TWO NONLINEAR DAYS IN URBINO 2025

May 22-23, 2025

Palazzo Passionei-Paciotti
Sala della Tartaruga
Via Valerio, 9, Urbino

This international conference aims to bring together researchers working in the field of partial differential equations and applications from all over the world to present their recent advances and to discuss future perspectives.

The conference will take place in the city center of Urbino, a World Heritage Site notable for its historical legacy of Renaissance culture and known as the cradle of Mathematical Renaissance. Graduate students and young researchers are particularly encouraged to attend.

Abstracts

Regularity results and maximum principles for quasilinear operators of mixed local non-local type

Carlo Alberto Antonini
Università degli Studi di Parma, Italy

In this talk, we will deal with mixed local-nonlocal quasilinear operators, modeled upon the sum of a p-Laplacian and a fractional (s,q)-Laplace operator, i.e., $-\Delta_p u + (-\Delta_q)^s u$.

We will review some recent results concerning local and global regularity of their solutions, and we will address the validity of maximum principles and Hopf Lemma for such operators.

Based on a joint work with M. Cozzi.

References

[1] C.A. Antonini, M. Cozzi, *Global gradient regularity and a Hopf lemma for quasilinear operators of mixed local-nonlocal type*, Journal of Differential equations, Volume 425, Pages 342-382(2025)

On a class of fractional elliptic reaction-diffusion systems: Analysis and Numerics

Maha Daoud
École nationale supérieure de techniques avancées, France

In this talk, we consider a class of nonlocal elliptic reaction-diffusion systems where the diffusion terms are governed by fractional Laplacian operators and the reaction terms depend on the gradient of the unknowns. These systems arise in a variety of applications such as, among others, Fluid dynamics and Engineering. First, we present some existence and nonexistence results. Second, we give a numerical answer to a question which naturally arises and which is, to our knowledge, open even in the case of a single equation.

Lectures on mappings of finite distortion

Luigi D'Onofrio
Università di Napoli 'Parthenope', Italy

In this talk we introduce the class of finite distortion maps. Connections with nonlinear elasticity models are also discussed. We study continuity properties, the behavior of maps on sets of zero measure, topological properties, the regularity of inverse maps, and many other aspects.

Critical normalized equations and bad deals: one for the price of two

Marco Gallo
Università Cattolica del Sacro Cuore, Italy

In this talk we will discuss the existence of *normalized* solutions for a Schrödinger-type equation

$$-\Delta u + \omega u = g(u) \quad \text{in } \mathbb{R}^N$$

where the solution u is required to additionally satisfy the *constraint* $\int_{\mathbb{R}^N} u^2 = m$. Here the mass $m > 0$ is *prescribed*, while the frequency $\omega > 0$ is part of the unknowns.

The case of interest is $g(t)$ with *critical* (in some sense) growth, both for $t \sim 0$ and $t \sim \infty$. That is, $g(t) \sim t^{1+\frac{4}{N}}$, but g is not assumed to be a power. We will see how this problem is particularly challenging and how a "lack of space" in the range of suitable masses will oblige us to implement new ideas.

The talk is based on joint works with S. Cingolani, N. Ikoma and K. Tanaka.

Principles of Nonsmooth Analysis

Giovanni Molica Bisci
Università San Raffaele Roma, Italy

We consider a linearly perturbed elliptic problem governed by the spectral fractional Laplacian and with mixed Dirichlet-Neumann boundary data. The nonlinearity is considered in the subcritical regime. By using linking and variational theorems of mixed type (one of the so called ∇-theorems), we prove the existence of multiple solutions when the perturbation parameter is close to the eigenvalues of the operator.

This is a joint work with A. Ortega, and L. Vilasi, Bull. Math. Sci. 14 (2024).

Normalized Schrődinger equations in unbounded domains

Riccardo Molle
Università degli Studi di Roma Tor Vergata, Italy

This talk concerns some recent results about the existence of positive solutions with fixed L^2-norm for nonlinear Schrődinger equations. Mass supercritical and mass subcritical regimes are considered. We quickly describe a nonexistence result and existence results on the whole space, where the absence of the boundary provides a fundamental tool to work with. Then the main topic will be some recent and ongoing results when the domain is unbounded and with a boundary.

Joint works with Luigi Appolloni and Sergio Lancelotti.

A case for the logarithmic Laplacian

Luigi Pollastro
Università degli Studi di Torino, Italy

In the past few decades the fractional Laplacian $(-\Delta)^s$ has capitalized the attention of the PDE community as an integrodifferential operator which is the nonlocal counterpart to the classical Laplacian.

Gaining popularity thanks to the seminal work of L. Caffarelli & L. Silvestre *"An extension problem related to the fractional Laplacian"*, it sees a widespread use thanks to its ability to model phenomena in which long term interactions between objects occur, leading to applications in particle physics, finance and population dynamics among others.

An interesting line of research is to investigate what happens to the operator as the index s approaches 0. It is in this instance that the logarithmic Laplacian L_Δ pops up, as the first order term in the Taylor expansion of the fractional Laplacian as s goes to 0.

The goal of this seminar is to present this fairly new operator, showcasing its basic features together with some open questions. Lastly, I will present symmetry results for an overdetermined and a rigidity problem involving the logarithmic Laplacian recently obtained in collaboration with N. Soave.

Critical Neumann problem in cones: instability and symmetry breaking

Camilla Polvara
Sapienza Università di Roma, Italy

We consider the critical Neumann problem in cones. We prove that the standard bubbles, which are the only radial solutions, become unstable for a class of nonconvex cones, which is defined through the first Neumann eigenvalue of the Laplace Beltrami operator on the domain D on the unit sphere which spans the cone. This immediately implies a symmetry breaking result for the minimizers of the Sobolev inequality. Actually, a bifurcation result from the standard bubbles can be proved.

We also present a quantitative Sobolev inequality of Bianchi-Egnell type, which holds in any cone, even if the minimizers are not the standard bubbles.

These results are contained in joint works with G. Ciraolo, F. Pacella and L. Provenzano.

On fractional Hardy-type inequalities in general open sets

Francesca Prinari
Università di Pisa, Italy

In this talk, we provide, in the case $sp > N$, an optimal lower bound for the Hardy constant $\mathfrak{h}_{s,p}(\Omega)$ of an open set $\Omega \subsetneq \mathbb{R}^N$, in the Sobolev-Slobodeckiĭ space, by means of the sharp Hardy constant $\mathfrak{h}_{s,p}$ of the punctured space $\mathbb{R}^N \setminus \{0\}$. The proof of this result exploits the characterization of Hardy's inequality in the fractional setting in terms of positive local weak supersolutions of the relevant Euler-Lagrange equation and relies on the construction of suitable supersolutions by means of the distance function from the boundary of Ω. Moreover, we compute the limit of $\mathfrak{h}_{s,p}$ as $s \nearrow 1$, as well as the limit when $p \nearrow \infty$.

Based on a joint work in collaboration with E. Cinti.

Critical dimensions for polyharmonic operators: The Pucci-Serrin conjecture for solutions of bounded energy

Frédéric Robert
Université de Lorraine, France

We prove a Pucci-Serrin conjecture on critical dimensions under a uniform bound on the energy. The method is based on the analysis of the Green's function of polyharmonic operators with "almost" Hardy potential.

Multiple critical points with fixed energy

Gaetano Siciliano
Università degli Studi di Bari Aldo Moro, Italy

In the talk we present a recent result on existence of solutions to elliptic equations by means of variational methods. In particular the Ljusternick-Schnirelmann Theory is implemented in order to prove multiplicity and bifurcation results. These results are indeed particular cases of a more general abstract framework that we present for a family of functionals depending on a parameter.

Maximum Principle for Variational Elliptic PDEs of $2m$ Order

Antonio Tarsia
Università di Pisa, Italy

It is well known how the Maximum Principle (MP) in general fails to hold for uniformly elliptic operators of order higher than two, even in smooth convex domains. In *D. Cassani - A. Tarsia, Adv. Nonlinear Anal. 2022*, it was shown in dimension $N = 2, 3$, by establishing a new Harnack type inequality, that the validity of the positivity preserving property can be restored when lower order derivatives are taken into account as a perturbation of the higher order differential operator. The restriction to the dimension was due to regularity issues which we develop here, extending the validity of the MP to any dimension and fairly general domains. Moreover, we show that the presence of inertial terms affects the range of the perturbation parameter, providing a balance between the positivity restoring effect of lower order derivatives and the mass energy. The method provided here is flexible with respect to the form of differential operators involved and thus suitable. In particular, we will observe that these techniques allow to obtain a MP for nonlinear operators such as the p-Laplacian or its generalizations.

Not only, but the same techniques, applied to the equations for which a MP is obtained, allow us to establish the behavior of the solution when the data does not have a constant sign.

On the long time behavior of solutions to NLS

Nicola Visciglia
Università di Pisa, Italy

We first introduce the nonlinear scattering operator associated with the Nonlinear Schroedinger Equations in the Euclidean setting and we recall some classical results. Then we move to some recent results concerning a smoothing effect of this operator.

The talk is based on a joint work with N. Burq, H. Koch, and N. Tzvetkov.