TWO NONLINEAR DAYS IN URBINO 2018

July 12-13, 2018

Aula Magna del Rettorato
Palazzo Bonaventura
via Saffi, 2

Abstracts

On some stationary Navier–Stokes type problems

Michel Chipot
University of Zurich, Switzerland

The goal of this talk is to present a simple proof of existence of solution to the stationary Navier–Stokes problem using a singular perturbation technique and to address some nonlocal issues related to it.

Non-uniformly elliptic problems

Cristiana De Filippis
University of Oxford, UK

I will discuss some new results on the regularity of minimizers of certain non-uniformly elliptic problems, focusing in particular to the manifold constrained case.

Multiplicity results for fractional Kirchhoff problems involving singular terms

Alessio Fiscella
Universidade Estadual de Campinas, Brazil

In this talk, we discuss about recent results and open questions for Kirchhoff problems driven by nonlocal fractional operators, involving critical Sobolev nonlinearities and different singular terms. Our variational problems present some difficulties due to the bi-nonlocal nature of the elliptic part, the lack of compactness at critical level and the possible non-differentiability of the related functional. For this, in order to state multiplicity results, we introduce different proof techniques based on approximation arguments and topological tools. Finally, we consider some interesting open questions. In particular, we see how the possible addiction of Hardy potential terms could affect the components of our problems.

Determining eigenvectors and eigenvalues of a self-adjoint operator and application to a Schrödinger–Poisson system

Otared Kavian
Université de Versailles, France

Given a self-adjoint $n \times n$ matrix A one may ask the following elementary question: can one define an appropriate functional J such that the eigenvectors $u_1, \dots, u_n$ of A are critical points of J?

Indeed the same question can be put forward for any diagonalizable matrix, or for any self-adjoint operator A acting on a Hilbert space which has a compact resolvent.

After showing that this can be achieved, as an application we investigate the Schrödinger–Poisson system with infinitely many states

$$\begin{aligned} -\Delta u_j + (V + \widetilde{V})u_j &= \lambda_j u_j \quad \text{in } \Omega \\ -\Delta V &= \sum_{j\geq 1} \alpha_j |u_j|^2 \quad \text{in } \Omega, \end{aligned}$$

where the coefficients α_j are positive and where one requires that in addition the sequence $(u_j)_{j\geq 1}$ is a Hilbert basis of $L^2(\Omega)$, while $u_j \in H_0^1(\Omega)$ and $V \in H_0^1(\Omega)$, with $\Omega \subset \mathbb{R}^N$ a bounded domain.

Minimal surfaces and their Gauss maps

Francesco Mercuri
University of Campinas, Brazil

If M is an oriented minimal surface in $\mathbb{R}^3$, the normal (Gauss) map $G : M \to S^2$, composed with stereographic projection, is a holomorphic map into $\mathbb{C}$. Since the middle of last century, researchers in the field asked which results of complex functions theory still hold for the normal map of a minimal surface. For example, there is a Picard type theorem for the Gauss map of a *complete* minimal surface?
In this talk, after recalling some basic facts in minimal surfaces theory, we will discuss some classical results in this line, some recent ones and some open problems.

Lipschitz estimates for every taste

Giuseppe Rosario Mingione
Università degli Studi di Parma, Italy

Lipschitz estimates play a crucial role in regularity problems. Several of the regularity problems arising in the theory of elliptic and parabolic operators can be faced only after knowing that solutions are Lipschitz continuous. In this talk I will present a survey of recent estimates for nonlinear elliptic problems, also considering the case of non-uniformly elliptic equations and functionals.

Representation Formulae and Liouville Theorems for second order elliptic inequalities

Enzo Mitidieri
Università degli Studi di Trieste, Italy

We discuss several representation formulae of solutions of general second order elliptic equations/inequalities. As a outcome we analyse the impact of these results with respect to the classical Liouville theorem. In particular we prove that the classical Liouville theorem is equivalent to the existence of a representation formula.

On existence and concentration ofsolutions to a class of quasilinear problems involving the 1-Laplace operator

Marcos Tadeu de Oliveira Pimenta
Universidade Estadual Paulista (UNESP), Brazil

In this work we use variational methods to prove results on existence and concentration of solutions to a problem in $\mathbb{R}^N$ involving the 1-Laplacian operator. A thorough analysis on the energy functional defined in the space of functions of bounded variation $BV(\mathbb{R}^N)$ is necessary, where the lack of compactness is overcome by using the Concentration-Compactness Principle of Lions.

On Green functions of second-order elliptic operators on Riemannian manifolds: the critical case

Yehuda Pinchover
Technion - Israel Institute of Technology, Israel

Let P be a second-order, linear, elliptic operator with real coefficients which is defined on a noncompact and connected Riemannian manifold M. It is well known that the equation $Pu = 0$ in M admits a positive supersolution which is not a solution if and only if P admits a unique positive minimal Green function on M, and in this case, P is said to be subcritical in M. If P does not admit a positive Green function but admits a global positive solution, then such a solution is called Agmon ground state of P in M, and P is said to be critical in M.

We prove for a critical operator P in M, the existence of a Green function which is dominated above by the ground state of P away from the singularity. Moreover, in a certain class of Green functions, such a Green function is unique, up to an addition of a product of the ground states of P and P^*. This result extends and sharpens the well known result of Peter Li and Luen-Fai Tam concerning the existence of a symmetric Green function for the Laplace–Beltrami operator on a smooth and complete Riemannian manifold M. This is a joint work with Debdip Ganguly.

L^p-theory for Schrödinger systems

Abdelaziz Rhandi
Università degli Studi di Salerno, Italy

In this talk we study for $p \in (1, \infty)$ the L^p-realization of the vector-valued Schrödinger operator $\mathcal{L}u := \mathrm{div}(Q\nabla u) + Vu$. Using a noncommutative version of the Dore–Venni theorem due to Monniaux and Prüss, and a perturbation theorem by Okazawa, we prove that L_p, the L^p-realization of $\mathcal{L}$, defined on the intersection of the natural domains of the differential and multiplication operators which form $\mathcal{L}$, generates a strongly continuous contraction semigroup on $L^p(\mathbb{R}^d; \mathbb{C}^m)$. We also study additional properties of the semigroup such as positivity, ultracontractivity, Gaussian estimates and compactness of the resolvent. We end the talk by giving several examples and counterexamples.

The talk is based on two joint works, the first one with Markus Kunze, Luca Lorenzi and Abdallah Maichine, and the second one in collaboration with Abdallah Maichine.

Gagliardo–Nirenberg inequality revisited

Tomas Roskovec
University of South Bohemia and Czech Technical University, Czech Republic

We study the classical proof by L. Nirenberg and show several weak points in this celebrated publication. We present the proof in an original way while excluding the invalid cases. In order to extend the scale from Lebesgue spaces to Hölder spaces, we also introduce new interpolation inequality. We also present some possibilities of extension to the finer scales of functional spaces.