

## TWO NONLINEAR DAYS IN URBINO 2019

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*Sala della Tartaruga  
Palazzo Passionei  
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### ABSTRACTS

*Controllability and positivity constraints in population dynamics with age structuring and degenerates diffusion*

**Idriss Boutaayamou**

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In this work, we deal with the controllability and positivity constraints of a population dynamics model with an interior degenerate diffusion. To this end, we show that the system can be steered between two positive steady states by controls preserving the positivity of the state trajectory. The proof follows the same strategy as the one used in the non-degenerate case.

*Pathological homeomorphisms and variational models of non-linear elasticity*

**Daniel Cameron Campbell**

University of Hradec Kralove, Czech Republic

We discuss recent results (examples) of pathological homeomorphisms and their relationship to models in non-linear elasticity. Especially we focus on the sign of the Jacobian and the sharp conditions for its stability.

*Existence of a standing wave for a ‘modified Schrödinger equation’*

**Anna Maria Candela**

Università degli Studi di Bari, Italy

Solutions of the quasilinear equation

$$(P) \quad -\operatorname{div}(A(x, u)|\nabla u|^{p-2}\nabla u) + \frac{1}{p} A_t(x, u)|\nabla u|^p + |u|^{p-2}u = g(x, u) \quad \text{in } \mathbb{R}^N,$$

are standing waves for a ‘modified Schrödinger equation’ if  $p = 2$  and  $N = 3$  which appears quite naturally in different fields of Mathematical Physics.

Taking  $G(x, t) = \int_0^t g(x, s)ds$ , suitable assumptions on  $A(x, t)$  and  $g(x, t)$  set off the variational structure of (P) and its related functional is

$$\mathcal{J}(u) = \frac{1}{p} \int_{\mathbb{R}^N} A(x, u)|\nabla u|^p dx + \frac{1}{p} \int_{\mathbb{R}^N} |u|^p dx - \int_{\mathbb{R}^N} G(x, u)dx,$$

which is  $C^1$  but not verifies the classical Palais–Smale condition on the Banach space  $X = W^{1,p}(\mathbb{R}^N) \cap L^\infty(\mathbb{R}^N)$  equipped with the intersection norm  $\|\cdot\|_X$ .

Following an approach which exploits the interaction between  $\|\cdot\|_X$  and the standard norm on  $W^{1,p}(\mathbb{R}^N)$ , introduced in joint papers with Giuliana Palmieri, and overcoming the lack of compactness by assuming that the problem has radial symmetry, we study the existence of critical points of  $\mathcal{J}$  restricted to  $X_r$ , subspace of the radial functions in  $X$ .

So, if  $p \geq 2$  and  $G(x, t)$  has a subcritical growth, we prove the existence of at least one weak bounded radial solution of  $(P)$  by applying a generalized version of the Ambrosetti–Rabinowitz Mountain Pass Theorem to  $\mathcal{J}$  on  $X_r$ .

This result is in a joint work with Addolorata Salvatore.

*On the Cohen-Macaulayness of Togliatti systems*

**Liena Colarte Gomez**

Universitat de Barcelona, Spain

The goal of this talk is to present some results on the arithmetic Cohen-Macaulayness (aCM) of some projective varieties arising from Togliatti systems. By means of invariant theory methods, we prove that the GT-variety associated to any GT-system generated by the monomials of degree  $d$  invariant under a linear diagonal action of the cyclic group of order  $d$ , is aCM.

*Stationary optimal harvesting strategies*

**Giuseppe Devillanova**

Politecnico di Bari, Italy

The talk deals with the stationary solution to the time dependent optimal harvesting problem addressed by Bressan, Coclite, etc. (see [1], [2]) in a multidimensional bounded, open, connected and smooth domain  $\Omega \subset \mathbb{R}^N$ , with  $N \geq 2$ , in a time interval  $[0, T]$

$$\begin{cases} \partial_t \varphi = \Delta \varphi - \varphi \mu + (h(x) - \varphi) \varphi & \text{in } ]0, T] \times \Omega \\ \partial_{\mathbf{n}} \varphi = 0 & \text{on } [0, T] \times \partial \Omega \\ \varphi(0, x) = \varphi_0(x) & \text{in } \Omega, \end{cases}$$

where the function  $h(x) \in L^\infty(\Omega)$  is the spatially heterogeneous environment capacity, the measure  $\mu$  is the strategy used while harvesting the amount of fish  $\varphi(x, t)$  in the point  $x$  at time  $t$ ,  $\mathbf{n} = \mathbf{n}(x)$  stands, for every point  $x \in \partial \Omega$ , for the unit outer normal vector to  $\partial \Omega$ , and  $\varphi_0(x)$  is the initial density of fish.

In particular, in [3] the authors deal with the leisure harvesting problem, i.e. with the problem of finding an harvesting strategy  $\mu^*$  which maximizes the functional

$$J_0(\mu) := \sup \int_{\Omega} u d\mu$$

(where the supremum is taken over the stationary solutions  $u$  of the problem) on a suitable subset of the set of smooth positive Radon measures  $\mu$ . They find that the maximization problem still makes sense and provide a *geometrical formulation* which actually is independent on the harvesting strategy. Furthermore, this new formulation admits a relaxed version and while, from one side, the relaxed problem always has a unique solution which, under suitable assumptions (expressed in terms of the environment capacity), is also a solution of the initial problem, on the other side, some examples of environment capacities are provided for which the corresponding original problem has no solution. Finally, on the space of smooth measures a notion of *convergence in sense of the strategies* is introduced and a comparison with the usual narrow convergence of measures is ruled out.

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*Atomic decomposition of predual  $B_*$  of the Bourgain-Brezis-Mironescu space and other big spaces*

**Luigi D’Onofrio**

Università ‘Parthenope’ di Napoli, Italy

A new function space  $B \subset L^1(Q_0)$ ,  $Q_0 = ]0, 1[^n$  was recently introduced by Bourgain-Brezis-Mironescu based on the seminorm

$$\|f\|_B = \sup_{0 < \varepsilon < 1} [f]_\varepsilon$$

and its subspace  $B_0 \subset B$  defined by the “little- $\circ$ ” condition

$$[f] = \limsup_{\varepsilon \rightarrow 0} [f]_\varepsilon = 0$$

where  $[f]_\varepsilon$  is defined with a suitable maximization procedure over families  $\mathcal{F}_\varepsilon$  of disjoint  $\varepsilon$ -cubes under the constraint  $\#\mathcal{F}_\varepsilon \leq \frac{1}{\varepsilon^{n-1}}$ .

Improving the constraint to  $\#\mathcal{F}_\varepsilon \leq 1$  the BMO-space is obtained, while, relaxing to  $\#\mathcal{F}_\varepsilon < \infty$  the BV-space is obtained.

In a forthcoming paper with L. Greco, K.M. Perfekt, C. Sbordone and R. Schiattarella, some new properties of the pair  $B_0, B$  are presented. In particular, we prove that, as in case of BMO, also predual of  $B$  or  $BV$  enjoy atomic decomposition. Moreover, the predual  $B_*$  of  $B$  is isometrically isomorphic to the dual of  $B_0$  and

$$\text{dist}_B(f, B_0) = [f].$$

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*On  $(p, q)$  problems with lack of compactness*

**Alessio Fiscella**

Universidade Estadual de Campinas, Brazil

The first part of the talk deals with systems driven by general  $(p, q)$  elliptic operators of Marcellini types, possibly involving critical Sobolev nonlinearities and Hardy potential terms. Our variational systems present some difficulties due to the double structure of the elliptic part combined with the lack of compactness at critical level. For this, in order to state existence results, we introduce different proof techniques based on concentration–compactness arguments and a suitable alternative of Lions type.

In the second part we discuss about degenerate Kirchhoff systems involving  $(p, q)$ -fractional operators, as well as critical Sobolev nonlinearities. Considering the bi-nonlocal framework of these systems, in order to state existence results we exploit a suitable tricky step analysis, successfully used for critical  $p$ -fractional problems. However, for critical  $(p, q)$ -fractional Kirchhoff systems the application of this strategy is fairly delicate because of the double structure of the functional norm. Indeed, we have to split the study on the behavior of the Palais–Smale sequences in different cases, mainly considering that  $1 < p < q < \infty$ .

The results presented in this talk are based on the papers [1, 2].

## REFERENCES

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- [2] A. FISCELLA AND P. PUCCI,  *$(p, q)$  systems with critical terms in  $\mathbb{R}^N$* , *Nonlinear Anal.* 177B (2018) 454–479.

*Asymptotic studies on growing tubular tissues***Yang Liu**

Tianjin University, China

Growth-induced wrinkling is commonly observed in biological tissues. In general, those functional patterns play a critical role in a biological system. In this talk, we study the growth-induced wrinkling for growing tubular tissues and aim to deduce some asymptotic expressions for the critical growth factor, critical mode number, and amplitude equation of a single wrinkling mode. The WKB method will be used to derive an explicit bifurcation condition, and the virtual work method is employed to deduce the amplitude equation. Based on the order analysis, we derived some asymptotic expression for the bifurcation threshold and the critical mode number, and a semi-analytical solution is obtained for the post-buckling mode. Then a parametric study is performed to real the effects of different geometrical and material parameters on wrinkling initiation and evolution. Finally, all analytical results are validated by the corresponding numerical solutions based on finite element method.

*Prescribed Gaussian and geodesic curvatures problem***Rafael Lopez Soriano**

University of Valencia, Spain

This talk is concerned with a Liouville type problem on compact surfaces with boundary. More precisely, this equation allows us to assign Gauss and geodesic curvatures under a conformal change of the metric. We derive existence using the variational structure of the problem and compactness of solutions analyzing the blow-up phenomenon. Joint works with A. Jevnikar (SNS Pisa), M. Medina (U. Granada), A. Malchiodi (SNS Pisa), D. Ruiz (Granada).

*On the action of topological groups on Sobolev spaces***Giovanni Molica Bisci**

Università degli Studi di Urbino Carlo Bo, Italy

In this talk we discuss the initial part of a joint research project with Patrizia Pucci devoted to the study of nonlinear critical problems on Carnot groups. In particular, critical equations are relevant for their relations with problems arising in differential geometry and in physics, where a lack of compactness occurs (see, for instance, [2, 3, 6, 7]). Motivated by this wide interest in the current literature, also in connection with the celebrated Brezis–Nirenberg problem and inspired by the recent papers [1] and [9], we present some existence results on unbounded domains of the Heisenberg group (see [4] and [5]). Certain open problems will be briefly presented.

## REFERENCES

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- [9] P. PUCCI, *Critical Schrödinger–Hardy systems in the Heisenberg group*, to appear in Discrete Contin. Dyn. Syst. Ser. S, Special Issue on the occasion of the 60th birthday of Vicentiu D. Rădulescu.

*On the Neumann fractional  $p$ -Laplacian*

**Dimitri Mugnai**

Università degli Studi della Tuscia, Italy

We develop some properties of the  $p$ -Neumann derivative for the fractional  $p$ -Laplacian in bounded domains. In particular, we prove the existence of a diverging sequence of eigenvalues and we introduce the evolution problem associated to such operators, studying the basic properties of solutions. Finally, we study some associated nonlinear problems with source.

*On the classification of monomial Togliatti systems*

**Marti Salat Molto**

Universitat de Barcelona, Spain

The study of Lefschetz properties has established multiple connections among different areas in mathematics such as commutative algebra, combinatorics or algebraic geometry. In particular it was recently discovered by Ottaviani, Mezzetti and Mir-Roig a link between ideals failing the weak Lefschetz property and varieties satisfying a Laplace Equation. These artinian ideals are called Togliatti systems. There have been many efforts in classifying these ideals and partial results have been achieved in the monomial case. Here, we address the problem on classifying monomial Togliatti systems focusing on the number of generators and give some results.

*Some results on the Nonlinear Schrödinger equation on metric graphs*

**Simone Secchi**

Università degli Studi di Milano ‘Bicocca’, Italy

I will present some results recently obtained in collaboration with Diego Noja and Sergio Rolando (Milano Bicocca) about solutions to a NLS equation on a metric graph.

*Pattern evolution in bending dielectric-elastic bilayers*

**Yipin Su**

National University of Ireland, Ireland

We propose theoretical and numerical analysis of smart bending deformation of dielectric-elastic bilayer in response to a voltage, based on the nonlinear theory of electro-elasticity and the associated linearized incremental field theory. We reveal that the mechanism allowing the bending angle of the bilayer can be tuned by adjusting the applied voltage. Furthermore, we investigate how much can the bilayer be bent before it loses its stability by buckling when one of its faces is under too much compression. We find that the physical properties of the two layers must be selected to be of the same order of magnitude to obtain a consequent bending without encountering buckling.