)$ is a continuous function, T-periodic in x and satisfying a suitable growth assumption weaker than the Ambrosetti-Rabinowitz condition.
The nonlocal operator $(-\Delta_x + m^2)^s$ can be realized as the Dirichlet to Neumann map for a degenerate elliptic problem posed on the half-cylinder $\mathcal{S}_T = (0,T)^N \times (0,\infty)$.
By using a variant of the Linking Theorem, we show that the extended problem in $\mathcal{S}_T$ admits a nontrivial solution $v(x,\xi)$ which is T-periodic in x. Moreover, by a procedure of limit as $m \to 0$, we also prove the existence of a nontrivial solution to (1) with $m = 0$.

On the Cauchy-Dirichlet problem for a general class of parabolic equations

Paolo Baroni
Università degli Studi di Parma

We consider a class of parabolic equations extending the evolutionary p-Laplacian in a natural way and we present some regularity results for solutions to the related Cauchy-Dirichlet problem. We shall also compare the local behaviour of solutions with the typical one of solutions to parabolic p-Laplace-type equations, both of singular and degenerate type, stressing differences and describing some open questions. The talk is based on a joint work with C. Lindfors (Aalto University, Helsinki).

Complete stickiness of nonlocal minimal surfaces for small values of the fractional parameter

Claudia Bucur
Università degli Studi di Milano

Nonlocal minimal surfaces are introduced in [2] as boundary of sets that minimize the fractional perimeter in a bounded and open set $\Omega \subset \mathbb{R}^n$, among sets with fixed exterior data. It is a known result that when Ω has a smooth boundary and the exterior data is a half-space, the s-minimal set is the same half-space. On the other hand, if one removes (even from far away) some small set from the half-space, for s small enough the s-minimal set completely sticks to the boundary, that is, the s-minimal set is empty inside Ω. In the paper [4], it is proved indeed that fixing the first quadrant of the plane as boundary data, the s-minimal set in $B_1 \subset \mathbb{R}^2$ is empty in B_1 for s small enough.

In this talk, we will present the behavior of s-minimal surfaces when the fractional parameter $s \in (0,1)$ is small, in a bounded and connected open set with C^2 boundary Ω. We classify the behavior of s-minimal surfaces with respect to the fixed exterior data. So, for s small and depending on the data at infinity, the s-minimal set can be either empty in Ω, fill all Ω, or possibly develop a wildly oscillating boundary.

Also, we will present the asymptotic behavior of the fractional mean curvature (see [3]) when $s \to 0^+$. In particular, as s gets smaller, the fractional mean curvature at any point on the boundary of a $C^{1,\gamma}$ set (for $\gamma \in (0,1)$) takes into account only the nonlocal contribution.

The results in this talk are obtained in the preprint [1] authored by myself, Luca Lombardini and Enrico Valdinoci.

References

[1] C. Bucur, L. Lombardini, and E. Valdinoci. Complete stickiness of nonlocal minimal surfaces for small values of the fractional parameter. *arXiv preprint arXiv:1612.08295*.

[2] L. Caffarelli, J.-M. Roquejoffre, and O. Savin. Nonlocal minimal surfaces. *Comm. Pure Appl. Math.*, 63(9):1111–1144, 2010.

[3] N. Abatangelo and E. Valdinoci. A notion of nonlocal curvature. *Numer. Funct. Anal. Optim.*, 35(7-9):793–815, 2014.

[4] S. Dipierro, O. Savin, and E. Valdinoci. Boundary behavior of nonlocal minimal surfaces. *arXiv preprint arXiv:1506.04282*, 2015.

Maximum principle results for a class of degenerate elliptic operators

Giulio Galise
Sapienza Università di Roma

In this talk I will present a joint work with I. Birindelli and H. Ishii concerning the limit of validity of the maximum principle and its consequences for fully nonlinear degenerate elliptic equations whose principal part is a partial sum of the eigenvalues of the Hessian. Some very unusual phenomena due to the degeneracy of the operators will be emphasized by means of explicit counterexamples. Moreover Lipschitz regularity and boundary estimates for solutions of the Dirichlet problem will be presented in the context of convex domains with the aim to obtain the existence of principal eigenfunctions.

Reduction of Quasilinear First Order PDE's to Partially or Fully Decoupled Systems

Matteo Gorgone
Università degli Studi di Messina

This presentation is about the decoupling problem of general quasilinear first order systems in two independent variables. We consider either the case of homogeneous and autonomous systems or the one of nonhomogeneous and/or nonautonomous systems. Necessary and sufficient conditions for the partial or full decoupling of the systems at hand are provided. The conditions involve the properties of eigenvalues and eigenvectors of the coefficient matrix, and provide the differential constraints whose integration leads to the decoupling transformation. Some applications of physical interest are also given.

Multiplicity issues for a nonlinear problem driven by the square root of the Laplacian

Alessandro Iacopetti
Università degli Studi di Torino

In this talk we show some new results about radial sign-changing solutions for the fractional Brezis–Nirenberg problem, which is the following semilinear elliptic problem:

$$\begin{cases} (-\Delta)^s u = \lambda u + |u|^{2^*-2}u & \text{in } B \\ u = 0, & \text{in } \mathbb{R}^N \setminus B, \end{cases} \tag{2}$$

where $s \in (0,1)$, $(-\Delta)^s$ is the fractional Laplacian, $2^* = \frac{2N}{N-2s}$ is the fractional critical Sobolev exponent, $N > 2s$, λ is a positive parameter and B is the unit ball of $\mathbb{R}^N$.

We will discuss the issue of existence of (least-energy) radial sign-changing solutions and study their asymptotic profile as $\lambda \to 0$, moreover we will try to shed some light on the problem concerning the structure of the nodal set of those solutions.

These results are contained in a paper in preparation, in collaboration with G. Cora.

Sign-changing solutions for fractional Schrödinger equations with vanishing potentials

Teresa Isernia
Università di Napoli 'Federico II'

In this talk we consider the following class of fractional Schrödinger equations

$$(-\Delta)^s u + V(x)u = K(x)f(u) \text{ in } \mathbb{R}^N$$

where $s \in (0,1)$, $N > 2s$, $(-\Delta)^s$ is the fractional Laplacian, V, K are positive continuous functions which vanish at infinity, and f is a continuous function.
By using a minimization argument and a quantitative deformation lemma, we obtain the existence of a sign-changing solution. Moreover, when the problem presents symmetry, we show that it has infinitely many nontrivial solutions.
The results in this talk are based on a joint work with V. Ambrosio, G.M. Figueiredo and G. Molica Bisci.

Magnetic Bourgain - Brezis - Mironescu limits

Andrea Pinamonti
Università di Trento

We discuss a Bourgain-Brezis-Mironescu type formula for a class of nonlocal magnetic spaces, building a bridge with the classical theory of magnetic Sobolev spaces.

Grand p-Harmonic Energy

Teresa Radice
Università di Napoli 'Federico II'

To every nonlinear differential expression there corresponds the so-called natural domain of definition. Usually, such a domain consists of Sobolev functions, sometimes with additional geometric constraints. There are, however, special nonlinear differential expressions (Jacobian determinants, div-curl products, etc.) whose special properties (higher integrability, weak-continuity, etc.) cannot be detected within their natural domain. We must consider them in a slightly larger class of functions. The grand Lebesgue space, denoted by $\mathscr{L}^{p)}(\mathbb{X})$, and the corresponding grand Sobolev space $\mathscr{W}^{1,p)}(\mathbb{X})$, turn out to be most effective. They were studied by many authors, largely in analogy with the questions concerning $\mathscr{L}^{p}(\mathbb{X})$ and $\mathscr{W}^{1,p}(\mathbb{X})$ spaces. The present paper is a continuation of these studies. We take on stage the grand p-harmonic energy integrals. These variational functionals involve both one-parameter family of integral averages and supremum with respect to the parameter. It is for this reason that the existence and uniqueness of the grand p-harmonic minimal mappings becomes a new (rather challenging) problem.

Lusin (N) condition and the distributional determinant

Roberta Schiattarella
Università di Napoli 'Federico II'

In Geometric Function Theory one usually assumes that $f \in W^{1,n}(\Omega, \mathbb{R}^n)$ (or that f belongs to some function space very close to $W^{1,n}$ like the closure of the grand-Sobolev space $W^{1,n)}$ and using this shows that the Jacobian is equal to the distributional Jacobian, i.e.

$$\int_\Omega \varphi(x) J_f(x)\, dx = -\int_\Omega f_1(x) J(\varphi(x), f_2(x), \ldots, f_n(x))\, dx$$

for every $\varphi \in C_C^\infty(\Omega)$. Our main aim is to show that Sobolev regularity $f \in W^{1,n}(\Omega, \mathbb{R}^n)$ can be essentially weakened if we a priori know that f is continuous and satisfies the Lusin (N) condition, i.e. for every $N \subset \Omega$ with $|N| = 0$ we have $|f(N)| = 0$.

As corollary we obtain various positive properties which were known before only with the stronger assumption $f \in W^{1,n}(\Omega, \mathbb{R}^n)$.

Multiplicity issues for a nonlinear problem driven by the square root of the Laplacian

Luca Vilasi
Università degli Studi di Messina

We focus on multiplicity issues for a parametric problem involving the spectral version of the fractional Laplacian under Dirichlet boundary conditions. By means of variational techniques - in particular, a local minimum result for abstract functionals - we estimate a bounded interval of eigenvalues for which the problem admits (at least) three essentially bounded weak solutions. Some applications are also shown. Joint work with G. Molica Bisci and D. Repovs.