Abstracts

Multiplicity results for a class of fractional equations

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We study the multiplicity of weak solutions to (possibly resonant) non-local equations involving the fractional $p$–Laplacian operator. More precisely, we study a Dirichlet problem involving a subcritical nonlinear term which does not satisfy the technical Ambrosetti–Rabinowitz condition and is driven by the fractional $p$–Laplacian operator. By framing this problem in an appropriate variational setting, we prove a multiplicity theorem.

References


Nonlinear Economic Dynamics ... and more

Gian Italo Bischi
Università degli Studi di Urbino ‘Carlo Bo’

In this lecture we give a summary of some nonlinear discrete-time dynamical systems studied in the framework of the research group on “Dynamic Models in Economics and Finance”. Starting from these models, ranging from oligopoly games with boundedly rational agents to systems with expectatoins and learning with fading memory, from binary choice games to endogenous business cycle models, we show how analytical, geometric and numerical methods have been combined to obtain a global qualitative analysis of some complex dynamic scenarios. Methods for the detection of contact bifurcations have been employed, such as those based on the critical sets in the analysis of iterated noninvertible maps, and other ones have been created ad hoc, such as the study of singularities that characterize the global bifurcations of maps with a vanishing denominator. Chaos synchronization phenomena, with on-off Intermittency, bubbling and riddled basins, observed in models with
invariant submanifolds with lower dimension than the phase space, that naturally arise in symmetric models with identical agents and evolutionary games, are described in terms of Milnor attractors, natural transverse Lyapunov exponents and minimal absorbing areas.

References


Some remarks about two-phase free boundary problems for degenerate operators

Fausto Ferrari

Alma Mater Studiorum - Università di Bologna

We discuss the notion of domain variations solution in the framework of two-phase free boundary problems governed by linear and nonlinear operators, possibly degenerate. In particular we deduce the natural jump condition on the free boundary for sublaplacians in Carnot groups and for the classical p-Laplace operator.
Fractional Kirchhoff type problems involving critical nonlinearities
Alessio Fiscella
Universidad Estadual de Campinas

In this talk we introduce existence, multiplicity and asymptotic behavior of non–trivial weak solutions for problems like

\[-M\left(\|u\|^2\right) \mathcal{L}_K u = \lambda f(x,u) \left[ \int_{\Omega} F(x,u(x)) dx \right]^r + |u|^{2^* - 2} u \quad \text{in } \Omega,\]

\[u = 0 \quad \text{in } \mathbb{R}^n \setminus \Omega,\]

where \(\Omega \subset \mathbb{R}^n\) is a bounded domain, \(n > 2s\) with \(s \in (0,1)\), \(2^* = \frac{2n}{n-2s}\), \(M\) is a continuous function, \(f\) satisfies suitable growth conditions, \(\lambda > 0\) and \(r \geq 0\) are real parameters, \(\mathcal{L}_K\) is a non–local operator defined as follows

\[\mathcal{L}_K u(x) = \frac{1}{2} \int_{\mathbb{R}^n} (u(x+y) + u(x-y) - 2u(x))K(y)dy, \quad \text{as } x \in \mathbb{R}^n.\]

As usual in elliptic problems involving critical nonlinearities, we must pay attention to the lack of compactness at critical level \(L^{2^*}(\Omega)\). For this we present different proof techniques, depending on the assumptions for problem (1), which allow us to overcome this lack and other difficulties deriving from the nonlocal nature of operator \(\mathcal{L}_K\).

Linear and semilinear problems involving \(\Delta_\lambda\)-laplacians
Alessia Kogoj
Alma Mater Studiorum - Università di Bologna

We present a survey of recent results related to \(\Delta_\lambda\)-laplacians, a class of degenerate elliptic operators containing for example Grushin-type operators.

In particular we show Pohozaev identities, Liouville theorems, Hardy inequalities, existence and longtime behavior of solutions of the related semi-linear degenerate parabolic equations.

These results are obtained in collaboration with Ermanno Lanconelli and Stefanie Sonner.

An inverse problems in Potential Theory and ‘spherical’ symmetry results
Ermanno Lanconelli
Alma Mater Studiorum - Università di Bologna

The Newtonian potential of an Euclidean ball \(B\) centered at the origin is proportional, outside \(B\), to the Newtonian potential of a mass concentrated at the origin. Viceversa, if \(D\) is a bounded open set containing the origin, having Newtonian potential proportional, outside \(D\), to the one of a mass concentrated at the origin, then \(D\) is an Euclidean ball with center at the origin. The first statement simply follows from the Gauss Mean Value property for the harmonic functions, while the second assertion is a theorem by Aharonov, Schiffer and Zalcman.

In this talk we present several results generalizing the previous ones. The key ingredient of our technique is the strong maximum principle for subharmonic functions. Thus our results easily extend to the heat operator, getting a characterizations of the Pini-Watson heat balls, as well as to the Laplace-Beltrami operators giving a characterization of the geodesic balls on harmonic Riemannian manifolds.
The results we show are contained in a joint work with Giovanni Cupini, of the University of Bologna.

**Elliptic problems involving the fractional Laplacian**

**Giovanni Molica Bisci**

Università ‘Mediterranea’ di Reggio Calabria

In the last years an always increasing interest has been shown towards nonlocal fractional problems (see, among others, the papers [1, 3, 4, 11] and the book [7]). Moving along this direction, the aim of this talk is to present some results on the existence and the multiplicity of weak solutions for nonlocal fractional equations whose simple prototype is

\[
\begin{aligned}
(-\Delta)^s u &= f(x, u) \quad \text{in } \Omega \\
\quad u &= 0 \quad \text{in } \mathbb{R}^n \setminus \Omega,
\end{aligned}
\]

where \( s \in (0, 1) \) is fixed, \((-\Delta)^s\) is the fractional Laplace operator, \( \Omega \subset \mathbb{R}^n, n > 2s \), is an open bounded set with continuous boundary and the nonlinearity \( f \) satisfies suitable growth assumptions. At this purpose we employ variational and topological methods (see [2, 5, 6, 8, 9, 10]).

**References**


**Existence theorems for fractional p–Laplacian problems**

**Patrizia Pucci**

Università degli Studi di Perugia

The talk focuses on the existence of nontrivial solutions of a nonlinear eigenvalue perturbed problem depending on a real parameter \( \lambda \) under homogeneous boundary conditions in bounded domains with Lipschitz boundary. The problem involves a weighted fractional \( p–\)Laplacian operator. Denoting by \( (\lambda_k)_k \) a sequence of eigenvalues obtained via mini–max methods and linking structures we prove the existence of (weak) solutions both when there exists \( k \in \mathbb{N} \) such that \( \lambda = \lambda_k \) and when \( \lambda \notin (\lambda_k)_k \). In the first part of the talk existence results are determined when the perturbation is the derivative of a globally positive function.
whereas, in the second part, the case when the perturbation is the derivative of a function that could be either locally positive or locally negative at 0 is taken into account. In both cases, the existence of solutions is achieved via linking methods. In the latter case however it is necessary to extend in several directions some results established in recent papers. These auxiliary properties are of independent interest.

On Two phase Free Boundary Problems with Distributed Sources

Sandro Salsa
Politecnico di Milano

We discuss some recent results obtained in a joint work with D. De Silva and F. Ferrari on existence and optimal regularity, in two phase free boundary problems governed by elliptic equations with bounded right hand side. At the same time we emphasize some questions that remain open.

On a Kirchhoff fractional problem

Francesco Tulone
Università degli Studi di Palermo

In this talk we present a class of nonlocal fractional Laplacian equations. More precisely, by using an appropriate analytical context on fractional Sobolev spaces, we establish the existence of one non-trivial weak solution for nonlocal fractional problems of Kirchhoff-type exploiting classical variational methods. The presented results are obtained in collaboration with G. Molica Bisci.

REFERENCES